Vertex decompositions of simplicial complexes from algebraic geometry

Date: November, 27

Time: 4pm

Location: 622 Math

Abstract: I'll first explain that semistandard Young tableaux, classically studied in algebraic combinatorics as a naked set, is naturally the set of facets of a simplicial complex, with Buch's "set-valued" tableaux as the lower-dimensional faces gluing things together. The combinatorial evidence that this is a natural thing to do is that the resulting complex is homeomorphic to a ball.

The cheapest way to prove a complex is homeomorphic to a ball is to show it is "vertex-decomposable". I'll recall this simple, inductive notion and explain the corresponding algebraic geometry (studied by Hodge if not before). In particular, from a suitable family of algebraic varieties related by nice degenerations, one is led naturally to vertex-decomposable balls. The Young tableaux example comes from "vexillary matrix Schubert varieties".

Then I'll describe two more such families: the simplicial ball of pipe dreams (from arbitrary matrix Schubert varieties), and subword complexes (from patches on general Schubert varieties). If time permits I will state a general criterion with which to demonstrate the niceness of the degenerations.

I won't assume any algebraic geometry knowledge. Much of this is joint with Ezra Miller and Alex Yong.