

Are Maximal Non-Hamiltonian Graphs Spanned by Thetas?

Date Tuesday, April 22

Time 3 pm

Location 303 Mudd

Abstract: One way to study Hamiltonicity of graphs is to consider (edge-)maximal non-Hamiltonian graphs. For instance, Ore's sufficient condition for a graph to be Hamiltonian can be rephrased as saying that in any (maximal) non-Hamiltonian graph of order n , there are two nonadjacent vertices whose degrees sum to less than n . In fact, the Bondy–Chvatal Theorem says that this property holds for every pair of nonadjacent vertices.

It is easy to show that every maximal non-Hamiltonian graph of order at least 3 is spanned by a figure-eight graph (the union of two cycles sharing a point, where we allow each cycle to degenerate to an edge). I conjecture that every 2-connected maximal non-Hamiltonian graph is spanned by a theta graph (the union of three internally disjoint paths between two vertices) and have shown that this holds for all graphs of order up to 20. I'll sketch this, proving a number of properties of a potential counterexample along the way.