Choice Set Effects

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Context Effects

- We will now think about context effects that are more general than simple reference dependence
- Standard Model: Adding options to a choice set can only affect choice in a very specific way
  - Either a new option is chosen or it isn't
  - Independence of Irrelevant Alternatives
- Work from economics, neuroscience and psychology suggest a different channel
  - Changing the choice set changes the *context* of choice
  - Context affects preferences
  - Can lead to violations of IIA

Observing Context Effects

- We are going to consider two examples

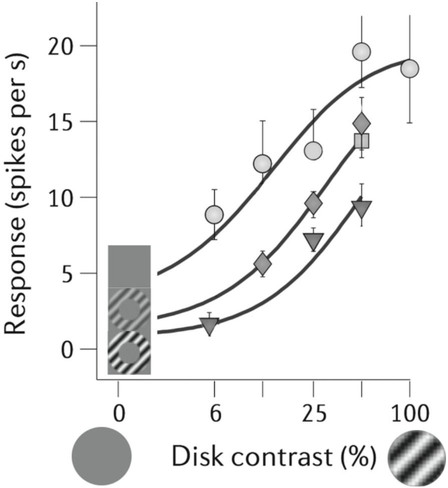
1. Stochastic Choice
   - Divisive Normalization: Louie, Khaw and Glimcher [2013]
2. Choice between multidimensional alternatives
   - Relative Thinking: Bushong, Rabin and Schwartzstein [2021]
   - Focussing: Koszegi and Szeidl [2013]
   - Salience: Bordalo, Gennaioli and Shleifer [2012]

- Then discuss an experiment designed to test between the last three models
  - Somerville [2022]

- The brain needs some way of representing (or encoding) stimuli
  - Brightness of visual stimuli
  - Loudness of auditory stimuli
  - Temperature etc.
- Typically, a given brain region will have the task of encoding a particular stimuli at a particular point in space and time
  - e.g. the brightness of a light at a particular point in the visual field
- How is this encoding done?
- A 'naive' mode: neural activity encodes the absolute value of the stimuli

$$\mu_i = KV_i$$

  - μ_i: neural activity in a particular region
  - V_i: The value of the related stimuli



- Encoding depends not only on the value of the stimuli, but also on the context [Carandini 2004]

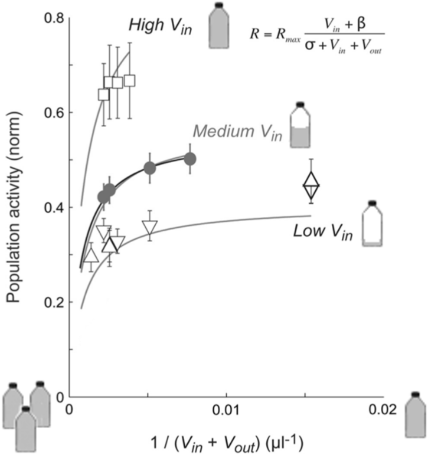
- Divisive Normalization:

$$\mu_i = K \frac{V_i}{\sigma_H + \sum_j w_j V_j}$$

  - σ_H: Normalizing constant (semi-saturation)
  - w_j: Weight of comparison stimuli j
  - V_j: Value of comparison stimuli j

- Why would the brain do this?
  - Efficient use of neural resources [Carandini and Heager 2011]
  - Neurons can only fire over a finite range
  - Want the same system to work (for example) in very bright and very dark conditions
  - Absolute value encoding is inefficient
    - In dark environments, everything encoded at the bottom of the scale
    - In light environments, everything encoded at the top of the scale
  - Normalization encodes relative to the mean of the available options
  - Encodes things near the middle of the scale.
- Allows stimuli to be encoded further apart from each other
  - Reduces errors that occur due to noise

