Tutte's Three-Edge-Coloring Conjecture

Date Tuesday, September 13

Time 4:30 pm

Location 303 Mudd

Abstract: The four-colour theorem is equivalent to the statement that every planar cubic graph with no cut-edge is 3-edge-colourable. What about non-planar cubic graphs? The Petersen graph is not 3-edge-colourable, and in 1966 Tutte conjectured that every cubic graph with no cut-edge that does not contain the Petersen graph as a minor is 3-edge-colourable.

In 1996 we proved this conjecture, but did not publish the result, for reasons that escape me. We are currently getting it all back together for publication; and this talk is an outline of our methods.

A graph is "apex" if deleting some vertex makes it planar; and "doublecross" if it can be drawn in the plane with crossings, but with only two crossings and both incident with the same region. Apex and doublecross cubic graphs do not have Petersen minors.

Our proof falls into three parts:

1. Proving that any minimal counterexample to Tutte's conjecture is "theta-connected" (vaguely, a cubic graph is theta-connected if every small edge-cut has a small number of vertices on one side); this part we did publish in 1996.

2. Proving that every theta-connected cubic graph with no Petersen minor is either apex or doublecross, except for one twenty-vertex graph; this is the main part of the lecture.

3. Proving that apex and doublecross graphs with no cut-edges are three-edge-colourable; this requires modifying the computer proof of the four-colour theorem.

This is joint work with Neil Robertson and Robin Thomas.