Behavioral Economics

Final Exam - Suggested Solutions

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Question 1 (35 pts)

We are going to consider preferences over compound lotteries. These are lotteries that give other lotteries as prizes. Let $\{p_1, q, p_2, r\}$ be the lottery that with probability p_1 gives the lottery q, and with probability p_2 gives the lottery r. For example, consider the lottery in which I flip a coin. If it comes down heads then I roll a die, and if I roll a 1 or 2 (out of 6) I give you \$5 (otherwise nothing). If it comes down heads, I roll a die, and if I get 1,2 ,3 or 4 I give you \$4 (otherwise nothing). We would write this as

$$
\begin{aligned}
& \{0.5, q, 0.5, r\} \\
q &= \frac{1}{3}\$5, \frac{2}{3}\$0 \\
r &= \frac{2}{3}\$4, \frac{1}{3}\$0
\end{aligned}
$$

Call this example a. We will also write $\{q\}$ for the situation in which the DM receives the lottery $\{q\}$ for sure. (i.e., in the above example, $\{q\}$ would be a 100% chance of getting $\frac{1}{3}\$5, \frac{2}{3}\$0$)

Here is one way of calculating the utility of the above lottery (1) calculate the expected utility $U(q)$ and $U(r)$. (2) calculate the utility of the compound lottery as

$$U(\{p_1, q, p_2, r\}) = p_1 U(q) + p_2 U(r)$$

We will call this recursive expected utility approach

Part 1

Assume that the utility of amount $u(x) = x$. Calculate the recursive expected utility of the lottery of example a

Answer By the definition of the recursive expected utility representation we have that

$$\begin{aligned}
U\left(\left\{\frac{1}{2}, q, \frac{1}{2}, r\right\}\right) &= \frac{1}{2}U(q) + \frac{1}{2}U(r) \\
&= \frac{1}{2}\left(\sum_x q(x)u(x)\right) + \frac{1}{2}\left(\sum_x r(x)u(x)\right) \\
&= \frac{1}{2}\left(\frac{1}{3}u(5) + \frac{2}{3}u(0)\right) + \frac{1}{2}\left(\frac{2}{3}u(4) + \frac{1}{3}u(0)\right) \\
&= \frac{1}{2}\left(\frac{1}{3}5\right) + \frac{1}{2}\left(\frac{2}{3}4\right) \\
&= \frac{1}{2}\left(\frac{5}{3} + \frac{8}{3}\right) \\
&= \frac{13}{6}
\end{aligned}$$

Part 2

Show that, for a recursive expected utility maximizer the compound lottery in example a is indifferent to receiving the lottery that gives \$5 with probability $\frac{1}{6}$, \$4 with probability $\frac{1}{3}$ and \$0 with probability $\frac{1}{2}$

Answer The recursive expected utility of the lottery s, where $s = \left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right)$ to prices $(5, 4, 0)$, it is just the expected utility of such a lottery

$$U(\{s\}) = U(s) = \frac{1}{6}u(5) + \frac{1}{3}u(4) + \frac{1}{2}u(0) = \frac{5}{6} + \frac{4}{3} = \frac{13}{6}$$

Part 3

Assume (for simplicity) that the lotteries we consider are over whole dollar amounts between \$0 and \$10. We say that preferences satisfy reduction of compound lotteries if, for ever compound lottery $\{p_1, q, p_2, r\}$

$$\{p_1, q, p_2, r\} \text{ is indifferent to the lottery } \{s\}$$

Where s is the lottery such that, from each $x \in \{0, 1, ...10\}$ $s(x) = p_1 q(x) + p_2 r(x)$ and $s(x)$, $q(x)$ and $r(x)$ are, respectively, the probability assigned to x by the lotteries s, q and r

Show that the recursive expected utility approach satisfies the reduction of compound lotteries

Answer We want to show that the recursive expected utility satisfies the reduction of compound lotteries.