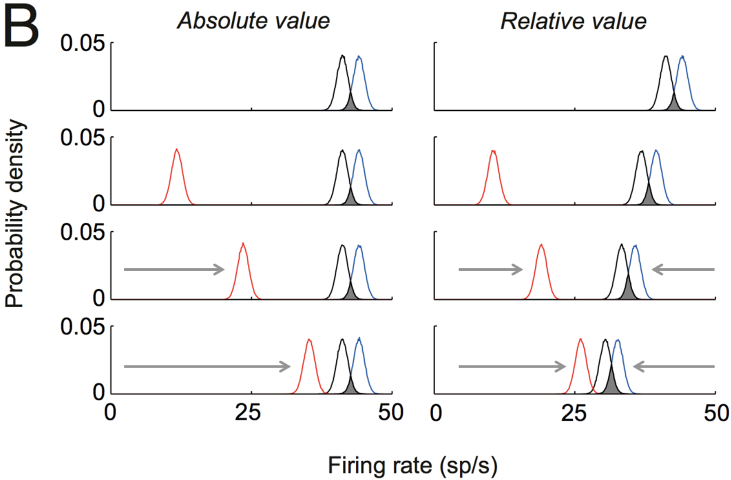
Divisive Normalization and Choice

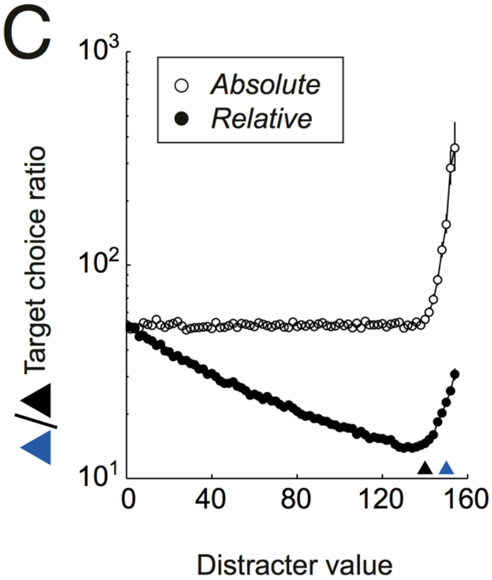


- There is also evidence that the value of choice alternatives is normalized [Louie et. al. 2011]

Divisive Normalization and Choice

- Why should normalization matter for choice?
- Does not change the ordering of the valuation of alternatives, so why should it change choice?
- Because choice is *stochastic*
- The above describes *mean* firing rates
- Choice will be determined by a draw from a random distribution around that mean
  - Claim that such stochasticity is an irreducible fact of neurological systems
- Probability of choice depends on the difference between the encoded value of each option
- Utility has a cardinal interpretation, not just an ordinal one


B
Absolute value
Relative value
Probability density
0.05
0
0
25
50
Firing rate (sp/s)


C
Absolute
Relative
Target choice ratio
10^3
10^2
10^1
0
40
80
120
160
Distracter value

- How do these predictions vary from standard random utility model?
- Luce model:

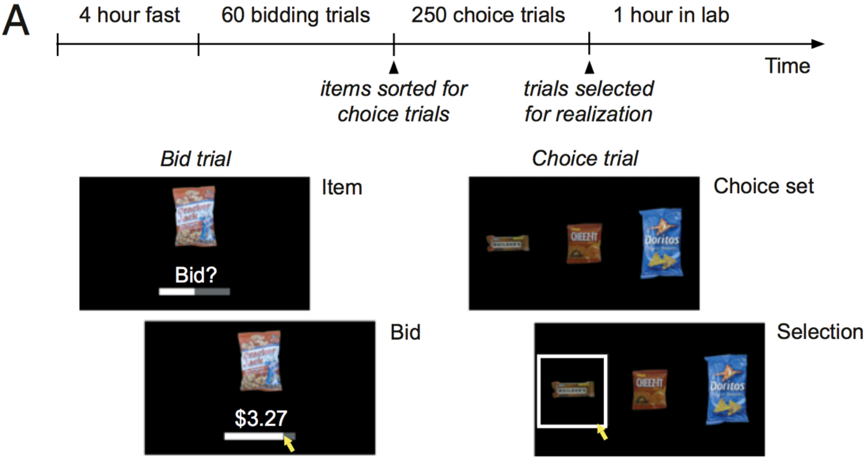
$$p(a|A) = \frac{u(a)}{\sum_{b \in A} u(b)}$$

