On Erdos conjecture on square-free sets

Date Tuesday, December 2

Time 4 pm (NOTE THE UNUSUAL TIME!!!)

Location Math 622 (NOTE THE UNUSUAL PLACE!!!)

Abstract: In 1986, Erdos raised the following question: A set A of integers is square-free if no subset of A sums up to a square. What is the largest cardinality of a square-free set contained in $\{1, ..., n\}$?

Let $SF(n)$ be the maximum cardinality in question. One can easily show that $SF(n) > n^{1/3}$ by considering the set $A := \{q, 2q, ..., kq\}$, where q is a prime $\sim n^{2/3}$ and $k \sim n^{1/3}$. Since $q > 1 + 2 + ... + k$, A is clearly square-free. Erdos conjecture that $SF(n)$ is close to this lower bound.

This problem has been considered by many researchers and there is a series of upper bounds on $SF(n)$: $o(n/logn)$ (Alon 1987), $n^{3/4+o(1)}$ (Lipkin 1988), $n^{2/3+o(1)}$ (Alon-Freiman 1988), $n^{1/2+o(1)}$ (Sarkozy 1994).

In this talk, we present the asymptotically sharp bound $n^{1/3+o(1)}$. The proof uses tools from additive combinatorics (structural/rigidity theorems) and analytic number theory.

The talk is based on the recent paper "Squares in sumsets" by H. Nguyen and V. Vu.