Econ 1820: Behavioral Economics

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Question 1 (20 pts)

Imagine you observed the following choices over bundles of American Flags and Red Coats

Observation	Price F	Price R	Amount F	Amount R
1	3	5	2	3
2	4	6	5	1
3	6	6	7	7

Part 1

Which bundles are revealed directly preferred to which other bundles?

Answer Consider the following auxiliary table

Prices for observation	Cost of B_1	Cost of B_2	Cost of B_3
1	**21**	20	56
2	26	**26**	70
3	30	36	**84**

Then by the definition of revealed directly preferred[1], we have that

$$B_2 R^D B_1 \quad B_3 R^D B_1 \quad B_3 R^D B_2 \quad B_1 R^D B_2$$

Part 2

Which bundles are revealed preferred to which other bundles?

Answer By the definition of revealed preferred to [2] we have that

[1]Definition 5 Lecture notes: A commodity bundle x^j is revealed directly preferred to a bundle x^k $[x^j R^D x^k]$ if $p^j x^k \leq p^j x^j$

[2]Again Definition 5 Lecture notes: A commodity bundle x^j is revealed preferred to a bundle x^k $[x^j R x^k]$ if we can find a sequence of bundles $x^1, \ldots, x^n$ such that $x^j R^D x^1 R^D \ldots R^D x^n R^D x^k$

$$B_2RB_1 \quad B_3RB_1 \quad B_3RB_2 \quad B_1RB_2$$

Part 3

Which bundles are revealed strictly preferred to which other bundles (assuming local non-satiation)?

Answer By the definition of strictly revealed preferred, assuming local non satiation[3] we have that

$$B_3SB_1 \quad B_3SB_2 \quad B_1SB_2$$

Part 4

Is this data rationalizable by a non-satiated utility function? If not, why not?

Answer No, it is not, because the data does not satisfy GARP and therefore due to the Afriat's theorem the data cannot be rationalizable by a non-satiated utility function. The data does not satisfy GARP because B_1RB_2 while B_1SB_2.

Part 5

What is the Houtmann Maks Index for the data (i.e. the size of the largest subset of the data that is rationalizable by a non-satiated utility function)

Answer As it can be seen from the previous question there is a cycle that involves observation 1 and 2. Excluding either of them we have a consistent data set, therefore the size of the largest subset that is rationalizable is 2, therefore the HM index is $\frac{2}{3}$

Question 2 (20 pts)

Consider the following game (the payoff of the row player are written first, and the column player second). Assume that $x < 2$

	A	B
A	$3,3$	$2,10$
B	$10,2$	x,x

Part 1

What are the Nash Equilibria of this game if people have selfish preferences?

[3] Definition 7 lecture notes: A commodity bundle x^j is revealed strictly preferred to a bundle x^k $[x^jSx^k]$ if $p^jx^k < p^jx^j$

Answer Assuming that $x < 2$ we have that the two Nash equilibria of the game are (A,B) and (B,A). This is because

$$u_1(A,B) = 2 > x = u_1(B,B) \quad u_2(A,B) = 10 > 3 = u_2(A,A)$$

$$u_1(B,A) = 10 > x = u_1(B,B) \quad u_2(B,A) = 2 > x = u_2(B,B)$$

Part 2

Can $\{A,B\}$ be an equilibrium of this game if players are inequality averse? i.e. they have the utility function

$$U_1(x_1,x_2) = x_1 - \alpha \max\{x_2 - x_1, 0\} - \beta \max\{x_1 - x_2, 0\}$$

Answer $\{A,B\}$ is an equilibrium of this game if neither of the two players want to deviate from playing $\{A,B\}$. Therefore we need the following conditions to hold

$$U_1(2,10) = 2 - 8\alpha \geq U_1(x,x) = x \Leftrightarrow \alpha \leq \frac{2-x}{8}$$

$$U_2(2,10) = 10 - 8\beta \geq U_2(3,3) = 3 \Leftrightarrow \beta \leq \frac{7}{8}$$