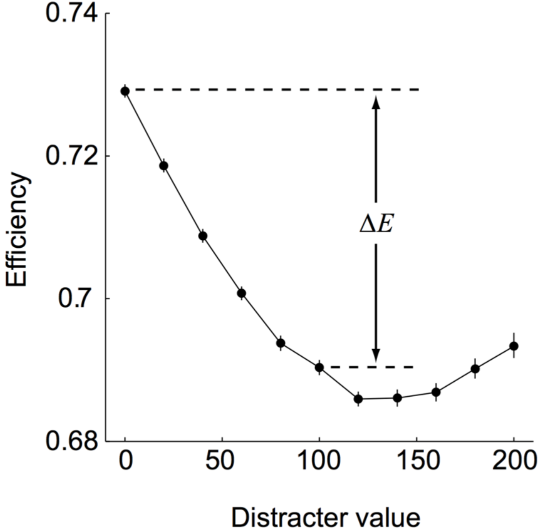
  - Implies that the relative likelihood of picking a over b is independent of the other available alternatives
  - Stochastic IIA
- More general RUM
  - Adding an alternative c can affect the relative likelihood of choosing a and b
  - But only because c itself is chosen
  - Can 'take away' probability from a or b
  - The amount c is chosen bounds the effect it can affect the choice of a or b

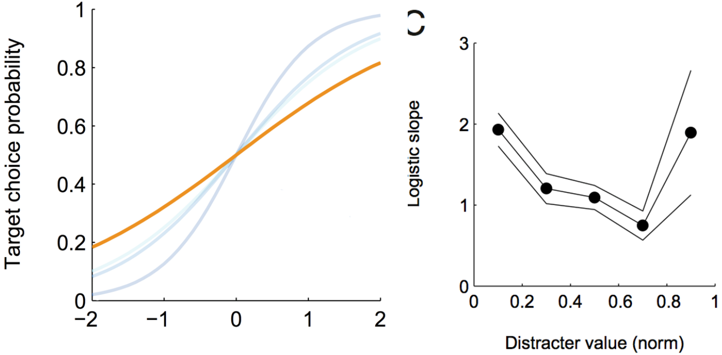
Experimental Evidence

- Subjects (40) took part in two tasks involving snack foods

1. Asked to *bid* on each of 30 different snack foods to elicit valuation
    - BDM procedure used to make things incentive compatible
2. Asked to make a choice from three alternatives
    - Target, alternative and distractor
    - 'True' value of each alternative assumed to be derived from the bidding stage


A
4 hour fast
60 bidding trials
250 choice trials
1 hour in lab
Time
items sorted for choice trials
trials selected for realization
Bid trial
Item
Bid?
Bid
$3.27
Choice trial
Choice set
Selection


0.74
0.72
0.7
0.68
Efficiency
ΔE
0
50
100
150
200
Distracter value


Target choice probability
1
0.8
0.6
0.4
0.2
0
−2
−1
0
1
2
Logistic slope
3
2
1
0
0
0.2
0.4
0.6
0.8
1
Distracter value (norm)

- But see "A neural mechanism underlying failure of optimal choice with multiple alternatives" by Chau et al. Nat Neurosci. 2014 Mar; 17(3): 463–470.

Choice with Multidimensional Alternatives

- In the model we just saw, adding a third ‘distractor’ changed the ‘distance’ between the value of two targets
  - Context changed apparent magnitude of the difference
- This could not be seen in 'standard' choice data
  - Is observable in stochastic choice

Choice with Multidimensional Alternatives

- Another data set in which such effects could be observed is choice over goods defined over multiple attributes
  - $c = \{c_1, ..., c_K\}$
- Utility is assumed additive,

$$U(c|A) = \sum_{k=1}^{K} w_k^A u_k(c_k)$$

  - $u_k(.)$ the true (context independent) utility on dimension k
  - w_k^A is a context dependent weight on dimension k
- Utility also assumed to be observable
  - Koszegi and Szeidl [2013] suggest how this can be done
- Context can change the distance between values on one dimension
  - Change the trade off relative to other dimensions

Choice with Multidimensional Alternatives

- Many recent papers make use of this framework
  - Bordalo, Gennaioli and Shleifer [2012, 2013]: Salience
  - Soltani, De Martino and Camerer [2012]: Range Normalization
  - Cunningham [2013]: Comparisons
  - Koszegi and Szeidl [2013]: Focussing
  - Bushong, Rabin and Schwartzstein [2021]: Relative thinking
  - Landry and Webb [2021]: Pairwise Divisive Normalization

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  - Bushong, Rabin and Schwartzstein [2015]: Relative thinking
  - Landry and Webb [2018]: Pairwise Divisive Normalization
- We will consider these three

