Econ 1820: Behavioral Economics

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Final 2014

Keep Calm and Apply Common Sense

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PLEASE ANSWER EACH QUESTION IN A DIFFERENT BOOK. PLEASE PUT YOUR NAME ON EACH BOOK

Question 1 (20 pts) Imagine you observed the following choices over bundles of American Flags and Red Coats

Observation	Price F	Price R	Amount F	Amount R
1	3	5	2	3
2	4	6	5	1
3	6	6	7	7

1. Which bundles are revealed directly preferred to which other bundles?
2. Which bundles are revealed preferred to which other bundles?
3. Which bundles are revealed strictly preferred to which other bundles (assuming local non-satiation)?
4. Is this data rationalizable by a non-satiated utility function? If not, why not?
5. What is the Houtmann Maks Index for the data (i.e. the size of the largest subset of the data that is rationalizable by a non-satiated utility function)

Question 2 (20 pts) Consider the following game (the payoff of the row player (1) are written first, and the column player (2) second). Assume that $x < 2$

	A	B
A	3,3	2,10
B	10,2	x,x

1. What are the Nash Equilibria of this game if people have selfish preferences?
2. Can $\{A, B\}$ be an equilibrium of this game if players are inequality averse? i.e, player 1 has the utility function

$$U(x_1, x_2) = x_1 - \alpha \max\{x_2 - x_1, 0\} - \beta \max\{x_1 - x_2, 0\}$$

with player 2 having the analogous preferences

3. Find values of x such that $\{B, B\}$ is an equilibrium under Rabin preferences. Remember, Rabin preferences *for player 1* are given by

$$U_1(x, y, z) = \pi_1(x, y) + f_1(x, y) f_2(y, z)$$

where x is the action of player 1, y is their belief over what player 2 will do and z is player 1's beliefs about what player 2 thinks player 1 will do. $\pi_1(x, y)$ is the payoff received by player 1 if player 1 plays x and player 2 plays y and $f_1(x, y)$ is the fairness function measuring how fair player 1 is being to player 2, defined as

$$f_1(x, y) = \frac{\pi_2(x, y) - \pi_2^e(y)}{\pi_2^h(y) - \pi_2^{\min}(y)}$$

where

- $\pi_2^e(y)$ is the equitable payoff: half way between the most that player 1 could give player 2 given they are playing y and the least they could have given them *without playing a strategy that leads to a pareto dominated outcome*
- $\pi_2^h(y)$ is the most that 1 could give 2 given they are playing y
- $\pi_2^{\min}(y)$ is the least that 1 could give 2 given that they are playing y

The fairness function of player 2 to player 1 (f_2) is defined analogously, as is the utility function for player 2 U_2.

Question 3 (30 pts) Consider the following simplified version of the 'Self Control at Work' paper we studied in class. In period 1 you will perform a number of data entry tasks. The effort cost of completing x tasks is given by αx^2, where $\alpha > 0$. In period 2, you will be paid according to how many tasks you have done. The (undiscounted) utility for receiving an amount of money y is equal to y. From the point of view of period 1, the utility from completing x tasks and getting money y is equal to

$$-ax^2 + \beta y$$

where $\beta \in [0, 1]$, while from the point of view of period 0 it is

$$-ax^2 + y$$

Assume that you are not restricted to completing whole numbers of tasks (so you can solve this problem using derivatives)

1. Assume that you get paid \$1 for each task (so if you complete x tasks you get $y = x$). If, in period 1, you are free to choose how much work to do, how much will you choose to do (as a function of α and β)?
2. Under the same assumptions, how much work would you choose to do if you could fix in period 0 the number of tasks you would do in period 1 (as a function of α)? Call this $x^*(\alpha)$. Assuming $\beta < 1$ is $x^*(\alpha)$ higher or lower than the effort level you would choose in period 1 for the same α?
3. Assume that $\alpha = 1$ and $\beta = \frac{1}{2}$. How much of your earnings would you be prepared to pay to commit to your preferred effort level in period 0? i.e. what is the largest amount T that you would be prepared to pay such that you would prefer to fix effort at $x^*(1)$ but only receive $x^*(1) - T$ in payment, rather than allow your period 1 self to choose effort levels.
4. Now consider the type of modified effort contract introduced in the 'Self Control at Work' paper: If you complete $x < x^*(1)$ tasks then your pay is only λx for $\lambda < 1$. If you complete $x \geq x^*(1)$ tasks then you get paid x. Having chosen a contract in Period 0, in Period 1 you choose how many tasks to complete. Show that if $\lambda = 0$ then in period 1 you will choose to produce $x^*(1)$ if $\beta \geq \frac{1}{2}$, but not otherwise (still assuming that $\alpha = 1$). Show that this implies that, if $\beta = \frac{3}{4}$, then in period 0 you would prefer the work contract in which $\lambda = 0$ to the work contract if $\lambda = 1$
5. Now again assume that $\beta = \frac{3}{4}$. With probability $1 - \varepsilon$ $\alpha = 1$, but with probability ε $\alpha = 2$.

