On reducing MAP Inference in Markov Random Fields to Max Weight Stable Set on Perfect Graphs

Date Tuesday, December 4

Time 3:00 pm

Location 303 Mudd

Abstract: Markov random fields (MRFs) provide a flexible framework to describe a wide range of problems, and have been used extensively in fields including speech recognition, computer vision and error-correcting codes. A discrete MRF is specified by a set of variables $X = \{X_1, \ldots, X_n\}$ with an associated probability distribution, p(x) := P(X = x) where $x = (x_1, \ldots, x_n)$, which factorizes as

$$p(x) = \frac{1}{Z} \prod_{c \in C} \psi_c(x_c).$$

C is a collection of hyperedges x_c of the variables in X, and Z is a normalizing constant called the partition function that ensures probabilities sum to 1.

Here we are interested in *Maximum a Posteriori* (MAP) inference, which is the discrete optimization task of solving for a most probable configuration, i.e. an assignment $x = x^*$ such that p(x) is maximized. This is NP-hard in general, leading to much interest in identifying cases where efficient exact solutions or good approximations may be obtained.

The MAP inference task is equivalent to finding

$$x^* = \arg\max_x \sum_{c \in C} \log \psi_c(x_c).$$

Problems of this form can be reduced to finding a maximum weight stable set (MWSS) in a weighted graph we construct, called a nand Markov random field (NMRF), based on the original model.

In this talk, we explain this approach and discuss when it leads to efficient solutions of the original MAP problem. Typically this is when the resulting NMRF is a perfect graph. We shall pay particular attention to binary pairwise models, i.e. cases where all X_i variables are binary and all hyperedges have cardinality at most 2.

Joint work with Tony Jebara.