Let $X = \{1,2,3,4,5,6,7,8,9,10\}$

$$\begin{aligned}
\{p_1,q,p_2,r\} \sim \{s\} &\Leftrightarrow U\left(\{p_1,q,p_2,r\}\right) = U\left(\{s\}\right) \\
&\Leftrightarrow U\left(\{p_1,q,p_2,r\}\right) = U\left(\{s\}\right) \\
&\Leftrightarrow p_1 U\left(\{q\}\right) + p_2 U\left(\{r\}\right) = U\left(\{s\}\right) \\
&\Leftrightarrow p_1\left(\sum_x q(x)u(x)\right) + p_2\left(\sum_x r(x)u(x)\right) = \left(\sum_x \left(p_1 q(x) + p_2 r(x)\right) u(x)\right) \\
&\Leftrightarrow \left(\sum_x p_1 q(x)u(x)\right) + \left(\sum_x p_2 r(x)u(x)\right) = \left(\sum_x \left(p_1 q(x) + p_2 r(x)\right) u(x)\right) \\
&\Leftrightarrow \left(\sum_x p_1 q(x)u(x) + p_2 r(x)u(x)\right) = \sum_x \left(p_1 q(x) + p_2 r(x)\right) u(x) \\
&\Leftrightarrow \sum_x \left(p_1 q(x) + p_2 r(x)\right) u(x) = \sum_x \left(p_1 q(x) + p_2 r(x)\right) u(x)
\end{aligned}$$

Part 4

Now consider another way of calculating the utility of a compound lottery. Let π be a cumulative probability weighting function. (1) use π to calculate the non-expected utility $\bar{U}(q)$ and $\bar{U}(r)$ of the lotteries q and r (i.e. using the cumulative probability weighting model) (2) calculate the non-expected utility as

$$U\left(\{p_1,q,p_2,r\}\right) = \pi(p_1)\bar{U}(q) + (1-\pi(p_1))\bar{U}(r)$$

if $\bar{U}(q) \geq \bar{U}(r)$, or

$$U\left(\{p_1,q,p_2,r\}\right) = \pi(p_2)\bar{U}(r) + (1-\pi(p_2))\bar{U}(q)$$

if $\bar{U}(r) > \bar{U}(q)$

We will call this the recursive non-expected utility approach.

Consider the recursive lottery in example (a). Show that the recursive non-expected utility approach does not necessarily satisfy the reduction of compound lotteries (Make life simple for yourself - assume $u(x) = x$ and remember that you can pick numbers for the probability weighting function, as long as $\pi(\frac{1}{6}) < \pi(\frac{1}{3}) < \pi(\frac{1}{2}) < \pi(\frac{2}{3})$)

Answer Consider the following subjective probabilities

$$\begin{aligned}
\pi\left(\frac{1}{6}\right) &= \frac{1}{9} \\
\pi\left(\frac{1}{3}\right) &= \frac{1}{4} \\
\pi\left(\frac{1}{2}\right) &= \frac{1}{2} \\
\pi\left(\frac{2}{3}\right) &= \frac{5}{6}
\end{aligned}$$

Then

$$\bar{U}(q) = \pi\left(\frac{1}{3}\right)u(5) + \pi\left(\frac{2}{3}\right)u(0) = 5\pi\left(\frac{1}{3}\right) = \frac{5}{3}$$

$$\bar{U}(r) = \pi\left(\frac{2}{3}\right)u(4) + \pi\left(\frac{2}{3}\right)u(0) = 4\pi\left(\frac{2}{3}\right) = \frac{10}{3}$$

Then since $\bar{U}(r) > \bar{U}(q)$, then

$$\begin{aligned}
U\left(\{p_1, q, p_2, r\}\right) &= \pi\left(\frac{1}{2}\right)\bar{U}(r) + \left(1 - \pi\left(\frac{1}{2}\right)\right)\bar{U}(q) \\
&= \pi\left(\frac{1}{2}\right)\frac{10}{3} + \left(1 - \pi\left(\frac{1}{2}\right)\frac{5}{3}\right) \\
&= \frac{1}{2}\frac{10}{3} + \frac{1}{2}\frac{5}{3} \\
&= \frac{1}{2}\left(\frac{5}{3} + \frac{10}{3}\right) \\
&= \frac{5}{2}
\end{aligned}$$

Part 5

If the probability weighting function is a power function, will the reduction of compound lotteries hold for the recursive lottery in example (a)? (if you get stuck, try it for $\pi(p) = p^2$.)