Part 3

Find the values of x such that $\{B,B\}$ is an equilibrium under Rabin preferences. Remember, Rabin preferences are

$$U_1(x,y,z) = \pi_1(x,y) + f_1(x,y)f_2(y,z)$$

where x is the action of player 1, y is their belief over what player 2 will do and z is player 1's belief about what player 2 thinks player 1 will do. $\pi_1(x,y)$ is the payoff received by player 1 if player 1 plays x and player 2 plays y and $f_1(x,y)$ is the fairness function measuring how fair player 1 is being to player 2, defined as

$$f_1(x,y) = \frac{\pi_2(x,y) - \pi_2^e(y)}{\pi_2^h(y) - \pi_2^{min}(y)}$$

where

- $\pi_2^e(y)$ is the equitable payoff: half way between the most that player 1 could give player 2 given that they are playing y and the least they could have given them without playing a strategy that leads to a pareto dominated outcome
- $\pi_2^h(y)$ is the most 1 could give 2 given they are playing y
- $\pi_2^{min}(y)$ is the least that 1 could give 2 given that they are playing y

U_2 and f_2 are defined analogously.

Answer We should ask the following question: If player 1 thought player 2 was going to play B, and thought that player 2 thought that they (player 1) would play B, would they prefer to play A or B? First let's think about $f_1(a_1, b_2)$, for $a_1 = \{A, B\}$ when $b_2 = B$. To calculate this we need:

- $\pi_2^h(B) = 10$
- $\pi_2^l(B) = x$
- $\pi_2^e(B) = 10$
- $\pi_2^{\min}(B) = x$

and so we have

$$f_1(a_1, B) = \frac{\pi_2(a_1, B) - \pi_2^e(B)}{\pi_2^h(B) - \pi_2^{\min}(B)}$$

$$f_1(A, B) = \frac{\pi_2(A, B) - \pi_2^e(B)}{\pi_2^h(B) - \pi_2^{\min}(B)}$$

$$f_1(A, B) = \frac{10 - 10}{10 - x} = 0$$

$$f_1(B, B) = \frac{\pi_2(B, B) - \pi_2^e(B)}{\pi_2^h(B) - \pi_2^{\min}(B)}$$

$$f_1(B, B) = \frac{x - 10}{10 - x} = -1$$

Now let's think about $\bar{f}_2(b_2, c_1)$. To calculate this, we need to figure out the following

- $\pi_1^h(B) = 10$
- $\pi_1^l(B) = x$
- $\pi_1^e(B) = 10$
- $\pi_1^{\min}(B) = x$

Thus

$$\begin{aligned} \bar{f}_2(B, B) &= \frac{\pi_1(B, B) - \pi_1^e(B)}{\pi_1^h(B) - \pi_1^{\min}(B)} \\ &= \frac{x - 10}{10 - x} = -1 \end{aligned}$$

Then $u_1(B, B, B) = \pi(B, B) + \bar{f}_2(B, B) f_1(B, B)$ So we have that

$$u_1(B, B, B)) = x - 1(-1) = x + 1$$

$$u_1(A, B, B)) = 2 - 1 \times 0 = 2$$

Therefore (B,B) is an equilibrium iff $x + 1 \geq 2$, iff $x \geq 1$

Question 3 (30 pts)

Consider the following simplified version of the "Self-Control" at Work's paper we studied in class. In period 1 you will perform a number of data entry tasks. The effort cost of completing x tasks is given by αx^2, where $\alpha > 0$. In period 2, you will be paid according to how many task you have done. The (undiscounted) utility for receiving an amount of money y is equal to y. From the point of view of period 1, the utility from completing x tasks and getting money y is equal to

$$-\alpha x^2 + \beta y$$

where $\beta \in [0, 1]$, while from the point of view of period 0 it is

$$-\alpha x^2 + y$$

Assume that you are not restricted to completing whole number of tasks (so you can solve this problem using derivatives).

