Behavioral Economics

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Final Exam

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Question 1 (35 pts) We are going to consider preferences over compound lotteries. These are lotteries that give other lotteries as prizes. Let $\{p_1, q, p_2, r\}$ be the lottery that with probability p_1 gives the lottery q, and with probability p_2 gives the lottery r. For example, consider the lottery in which I flip a coin. If it comes down heads then I roll a die, and if I roll a 1 or 2 (out of 6) I give you \$5 (otherwise nothing). If it comes down heads, I roll a die, and if I get 1,2 ,3 or 4 I give you \$4 (otherwise nothing). We would write this as

$$\begin{aligned} & \{0.5, q, 0.5, r\} \\ q &= \frac{1}{3}\$5, \frac{2}{3}\$0 \\ r &= \frac{2}{3}\$4, \frac{1}{3}\$0 \end{aligned}$$

Call this example a. We will also write $\{q\}$ for the situation in which the DM receives the lottery $\{q\}$ for sure. (i.e., in the above example, $\{q\}$ would be a 100% chance of getting $\frac{1}{3}\$5, \frac{2}{3}\$0$)

Here is one way of calculating the utility of the above lottery (1) calculate the expected utility $U(q)$ and $U(r)$. (2) calculate the utility of the compound lottery as

$$U(\{p_1, q, p_2, r\}) = p_1 U(q) + p_2 U(r)$$

We will call this recursive expected utility approach

1. Assume that the utility of amount $u(x) = x$. Calculate the recursive expected utility of the lottery of example a

2. Show that, for a recursive expected utility maximizer the compound lottery in example a is indifferent to receiving the lottery that gives \$5 with probability $\frac{1}{6}$, \$4 with probability $\frac{1}{3}$ and \$0 with probability $\frac{1}{2}$

3. Assume (for simplicity) that the lotteries we consider are over whole dollar amounts between \$0 and \$10. We say that preferences satisfy reduction of compound lotteries if, for ever compound lottery $\{p_1, q, p_2, r\}$

$$\{p_1, q, p_2, r\} \text{ is indifferent to the lottery } \{s\}$$

Where s is the lottery such that, from each $x \in \{0, 1, ...10\}$ $s(x) = p_1 q(x) + p_2 r(x)$ and $s(x)$, $q(x)$ and $r(x)$ are, respectively, the probability assigned to x by the lotteries s, q and r

Show that the recursive expected utility approach satisfies the reduction of compound lotteries

4. Now consider another way of calculating the utility of a compound lottery. Let π be a cumulative probability weighting function. (1) use π to calculate the non-expected utility $\bar{U}(q)$ and $\bar{U}(r)$ of the lotteries q and r (i.e. using the cumulative probability weighting model) (2) calculate the non-expected utility as

$$U(\{p_1, q, p_2, r\}) = \pi(p_1)\bar{U}(q) + (1 - \pi(p_1))\bar{U}(r)$$

if $\bar{U}(q) \geq \bar{U}(r)$, or

$$U(\{p_1, q, p_2, r\}) = \pi(p_1)\bar{U}(r) + (1 - \pi(p_1))\bar{U}(q)$$

if $\bar{U}(r) > \bar{U}(q)$

We will call this the recursive non-expected utility approach.

Consider the recursive lottery in example (a). Show that the recursive non-expected utility approach does not necessarily satisfy the reduction of compound lotteries (Make life simple for yourself - assume $u(x) = x$ and remember that you can pick numbers for the probability weighting function, as long as $\pi(\frac{1}{6}) < \pi(\frac{1}{3}) < \pi(\frac{1}{2}) < \pi(\frac{2}{3})$)

5. If the probability weighting function is a power function, will the reduction of compound lotteries hold for the recursive lottery in example (a)? (if you get stuck, try it for $\pi(p) = p^2$.)