Answer Assume that $\bar{U}(r) > \bar{U}(q)$

$$\begin{aligned}
U\left(\{p_1, q, p_2, r\}\right) &= \pi\left(\frac{1}{2}\right)\bar{U}(r) + \left(1 - \pi\left(\frac{1}{2}\right)\right)\bar{U}(q) \\
&= \pi\left(\frac{1}{2}\right)\left(4\pi\left(\frac{2}{3}\right)\right) + \left(1 - \pi\left(\frac{1}{2}\right)\right)\left(5\pi\left(\frac{1}{3}\right)\right) \\
&= \left(\frac{1}{2}\right)^{\alpha} 4\left(\frac{2}{3}\right)^{\alpha} + \left(1 - \left(\frac{1}{2}\right)^{\alpha}\right)5\left(\frac{1}{3}\right)^{\alpha} \\
&= 4\left(\frac{1}{2}\frac{2}{3}\right)^{\alpha} + 5\left(1 - \left(\frac{1}{2}\right)^{\alpha}\right)\left(\frac{1}{3}\right)^{\alpha} \\
&= 4\left(\frac{1}{3}\right)^{\alpha} + 5\left(1 - \left(\frac{1}{2}\right)^{\alpha}\right)\left(\frac{1}{3}\right)^{\alpha}
\end{aligned}$$

While $U(s)$ is given by

$$\begin{aligned}
U(s) &= 5\pi\left(\frac{1}{6}\right) + 4\pi\left(\frac{1}{3}\right) + 0\pi\left(\frac{1}{2}\right) \\
&= 5\left(\frac{1}{6}\right)^{\alpha} + 4\left(\frac{1}{3}\right)^{\alpha}
\end{aligned}$$

Then these are equal to each other if and only if

$$\begin{aligned}
\left(1-\left(\frac{1}{2}\right)^{\alpha}\right)\left(\frac{1}{3}\right)^{\alpha}=\left(\frac{1}{6}\right)^{\alpha} &\Leftrightarrow \left(1-\left(\frac{1}{2}\right)^{\alpha}\right)\left(\frac{1}{3}\right)^{\alpha}=\left(\frac{1}{3}\right)^{\alpha}\left(\frac{1}{2}\right)^{\alpha} \\
&\Leftrightarrow \left(1-\left(\frac{1}{2}\right)^{\alpha}\right)=\left(\frac{1}{2}\right)^{\alpha} \\
&\Leftrightarrow 1-\left(\frac{1}{2}\right)^{\alpha}=\left(\frac{1}{2}\right)^{\alpha} \\
&\Leftrightarrow \frac{1}{2}=\left(\frac{1}{2}\right)^{\alpha} \\
&\Leftrightarrow \alpha=1
\end{aligned}$$

Question 2 (45 pts)

Consider a decision maker who is choosing over what menu they want to choose from tomorrow. These menus can consist of subsets of three items: apples (a), bourbon (b) and (c) cigarillos . Say that the decision maker has a utility function u such

$$\begin{aligned}
u(a) &= 1 \\
u(b) &= 2 \\
u(c) &= 3
\end{aligned}$$

Say that the decision maker is standard: i.e. from any menu they will choose the best object in that menu, and so value the menu according to its best option. Let $\trianglerighteq$ represent preferences over menus

Part 1

Calculate the utility of the 7 possible menus that can be constructed from subsets of $\{a, b, c\}$
The power set $2^{\{a,b,c\}}/\emptyset$ is:

- $\{a\}, \{b\}, \{c\}$

-$\{a, b\}, \{a, c\}, \{b, c\}$

-$\{a, b, c\}$.

Where - $U(\{a\}) = 1, U(\{b\}) = 2, U(\{c\}) = 3$

-$U(\{a, b\}) = 2, U(\{a, c\}) = 3, U(\{b, c\}) = 3$

-$U(\{a, b, c\}) = 3$.

Part 2

Notice that we can write the utility of a menu X as

$$U(X) = \max_{x \in X} u(x)$$

Verify that a preference function $\trianglerighteq$ that can be represented by this utility function satisfies the property that, if $X \trianglerighteq Y$, then $X \sim X \cup Y$ (If you can show this for general case, at least show its true for the 7 menus you looked at in part 1)

Answer

Claim *If* $X \succ Y$ *then* $X \sim X \cup Y$

Proof. The set of choices is A, the set of menus if $\mathcal{A} = 2^X/\emptyset$.

Let $X, Y \in \mathcal{A}$. If $\trianglerighteq$ is represented by $U(X) = max_{x \in X} u(x)$ then

$X \trianglerighteq Y \iff U(X) \geq U(Y)$

Now this implies that $x^* = argmax_{x \in X} u(x)$, and $y^* = argmax_{y \in Y} u(y)$ then $u(x^*) \geq u(y^*)$ and by definition of maximum $u(x^*) \geq u(y)$ for all $y \in Y$.

