Almost independent sets in the Kneser graph and in graph powers are nearly independent

Date Tuesday, April 30

Time 3 pm

Location 303 Mudd

Abstract:

Let $G_n = (V, E)$ be the Kneser graph $K(n, cn)$ for some $c > 0$. We show that for any $\epsilon > 0$ there exists a $\delta = \delta(\epsilon) > 0$ so that any set of vertices in G_n that spans less than $\delta|E|$ edges can be turned into an independent set by removing at most $\epsilon|V|$ vertices. A similar result holds for (weak) graph powers.

As an immediate corollary, we resolve the main open question in past work of the authors with Dinur [GAFA 2008]. We show that given any independent set in G_n and $\epsilon > 0$, we can remove at most $\epsilon|V|$ of its vertices, so that the result is contained in a junta on some $k = k(\epsilon)$ coordinates which is itself an independent set. Previously, it was known how to show containment in a junta that is *almost* an independent set, i.e., contains few edges.

The proof is based on ideas from Fox's recent triangle removal lemma (as well as the followup work by Hatami, Sachdeva, and Tulsiani).

Based on work in progress with Ehud Friedgut.