Relative Thinking

- In the Louie et al. [2013] paper, normalization was relative to the *mean* of the value of the available options
- There is also a long psychology literature which suggests that *range* can play an important role in normalization
- A given absolute difference will seem *smaller* if the total range under consideration seems *larger*
- Bushong et al. [2021] suggest conditions on the weights w_k^A to capture this effect .

$$U(c|A) = \sum_{k=1}^{K} w_k^A u_k(c_k)$$

Relative Thinking: Assumptions

1. $w_k^A = w(\Delta_k(A))$ where

$$\Delta_k(A) = \max_{a \in A} u_k(a_k) - \min_{a \in A} u_k(a_k)$$

   - The weight given to dimension k depends on the range of values in this dimension

2. $w_k^A(\Delta)$ is diffable and decreasing in Δ
   - A given absolute difference receives less weight as the range increases

3. $w_k^A(\Delta)\Delta$ is strictly increasing, with $w(0)0 = 0$
   - The change in weight cannot fully offset a change in absolute difference

4. $\lim_{\Delta \to \infty} w(\Delta) > 0$
   - Absolute differences still matter even as the range goes to infinity

- An example of such a function

$$w_k^A(\Delta) = (1-\rho) + \rho \frac{1}{\Delta^\alpha}$$

- Bushong et al. [2021] do not fully characterize the behavioral implications of their model
  - Potentially interesting avenue for future research
- However, some of the implications are made clear in the following examples

Example 1

$$c = \left\{ \begin{array}{c} 2 \\ 3 \\ 0 \end{array} \right\}, \; c' = \left\{ \begin{array}{c} 0 \\ 0 \\ 5 \end{array} \right\}$$

- Assume these payoffs are in utility units
- What will the DM choose?
- They would choose c, despite the fact that the 'unweighted' utility of the two options is the same

$$2w(2) + 3w(3) > 5w(3) > 5w(5)$$

- DM favors benefits spread over a large number of dimensions

Example 2

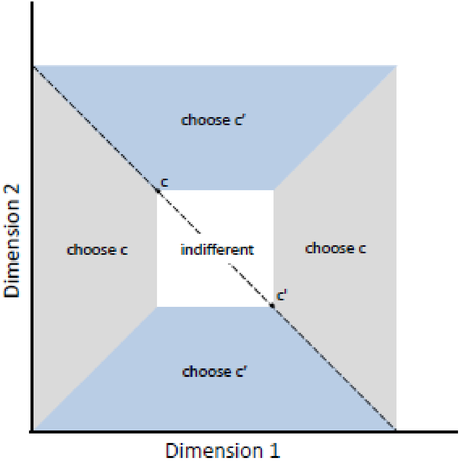
$$c = \left\{ \begin{array}{c} 2 \\ 1 \end{array} \right\}, c' = \left\{ \begin{array}{c} 1 \\ 2 \end{array} \right\}$$

- Say that, in the choice set $\{c, c'\}$ the DM is indifferent between the two.
- What would they choose from

$$c = \left\{ \begin{array}{c} 2 \\ 1 \end{array} \right\}, c' = \left\{ \begin{array}{c} 1 \\ 2 \end{array} \right\}, c'' = \left\{ \begin{array}{c} 2 \\ 0 \end{array} \right\}$$

- They would choose c
- The introduction of c'' increases the range of dimension 2, but not dimension 1
- Reduces the weight on the dimension in which c' has the advantage
- This is an example of the asymmetric dominance effect

Example 2



choose c'
c
choose c
indifferent
choose c
c'
choose c'
Dimension 2
Dimension 1

Focussing

- Somewhat confusingly, Koszegi and Szeidl consider a model which
  - Has the same set up
  - Makes opposite assumption about W
- Specifically, $w_k^A(\Delta)$ is **incresing** in Δ
- Dimensions with higher range get a higher weight
- This is a model of *focussing*

Focussing

- What is the justification for this?
- Basically some sort of 'editing' procedure
- You are trying to simplify choice by focussing your attention on important dimensions
- Similar in spirit to
  - Gabaix [2014]: Sparsity
  - Bordallo et al [2012]: Salience
  - Rubinstein [1988]: Similarity
- Unsurprisingly, Implications are the opposite of the relative thinking model
- Model biases choices towards options that focus their benefits in a small number of dimensions

