Edge Intersection Graphs of Single Bend Paths in a Grid

Date Tuesday, February 15

Time 4 pm

Location 317 Mudd

Abstract:

Let $\mathcal{P}$ be a collection of nontrivial simple paths on a host graph H. The edge intersection graph of $\mathcal{P}$, denoted by $EP_H(\mathcal{P})$, has vertex set that corresponds to the members of $\mathcal{P}$, where two vertices are joined by an edge if and only if the corresponding members of $\mathcal{P}$ share at least one common edge in H. An undirected graph Γ is called an edge intersection graph of paths in a tree (EPT) if $\Gamma = EP_T(\mathcal{P})$ for some $\mathcal{P}$ and tree T. Similarly, Γ is called an edge intersection graph of paths in a grid (EPG) if $\Gamma = EP_G(\mathcal{P})$ for some $\mathcal{P}$ and grid G. The EPT and EPG graphs can be useful in network and circuit applications, where scheduling and layout problems are often equivalent to coloring an EPT or EPG graph.

In this lecture, we will survey the mathematical and algorithmic results on various types of EPT and EPG graphs and some of their generalizations, together with several restrictions on the representations. The class of EPT graphs was first investigated by Golumbic and Jamison in two papers appearing in 1985, and subsequently, further research has been carried out by a number of algorithmic graph theorists. In a series of papers during the past 5 years, Golumbic, Lipshteyn and Stern studied the hierarchy of related EPT graph classes giving some structure theorems.

ery recently, Golumbic, Lipshteyn and Stern introduced EPG graphs, proving that every graph is an EPG graph, and then turning their attention to the subclass of graphs that admit an EPG representation in which every path has at most a single bend, called B_1-EPG graphs. They proved that any tree is a B_1-EPG graph and gave a structural property that enables generating non B_1-EPG graphs. A characterization of the representation of cliques and chordless 4-cycles in B_1-EPG graphs was given, and also prove that single bend paths on a grid have Strong Helly number 3 when the paths satisfy the usual Helly property, and otherwise have Strong Helly number 4.

We will also survey recent results by our colleagues Andrei Asinowski, Andrew Suk and Bernard Ries on edge intersection graphs of systems of

paths on a grid with a bounded number of bends and some further research by a team in Germany. We conclude with some open problems and future work.