Recent progress in distinct distances probelms

Date Tuesday, November 19

Time 3 pm

Location 303 Mudd

Abstract: During 2013, significant progress has been obtained for several problems that are related to the Erdos distinct distances problem. In this talk I plan to briefly describe some of these results and the tools that they rely on. I will focus on the following two results.

Let P and P' be two sets of points in the plane, so that P is contained in a line L, P' is contained in a line L', and L and L' are neither parallel nor orthogonal. Then the number of distinct distances determined by the pairs of $P \times P'$ is $\Omega(\min |P|^{2/3}|P'|^{2/3}, |P|^2, |P'|^2)$. In particular, if $|P| = |P'| = m$, then the number of these distinct distances is $\Omega(m^{4/3})$, improving upon the previous bound $\Omega(m^{5/4})$ of Elekes.

In the seoncd result, we study the structure of planar point sets that determine a small number of distinct distances. Specifically, we show that if a set P of n points determines $o(n)$ distinct distances, then no line contains $\Omega(n^{7/8})$ points of P and no circle contains $\Omega(n^{5/6})$ points of P.

In both cases, we rely on a bipartite and partial variant of the Elekes-Sharir framework, which has been used by Guth and Katz in their 2010 solution of the general distinct distances problem. We combine this framework with some basic algebraic geometry, with a theorem from additive combinatorics by Elekes, Nathanson, and Ruzsa, and with a recent incidence bound for plane algebraic curves by Wang, Yang, and Zhang.

The first result is a joint work Micha Sharir (Tel Aviv) and Jzsef Solymosi (UBC). The second is a joint work with Joshua Zahl (MIT) and Frank de Zeeuw (EPFL).