Theorem

For any $F > f > 0$ and suppose that for some $c \in C$

1. *The advantages of c over any alternative in C are all greater than F*
2. *The disadvantages of c over any alternative in C is less than f*
   Then the DM will not choose any $c' \in C \backslash c$ such that

$$U(c') < U(c) + F\left[\frac{W(F)}{W(f)} - 1\right]$$

where $U(.)$ is the unweighted utility

- If the advantages of c are focussed in a small number of attributes it wiill be chosen

Salience Theory

- Basic Idea: Attention is not spread evenly across the environment
- Some things draw our attention whether we like it or not
  - Bright lights
  - Loud noises
  - Funky dancing
- The things that draw our attention are likely to have more weight in our final decision
- Notice here that attention allocation is *exogenous* not *endogenous*
  - Potentially could be thought of as a reduced form for some endogenous information gathering strategy

- Bordalo, Gennaioli and Shleifer [2013] formulate salience in the following way

$$\begin{aligned} U(c|A) &= \sum_{k=1}^{K} w_{k,c}^{A} u_k(c_k) \\ &= \sum_{k=1}^{K} w_{k,c}^{A} \theta_k c_k \end{aligned}$$

  - θ_k is the 'true' utility of dimension k
  - $w_{k,c}^{A}$ is the 'salience' weight of dimension k for alternative c
- Notice that the weight that dimension k receives may be different for different alternatives

Determining Salience

- How are the weights determined?
- First define a 'Salience Function'

$$\sigma(c_k, \bar{c}_k)$$

  - $\bar{c}_k$ is the reference value for dimension k (usually, but not always, the mean value of dimension k across all alternatives)
  - $\sigma(c_k, \bar{c}_k)$ is the salience of alternative c on dimension k

- Properties of the Salience function
  1. Ordering: $[\min(c_k, \bar{c}_k), \max(c_k, \bar{c}_k)] \supset [\min(c'_k, \bar{c}'_k), \max(c'_k, \bar{c}'_k)] \Rightarrow \sigma(c'_k, \bar{c}'_k) \leq \sigma(c_k, \bar{c}_k)$
  2. Diminishing Sensitivity: $\sigma(c_k + \varepsilon, \bar{c}_k + \varepsilon) < \sigma(c_k, \bar{c}_k)$
  3. Reflection: $\sigma(c'_k, \bar{c}'_k) > \sigma(c_k, \bar{c}_k) \Rightarrow \sigma(-c'_k, -\bar{c}'_k) > \sigma(-c_k, -\bar{c}_k)$

- An example of a salience function

$$\sigma(c_k, \bar{c}_k) = \frac{|c_k - \bar{c}_k|}{|c_k| + |\bar{c}_k|}$$

- Note:
  - Shares some features with both the previous approaches we have seen
  - Normalization by the mean
  - Diminishing sensitivity (but relative to zero, rather than the range)

From Salience to Decision Weights

- Use $\sigma(c_k, \bar{c}_k)$ to rank the salience of different dimensions for good c
    - $r_{k,c}$ is the salience rank of dimension k (1 is most salient)
- Assign weight $w^A_{k,c}$ as
$$\frac{\delta^{r_{k,c}}}{\sum_j \theta_j \delta^{r_{j,c}}}$$
- Then plug into
$$\sum_{k=1}^{K} w^A_{k,c} \theta_k c_k$$
- More salient dimensions get a higher decision weight
- δ indexes degree to which subject is affected by salience
    - lower δ, more affected by salience
- Note salience of a dimension is good specific

Application: Choice Under Risk

- Bordalo et al [2012] apply the salience model to choice under risk
- Choice objects are lotteries on a subjective state space
- Dimensions are states of the world
  - c_k is the utility provided by lottery c in state of the world k
  - θ_k is the objective probability of state of the world k
- Someone who does not have salience effects maximizes expected utility
- Salience leads to probability weighting
- Note: in binary choices, assume that each alternative has the same salience for each state
- e.g.

$$\sigma(c_k, c'_k) = \frac{|c_k - c'_k|}{|c_k| + |c'_k| + \lambda}$$

- Example: Salience and the Allais Paradox
- Allais Paradox: Consider the following pairs of choices:

$$
\begin{aligned}
c &= (0.33 : 2500; 0.01 : 0; 0.66 : 2400) \\
\text{or } c' &= (0.34 : 2400; 0.66 : 2400)
\end{aligned}
$$

$$
\begin{aligned}
\bar{c} &= (0.33 : 2500; 0.01 : 0; 0.66 : 0) \\
\text{or } \bar{c}' &= (0.34 : 2400; 0.66 : 0)
\end{aligned}
$$