Question 2 (45 pts) Consider a decision maker who is choosing over what menu they want to choose from tomorrow. These menus can consist of subsets of three items: apples (a), bourbon (b) and (c) cigarillos . Say that the decision maker has a utility function u such

$$\begin{aligned} u(a) &= 1 \\ u(b) &= 2 \\ u(c) &= 3 \end{aligned}$$

Say that the decision maker is standard: i.e. from any menu they will choose the best object in that menu, and so value the menu according to its best option. Let $\trianglerighteq$ represent preferences over menus

1. Calculate the utility of the 7 possible menus that can be constructed from subsets of $\{a, b, c\}$

2. Notice that we can write the utility of a menu X as

$$U(X) = \max_{x \in X} u(x)$$

Verify that a preference function $\trianglerighteq$ that can be represented by this utility function satisfies the property that, if $X \trianglerighteq Y$, then $X \sim X \cup Y$ (If you can show this for general case, at least show its true for the 7 menus you looked at in part 1)

3. Does $\trianglerighteq$ satisfy set betweenness (again, do the general case if you can, or if not, then show its true for the 7 menus in part 1)

4. Now consider a decision maker who does not know what sort of mood they will be in tomorrow. With a 50% chance they think that they will want to be unhealthy, in which case they will have the utility function u (from section 1 above). with a 50% chance they think that they will wake up wanting to be healthy, in which case they will have the utility function v

$$\begin{aligned} v(a) &= 3 \\ v(b) &= 2 \\ v(c) &= 1 \end{aligned}$$

They calculate the utility of a menu by calculating the expected utility of that menu: i.e., for a menu containing $\{a, b\}$, there is a 50% chance that they will wake up with utility function u. In this case b is better than a, and so they will choose b and get utility $u(b)$. With 50% chance they will wake up with utility function v, in which case a is better than b and, they will choose a and get utility $v(a)$. Thus the utility of this set is $\{a, b\}$ is given by $U(\{a, b\}) = 0.5u(b) + 0.5v(a)$

Calculate the utility for this decision maker of the 7 possible menus that can be constructed from $\{a, b, c\}$

5. Do the preferences over menus of this decision maker satisfy the condition described in (2) above?

6. Do they satisfy set betweenness?

7. Now consider a general description of this type of preferences (sometimes called a preference for flexibility): Let Ω be a set of alternatives, and assume that the decision maker has a set of moods M. Each mood occurs with probability $p(m)$, and each mood gives rise to a utility function u_m over the objects in Ω. For any subset X of Ω, the utility of that subset is calculated as

$$U(X) = \sum_{m \in M} p(m) \max_{x \in X} u_m(x)$$

where $\max_{x \in X} u_m(x)$ is the highest utility obtainable in X according to the utility function u_m

Show that a decision maker who assesses menus in this way will satisfy the following condition:

$$\begin{aligned} X &\supseteq Y \\ &\Rightarrow X \trianglerighteq Y \end{aligned}$$

8. Show that they will also satisfy the following condition

$$\begin{aligned} X &\sim X \cup Y \\ \text{implies that, for any } Z &\subset \Omega \\ X \cup Z &\sim X \cup Y \cup Z \end{aligned}$$

9. We sometimes describe a decision maker as sophisticated if $X \cup \{x\} \triangleright X$ if and only if x will be chosen from the menu $X \cup \{x\}$. Will the preferences described at the start of the question satisfy this description of sophistication?

10. Show that the preferences in section 7 will not satisfy sophistication (i.e., there is a chance that $X \cup \{x\} \rhd X$, but x would not be chosen from the second stage menu). Can you think of a new definition of sophistication that would be satisfied by these preferences?

Question 3 (20 pts) Consider the following game (sometimes called the Nash Bargaining game). Two players have to share \$10. Each player makes a bid b_1 and b_2, which can be any number between 0 and 10. If $b_1 + b_2 \leq 10$, then each player receives their bid. If $b_1 + b_2 > 10$ then each player receives zero. These bids are made simultaneously. Assume that utility is linear in money.

1. Show that assuming standard preferences, a pair of strategies $\{b_1, b_2\}$ is a Nash Equilibrium if $b_1 + b_2 = 10$. Are these the only Nash Equilibria of this game? (Remember, a Nash Equilibrium is a pair of strategies $\{b_1, b_2\}$ such that b_1 is the best that player 1 can do, given b_2, and b_2 is the best that player 2 can do given b_1)
2. Imagine that player 1 has standard preferences, and player 2 has inequality averse preferences with $\alpha > 0$. Show that there is a threshold for $\bar{b}$ such that, if $b_2 < \bar{b}$, then $\{b_1, b_2\}$ such that $b_1 + b_2 = 10$ is not a Nash Equilibrium. Calculate $\bar{b}$ as a function of α
3. Again imagine that player 1 has standard preferences, and player 2 has inequality averse preferences. Is it always the case that, if $b_2 > \bar{b}$, then $\{b_1, b_2\}$ such that $b_1 + b_2 = 10$ is a Nash Equilibrium of the game? What if $\beta > 0.5$?