Polynomial Bounds for the Grid-Minor Theorem

Date Friday, February 7

Time 3 pm

Location 317 Mudd

Abstract:
One of the key results in Robertson and Seymour’s seminal work on graph minors is the Grid-Minor Theorem (also called the Excluded Grid Theorem). The theorem states that for every fixed-size grid $H$, every graph whose treewidth is large enough, contains $H$ as a minor. This theorem has found many applications in graph theory and algorithms.

Let $f(k)$ denote the largest value, such that every graph of treewidth $k$ contains a grid minor of size $f(k)$. The best current quantitative bound, due to recent work of Kawarabayashi and Kobayashi, and Leaf and Seymour, shows that:

$$f(k) = \Omega\left(\sqrt{\frac{\log k}{\log \log k}}\right).$$

In contrast, the best known upper bound implies that:

$$f(k) = O\left(\sqrt{\frac{k}{\log k}}\right).$$

In this talk we present the first polynomial relationship between treewidth and grid-minor size, by showing that:

$$f(k) = \Omega(k^d)$$

for some fixed constant $d > 0$.

Joint work with Chandra Chekuri.