- Typical choice is c' over c but $\bar{c}$ over $\bar{c}'$
- Inconsistent with expected utility theory
- Can be explained by salience

- Consider choice 1

$$\begin{aligned} c &= (0.33 : 2500; 0.01 : 0; 0.66 : 2400) \\ \text{or } c' &= (0.34 : 2400; 0.66 : 2400) \end{aligned}$$

- Represent by the following state space:

State	c	c'
s_1	2500	2400
s_2	0	2400
s_3	2400	2400

- State s_2 is the most salient state, receives most weight
- c' chosen if

$$\delta 0.33 \times 100 < 0.01 \times 2400$$

- More susceptible to salience, the more likely to choose c'

- Consider choice 2

$$\begin{aligned} \bar{c} &= (0.33 : 2500; 0.01 : 0; 0.66 : 0) \\ \text{or } \bar{c}' &= (0.34 : 2400; 0.66 : 0) \end{aligned}$$

- *Assume independence* and represent by the following state space:

State	$\bar{c}$	$\bar{c}'$
s_1	2500	2400
s_2	2500	0
s_3	0	2400
s_4	0	0

- Salience ranking is s_2, then s_3, then s_1
- Now the upside of $\bar{c}$ is most salient
- $\bar{c}'$ chosen if

$$0.33 \times 0.66 \times 2500 - \delta 0.67 \times 0.34 \times 2400 + \delta^2 0.33 \times 0.34 \times 100 < 0$$

- Which is never true for $\delta \geq 0$

Somerville [2022]

- So now we have three potential models on the table
  - Focussing
  - Relative thinking
  - Salience
- All have different implications
- Which one best matches the data?
- This question is addressed experimentally in a recent ECMA paper

- Subjects asked to choose between options defined by (q_j, p_j) where
  - q_j is the quality of good j in monetary units
  - p_j is the price
- Basic choice is between

$$(q_h, p_h) \text{ and } (q_l, p_l)$$

with $q_h > q_l$

- Assume that choices are driven by the utility function

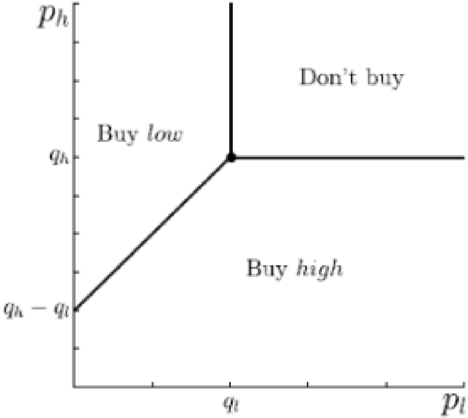
$$\Delta_q^{\gamma} q_i - \Delta_p^{\gamma} p_i$$

- where Δ_x is the range of attribute x in the choice set and γ is a parameter
  - $\gamma \in (-1, 0)$ - relative thinking
  - $\gamma > 0$ - focussing
- What happens in binary choice?
- Choose h if

$$\begin{aligned} \Delta_q^{\gamma} q_h - \Delta_p^{\gamma} p_h &> \Delta_q^{\gamma} q_l - \Delta_p^{\gamma} p_l \\ &\Rightarrow \Delta_q^{\gamma+1} > \Delta_p^{\gamma+1} \end{aligned}$$

- So choice independent of γ

- What about if we add the option to buy nothing?
- Treat this as a third alternative with values $(0, 0)$
- What should happen in the 'standard' case?
- i.e. if $\gamma = 0$?



- Introduce new 'Don't buy' regon
- Should not switch choices between 'buy low' and 'buy high'

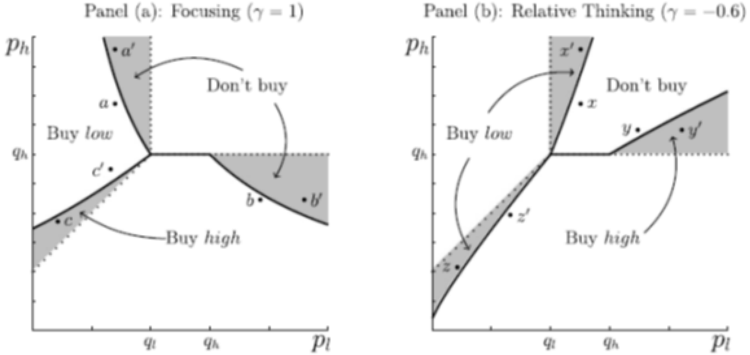
- What about if $\gamma \neq 0$?
- Remember, h is chosen if

