Graph ideals and the Gyárfás-Sumner conjecture

Date Tuesday, September 11

Time 3:30 pm

Location 303 Mudd

Abstract: An IDEAL of graphs means a class of graphs closed under a containment relation. One attractive feature of minor containment for graphs is that not only do we get excluded minor theorems, we get excluded minor-ideal theorems; for instance, the minor-ideals that do not include the minor-ideal of all planar graphs are precisely those that have a bound on the tree-width of their members. There are many other excluded minor-ideal theorems.

But if we use induced subgraph containment instead, there were no excluded is-ideal theorems known except Ramsey's theorem, until a few months ago. ("is-ideal" means induced subgraph ideal.) In joint work with Chudnovsky, Norin and Reed, we found the first non-trivial excluded is-ideal theorem. Then, we found a second, quite different; it describes the structure of the is-ideals not including the is-ideal of all disjoint unions of cliques and the is-ideal of all complete multipartite graphs. This we discovered by accident; it grew out of work on extending the Gyarfas-Sumner conjecture, as follows.

Some tournaments H are "heroes"; they have the property that all tournaments not containing H as a subtournament have bounded chromatic number (colouring a tournament means partitioning its vertex-set into transitive subsets). In joint work with eight authors, we found all heroes explicitly. That was great fun, and it would be nice to find an analogue for graphs instead of tournaments.

Sadly, the problem is too trivial for graphs, if we only exclude one graph H; but it becomes fun again if we exclude a finite set of graphs. The Gyarfas-Sumner conjecture says that if we exclude a forest and a clique then chromatic number is bounded. So what other combinations of excluded subgraph will give bounded chromatic (or cochromatic) number? Joint work with Maria Chudnovsky.