Then $Z = X \cup Y$ means that $z^* = argmax_{z \in Z} u(z)$ is necessarily equivalent to x^*, $u(z^*) = u(x^*)$.

To see this, assume this is false (i.e. $u(z^*) > u(x^*)$) then it has to be the case that either $z^* \in Y$ such that $u(z^*) > u(x^*)$ which contradicts the assumption that $u(x^*) \geq u(y^*)$ or $z^* \in X$ such that $u(z^*) > u(x^*)$ which contradicts the definition that $u(x^*) \geq u(x)$ for all $x \in X$.

We must conclude that $u(x^*) = u(z^*)$ so that $X \sim Z \equiv X \cup Y$. ∎

Part 3

Does $\trianglerighteq$ satisfy set betweenness (again, do the general case if you can, or if not, then show its true for the 7 menus in part 1)

Answer

Axiom[Set Betweenness]. *If* $X \trianglerighteq Y \implies X \trianglerighteq X \cup Y \trianglerighteq Y$ *for all* $X, Y \in \mathcal{A}$.

Claim $\trianglerighteq$ satisfy set betweeness.

Proof. $X \trianglerighteq Y \implies X \sim X \cup Y$ that means $X \trianglerighteq X \cup Y$ (and $X \cup Y \trianglerighteq X$).

Also, we have that $Z = X \cup Y \trianglerighteq Y$ since $u(z^*) = u(x^*) \geq u(y)$ for all $y \in Y$ by assumption. Then we conclude that $X \trianglerighteq X \cup Y \trianglerighteq Y$, so the $\trianglerighteq$ satisfies set betweenness. ∎

Part 4

Now consider a decision maker who does not know what sort of mood they will be in tomorrow. With a 50% chance they think that they will want to be unhealthy, in which case they will have the utility function

u (from section 1 above). with a 50% chance they think that they will wake up wanting to be healthy, in which case they will have the utility function v

$$\begin{aligned} v(a) &= 3 \\ v(b) &= 2 \\ v(c) &= 1 \end{aligned}$$

They calculate the utility of a menu by calculating the expected utility of that menu: i.e., for a menu containing $\{a, b\}$, there is a 50% chance that they will wake up with utility function u. In this case b is better than a, and so they will choose b and get utility $u(b)$. With 50% chance they will wake up with utility function v, in which case a is better than b and, they will choose a and get utility $v(a)$. Thus the utility of this set is $\{a, b\}$ is given by $U(\{a, b\}) = 0.5u(b) + 0.5v(a)$

Calculate the utility for this decision maker of the 7 possible menus that can be constructed from $\{a, b, c\}$

Answer - $U(\{a\}) = 0.5(1) + 0.5(3); U(\{b\}) = 0.5(2) + 0.5, (2); U(\{c\}) = 0.5(3) + 0.5(1)$
-$U(\{a, b\}) = 0.5(2) + 0.5(3); U(\{a, c\}) = 0.5(3) + 0.5(3); U(\{b, c\}) = 0.5(3) + 0.5(2)$
-$U(\{a, b, c\}) = 0.5(3) + 0.5(3)$.

Part 5

Do the preferences over menus of this decision maker satisfy the condition described in (2) above?

Observe that $U(\{a, b\}) > U(\{c\})$ but $U(\{a, b\}) < U(\{a, b, c\})$ which violates condition (2) ($U(\{a, b\}) = U(\{a, b, c\})$).

Part 6

Do they satisfy set betweenness?

Observe that $U(\{a\}) = U(\{b\})$ then $U(\{a\}) < U(\{a, b\})$ and $u(\{b\}) < U(\{a, b\})$ to that set betweenness is violated.

Part 7

Now consider a general description of this type of preferences (sometimes called a preference for flexibility): Let Ω be a set of alternatives, and assume that the decision maker has a set of moods M. Each mood occurs with probability $p(m)$, and each mood gives rise to a utility function u_m over the objects in Ω. For any subset X of Ω, the utility of that subset is calculated as

$$U(X) = \sum_{m \in M} p(m) \max_{x \in X} u_m(x)$$

where $\max_{x \in X} u_m(x)$ is the highest utility obtainable in X according to the utility function u_m

Show that a decision maker who assesses menus in this way will satisfy the following condition:

$$\begin{aligned} X &\supseteq Y \\ &\Rightarrow X \trianglerighteq Y \end{aligned}$$

Answer:

If $X \supseteq Y$ then if $y \in Y$ then $y \in X$, in particular for any $m \in M$ $y^* = argmax_{y \in Y} u_m(y)$ it follows that $y^* \in X$ and $u(x^*) = max_{x \in X} u_m(x) \geq u(y^*)$.