$$\Delta_q^\gamma (q_h - q_l) > \Delta_p^\gamma (p_h - p_i)$$

- Imagine that, without the option not to buy, the DM was indifferent, so

$$(q_h - q_l) = (p_h - p_i)$$

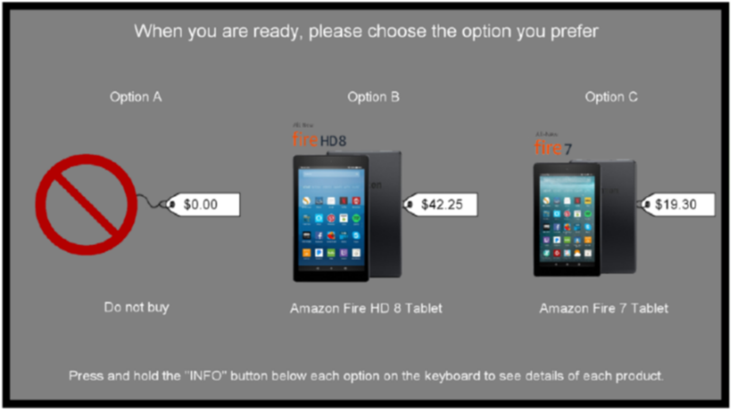
- Assume also, that we are in the region where both is prefered to not buying, so $q_h > p_h > p_l$
- So adding the option $(0, 0)$ increases the range of q more that the range of p
    - For relative thinking, implies that $\Delta_q^\gamma (q_h - q_l) < \Delta_p^\gamma (p_h - p_i)$
    - For focussing imples $\Delta_q^\gamma (q_h - q_l) > \Delta_p^\gamma (p_h - p_i)$


Panel (a): Focusing ($\gamma = 1$)
p_h
a'
a
Don't buy
Buy *low*
q_h
c'
c
b
b'
Buy *high*
q_l
q_h
p_l
Panel (b): Relative Thinking ($\gamma = -0.6$)
p_h
x'
x
Don't buy
Buy *low*
y
y'
q_h
z'
z
Buy *high*
q_l
q_h
p_l

Predictions

- Note that utility maximization model implies that an increase in the price of either good must weakly decrease the probability of buying one of the goods
  - let π_j be probability of buying good j
  - $\pi_h + \pi_l$ decreasing in both p_i and p_j
- Not true for the relative thinking model
- In the 'high decoy' region (i.e. the buy low/dont buy border)
  - An increase in p_h leads to an increase in π_l
- In the low decoy region
  - An increase in p_l leads to an increase in π_h

- Experiment conducted using real goods
- Two tasks
  - Valuation (using BDM)
  - Choice (with or without the option not to buy)


When you are ready, please choose the option you prefer
Option A
Option B
Option C
fire HD8
fire 7
$0.00
$42.25
$19.30
Do not buy
Amazon Fire HD 8 Tablet
Amazon Fire 7 Tablet
Press and hold the "INFO" button below each option on the keyboard to see details of each product.

Experiment

TABLE III

CHANGE IN CHOICE PROBABILITIES IN RESPONSE TO PRICE INCREASES.

	(i) Surplus Maximization $\gamma = 0$	(ii) Focusing $\gamma > 0$	(iii) Relative Thinking $\gamma < 0$	(iv) Data
		(a) $\uparrow$ p_h in the *h*-decoy region		
Buy *high*	⇩	⇩	⇩	⇩
Buy *low*	⇧	⇧ or ⇩	⇧	⇧
Don't buy	⇧	⇧	⇧ or ⇩	⇩
		(b) $\uparrow$ p_l in the *l*-decoy region		
Buy *high*	⇧	⇧ or ⇩	⇧	⇧
Buy *low*	⇩	⇩	⇩	⇩ †
Don't buy	⇧	⇧	⇧ or ⇩	⇩
		(c) $\uparrow$ p_h in the *o*-decoy region*		
Buy *high*	⇩	⇩	⇧ or ⇩	⇧
Buy *low*	⇩	⇧ or ⇩	⇩	⇩
Don't buy	⇧	⇧	⇧	⇧ †