The structure of graphs with no cycles of length divisible by three

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Abstract: For a graph G, let $\#E(G)$ be the number of stable sets of even size and let $\#O(G)$ be the number of stable sets of odd size. We are interested in the relation between the structure of a graph and the value of $|\#E(G) - \#O(G)|$. As it turns out, $|\#E(G) - \#O(G)|$ has small value in many natural situations, and the simplest examples where $|\#E(G) - \#O(G)|$ is larger than 1 are complete graphs, and cycles of length divisible by 3. Kalai and Meshulam made a conjecture that if $|\#E(H) - \#O(H)|$ is bounded for all induced subgraphs H of G, then the chromatic number of G is bounded; they also conjectured that if G has no induced cycle of length divisible by 3, then G has bounded chromatic number. These two conjectures lead us to study the following question: is it true that if a graph G has no induced cycle of length divisible by 3, then $|\#E(G) - \#O(G)|$ is at most one? In this talk we discuss a special case of this problem, and give a complete structural description of graphs with no cycle of length divisible by 3 (not necessarily induced). The proof of the conjecture in this special case follows directly from the results of the decomposition. Joint work with Maria Chudnovsky, Alex Scott and Paul Seymour.