Now since $u_m(x^*) \geq u_m(y^*)$ for all $m \in M$ it follows that $\sum_m p(m) max_{x \in X} u_m(x) \geq \sum_m p(m) max_{y \in Y} u_m(y)$ then $X \trianglerighteq Y$.

Part 8

Show that they will also satisfy the following condition

$$\begin{aligned} X &\sim X \cup Y \\ \text{implies that, for any } Z &\subset \Omega \\ X \cup Z &\sim X \cup Y \cup Z \end{aligned}$$

Answer:

Intuitively this condition is like independence that is related to linearity, however it is a special kind of independence that works across menus with the standard representation.

If $X \sim X \cup Y$ then $\sum_m p(m) max_{x \in X} u_m(x) = \sum_m p(m) max_{y \in X \cup Y} u_m(y) \iff \sum_m p(m) U_m(X) = \sum p(m) U_m(X \cup Y)$ where $U_m(X) = max_{x \in X} u_m(x)$ is the standard representation.

Then it is clear that $U_m(X \cup Z) = U_m(X \cup Y \cup Z)$ for all $m \in M$. To see this is true, assume without loss of generality that $U_m(X \cup Y \cup Z) \geq U_m(X \cup Y)$ (the other possible inequality is ruled out by the fact that $X \cup Y \cup Z \supseteq X \cup Y$) then $\exists \overline{z} \in X \cup Y \cup Z / X \cup Z$ that is a $\overline{z} \in Y$ such that $u_m(\overline{z}) \geq u_m(z)$ for all $z \in X \cup Z$. But by assumption $X \sim X \cup Y$ there is at least one element $x \in X$ such that $u_m(x) \geq u_m(y)$ for all $y \in Y$, then it must be the case that $\exists z \in X \cup Z$ such that $u_m(z) \geq u_m(y)$ for all $y \in Y$. This is a contradiction. Then we conclude that $U_m(X \cup Z) = U_m(X \cup Y \cup Z)$.

The last part of the proof just follows from the linearity of the preferences since $U_m(X \cup Z) = U_m(X \cup Y \cup Z) \implies \sum_m p(m) U_m(X \cup Z) = \sum_m p(m) U_m(X \cup Y \cup Z)$.

Then we have that $X \cup Z \sim X \cup Y \cup Z$.

Part 9

We sometimes describe a decision maker as sophisticated if $X \cup \{x\} \triangleright X$ if and only if x will be chosen from the menu $X \cup \{x\}$. Will the preferences described at the start of the question satisfy this description of sophistication?

Answer:

Yes, since $U(X \cup \{x\}) > U(X) \iff u(x) > u(x^*)$ where $x^* = argmax_{x' \in X} u(x')$. This is clearly equivalent to stating that x will be chosen from $X \cup \{x\}$.

Part 10

Show that the preferences in section 7 will not satisfy sophistication (i.e., there is a chance that $X \cup \{x\} \triangleright X$, but x would not be chosen from the second stage menu). Can you think of a new definition of sophistication that would be satisfied by these preferences?

Answer - $U(\{a\}) = 0.5(1) + 0.5(3); U(\{b\}) = 0.5(2) + 0.5,(2); U(\{c\}) = 0.5(3) + 0.5(1)$

-$U(\{a,b\}) = 0.5(2) + 0.5(3); U(\{a,c\}) = 0.5(3) + 0.5(3); U(\{b,c\}) = 0.5(3) + 0.5(2)$

-$U(\{a,b,c\}) = 0.5(3) + 0.5(3)$.

Observe in the example (5), that:

$U(\{a,b\} \cup \{c\}) = U(\{a,b,c\}) = 3 > U(\{a,b\}) = 2.5$

But c is only chosen with probability 0.5 when the utility is u, $u(c) = 3$ (and not v).

In particular, c is not chosen with probability 0.5 then it is not sophisticated in the usual sense.

Now define $P(a|A)$ as the probability of choosing a from A, then sophistication in the flexibility case means that if $U(X \cup \{x\}) > U(X)$ for U defined in (7) then $P(x|X \cup \{x\}) > 0$.