(a) How much would you choose to work in period 1 if $\alpha = 1$ and $\lambda = 0$?

(b) How much would you choose to work in period 1 if $\alpha = 2$ and $\lambda = 0$? (remember that the contract pays λx for $x < x^*(1)$)

(c) How much would you choose to work in period 1 if $\alpha = 1$ and $\lambda = 1$?

(d) How much would you choose to work in period 1 if $\alpha = 2$ and $\lambda = 1$?

6. Use the above results to derive an expression for $\bar{\varepsilon}$ such that, if $\varepsilon > \bar{\varepsilon}$ then in period 0 you would prefer the work contract with $\lambda = 1$, but if $\varepsilon < \bar{\varepsilon}$ you would prefer the contract with $\lambda = 0$ (you don't have to solve the expression explicitly as the fractions are messy)

Question 4 (30 pts) Imagine that we observe Gertrude choosing between different ways of getting to work.

1. There are three possible routes, A, B and C, and as the economist we get to observe how Gertrude chooses from (A, B), (B, C), (A, C) and (A, B, C). For each of the following choice procedures, prove whether or not the resulting choices will satisfy property α (i..e. if x is chosen from a set S and $x \in T \subset S$ then x is chosen from T),

   (a) She calculates the average time taken on each available route and chooses the quickest (assume there are no ties)

   (b) She calculates the average time taken on each available route and chooses the slowest (assume there are no ties)

   (c) She calculates the average time taken on each available route and chooses the second quickest of all available routes (assume there are no ties)

2. Now imagine that Gertrude uses the following procedure: She prefers shorter routes to longer routes, and routes with less traffic to more. C is shorter than A and A is shorter than B. B has less traffic than route C, but she does not know how much traffic there is on route A. In any given choice set, Gertrude first rules out any routes such that there is an available route which definitely has less traffic. Then of the remaining routes she chooses the shortest

   (a) Under this procedure, figure out what Gertrude will choose from each choice set

   (b) Do the resulting choices satisfy condition α?

3. Here is a more general way of describing the procedure above: The decision maker has two rationales P_1 and P_2 (for example traffic and length). P_2 is antisymmetric (if xP_2y and yP_2x then $y = x$), transitive (xP_2y, yP_2z implies xP_2z) and complete (xP_2y or yP_2x for all x, y). P_1 is antisymmetric, but not necessarily complete and transitive. When choosing from a set S the decision maker first identifies all the elements that are not dominated by some other element according to P_1 - i.e. the set S_1 such that

$$S_1 = \{x \in S | \text{ there is no } y \in S \text{ such that } y \neq x \text{ and } yP_1x\}$$

Then chooses the best thing from this set according to P_2, so if $C(S)$ is the choice function

$$C(S) = \{x \in S_1 | \text{there is no } y \in S_1 \text{ such that } y \neq x \text{ and } yP_2x\}$$

We call this the rational shortlist method. Show that the procedure described in 2 is a rational shortlist method

4. Imagine that we observed Gertrude make the following choices:

$$\begin{aligned} C(\{A, B\}) &= B \\ C(\{A, C\}) &= A \\ C(\{B, C\}) &= B \\ C(\{A, B, C\}) &= A \end{aligned}$$

Show that this behavior is not consistent with the rational shortlist method (i.e. there is no P_1, P_2 that would rationalize these choices according to the rational shortlist method)

5. Here are two properties of choices

   - **WEAK WARP:** If x is chosen from $\{x, y\}$ and is also chosen from $\{x, y, z_1, ..., z_n\}$, then y is not chosen from any set consisting of x, y and some subset of $\{z_1, .., z_n\}$. i.e.

$$\begin{aligned} \{x, y\} &\subset S \subset T \\ C(\{x, y\}) &= C(T) = x \\ &\Rightarrow y \neq C(S) \end{aligned}$$

- **EXPANSION:** if x is chosen from each of two sets is also chosen from their union. i.e.

$$\begin{aligned} x &= C(S) = C(T) \\ &\Rightarrow x = C(S \cup T) \end{aligned}$$

Show that

(a) If choices satisfy property α they will also satisfy expansion and weak WARP (remember that we are assuming a choice function here, not a choice correspondence)

(b) If choices are made according to the rational shortlist method then they will satisfy expansion and weak WARP.