Question 3 (20 pts)

Consider the following game (sometimes called the Nash Bargaining game). Two players have to share \$10. Each player makes a bid b_1 and b_2, which can be any number between 0 and 10. If $b_1 + b_2 \leq 10$, then each player receives their bid. If $b_1 + b_2 > 10$ then each player receives zero. These bids are made simultaneously. Assume that utility is linear in money.

Part 1

Show that assuming standard preferences, a pair of strategies $\{b_1, b_2\}$ is a Nash Equilibrium if $b_1 + b_2 = 10$. Are these the only Nash Equilibria of this game? (Remember, a Nash Equilibrium is a pair of strategies $\{b_1, b_2\}$ such that b_1 is the best that player 1 can do, given b_2, and b_2 is the best that player 2 can do given b_1)

A pair of strategies $\{b_1, b_2\}$ is a Nash Equilibrium if $b_1 + b_2 = 10$. We need to show that there is not a profitable deviation for any of the two players if they are playing $\{b_1, b_2\}$ such that $b_1 + b_2 = 10$. Clearly, none of the players has incentives to offer a $b'_i < b_i$ since given the other player's strategy they are better off bidding as high as possible as long as $b_1 + b_2 \leq 10$.

It is also straightforward that, as long as $b_i > 0$ for $i = 1, 2$ they are better off by bidding $\{b_1, b_2\}$ such that $b_1 + b_2 = 10$ than bidding $b'_i > b_i$, since that would imply both of them getting 0, while before $u_i = b_i > 0$. Finally even if one of the subjects is bidding 10 and the other 0, if we consider standard preferences the subject that is receiving 0 has no incentives to deviate, since it would get exactly the same payoffs.

Playing 10,10 is also a Nash equilibrium and both get zero. Any unilateral deviation won't change the deviant payoffs and therefore no incentives to deviate from it.

Part 2

Imagine that player 1 has standard preferences, and player 2 has inequality averse preferences with $\alpha > 0$. Show that there is a threshold for $\bar{b}$ such that, if $b_2 < \bar{b}$, then $\{b_1, b_2\}$ such that $b_1 + b_2 = 10$ is not a Nash

Equilibrium. Calculate $\bar{b}$ as a function of α

Subject 1 has standard preferences (assuming linearity)

$$u_1(x_1, x_2) = x_1$$

while the preference for subject 2 is given by

$$u_2(x_1, x_2) = x_2 - \alpha \max\{x_1 - x_2, 0\} - \beta \max\{x_2 - x_1, 0\}$$

Assume $x_1 > x_2$ and $x_1 + x_2 = 10$ then the utility of subject 2 collapses to

$$u_2(x_1, x_2) = x_2 - \alpha(x_1 - x_2)$$

if subject 2 rejects the offer he gets 0, therefore we must have that if the subject accepts the split it should be that

$$\begin{aligned}
u_2(x_1, x_2) = x_2 - \alpha(x_1 - x_2) \geq 0 \quad &\Leftrightarrow \quad x_2 - \alpha(10 - 2x_2) \geq 0 \\
&\Leftrightarrow \quad x_2 + 2\alpha x_2 - 10\alpha \geq 0 \\
&\Leftrightarrow \quad x_2(1 + 2\alpha) \geq 10\alpha \\
&\Leftrightarrow \quad x_2 \geq \frac{10\alpha}{1 + 2\alpha}
\end{aligned}$$

Then, $\bar{b} = \frac{10\alpha}{1+2\alpha}$

Part 3

Again imagine that player 1 has standard preferences, and player 2 has inequality averse preferences. Is it always the case that, if $b_2 > \bar{b}$, then $\{b_1, b_2\}$ such that $b_1 + b_2 = 10$ is a Nash Equilibrium of the game? What if $\beta > 0.5$?

Assume that $x_2 > x_1$ and $x_1 + x_2 = 10$, $x_2 > 10 - x_2$, that implies that $x_2 > 5$ and $x_1 < 5$. Then the utility function of subject 2 collapses to

$$u_2(x_1, x_2) = x_2 - \beta(x_2 - 10 - x_2) = x_2 - 10\beta$$

. If $x_1 < 5$, subject 1 always can bid 5 and get an utility of 5; therefore, it must be the case that, if he is willing to bid more is because $x_2 - 10\beta > 5$, which no matter x_1 it won't be possible if $\beta > 0.5$