Part 1

Assume that you get paid \$1 for each task (so if you complete x tasks you get $y = x$). If, in period 1, you are free to choose how much work to do, how much will you choose to do (as a function of α and β)

Answer By the value function from the point of view of period 1, and with the wages scheme $y = x$: $V_1(x) = -\alpha x^2 + \beta x$, for $x \geq 0$ this is a concave function with a unique maximum that is achieved at the x that satisfies the first order conditions of

$$FOC : \frac{\partial V_1(x)}{\partial x} = -2\alpha x + \beta = 0$$

$$x \geq 0$$

For the values allowed for α, β, $x \geq 0$ always so we can ignore the positivity constraint:

Then

$x^+(\alpha, \beta) = \frac{\beta}{2\alpha} \geq 0.$

Part 2

Under the same assumptions, how much work would you choose to do if you could fix in period 0 the number of tasks you would do in period 1 (as a function of α)? Call this $x^*(\alpha)$. Assuming $\beta < 1$ is $x^*(\alpha)$ higher or lower than the effort level you would choose in period 1 for the same α?

Answer The utility from the point of view of time 0 is $V_0(x) = -\alpha x^2 + x$ under the same wage scheme as in Part 1. The first order condition of the function is

$$\frac{\partial V_0(x)}{\partial x} = -2\alpha x + 1 = 0$$

With $x^*(\alpha) = \frac{1}{2\alpha}$.
Assume $\beta < 1$, then it is clear that since $x^*(\alpha) = \beta x^+(\alpha, \beta)$ it follows that $x^*(\alpha) > x^+(\alpha, \beta)$.

Part 3

Assume that $\alpha = 1$ and $\beta = \frac{1}{2}$. How much of your earnings would you be prepared to pay to commit to your preferred effort level in period 0? i.e. what is the largest amount T that you would be prepared to pay such that you would prefer to fix effort at $x^*(1)$ but only receive $x^*(1) - T$ in payment, rather than allow your period 1 self to choose effort levels

Answer Redefine the value function of period 0 as $V_0^T(T) = -x^*(1)^2 + (x^*(1) - T)$. And the value function of allowing period 1 self to fix the effort $V_0^+ = -x^+(1, \frac{1}{2})^2 + x^+(1, \frac{1}{2})$, with $x^+(1) = \frac{1}{2}$ and $x^+(1, \frac{1}{2}) = \frac{1/2}{2} = \frac{1}{4}$.
$V_0^T(T) = V_0^+ \implies -\frac{1}{4} + (\frac{1}{2} - T) = -\frac{1}{16} + \frac{1}{4} \implies T = \frac{1}{16}$.

Part 4

Now consider the type of modified effort contract introduced in 'Self Control at Work' paper: If you complete $x < x^*(1)$ tasks then your pay is only λx for $\lambda < 1$. If you complete $x \geq x^*(1)$ tasks then you get paid x. Having chosen a contract in Period 0, in Period 1 you choose how many tasks to complete.
Show that if $\lambda = 0$ then in period 1 you will choose to produce $x^*(1)$ if $\beta \geq \frac{1}{2}$, but not otherwise (still assuming $\alpha = 1$).
Show that this implies that if $\beta = \frac{3}{4}$, then in period 0 you would prefer the work contract in which $\lambda = 0$ to the work contract if $\lambda = 1$.

Answer
The wages scheme is $y = \begin{cases} x & if \quad x \geq x^*(1) = \frac{1}{2\alpha} \\ \lambda x & otherwise. \end{cases}$.
For $\lambda = 0$, $\alpha = 1$
$y = \begin{cases} x & if \quad x \geq x^*(1) = \frac{1}{2} \\ 0 & otherwise. \end{cases}$
$V_1^\lambda(x) = (-\alpha x^2 + \beta x)\mathbb{I}(x \geq \frac{1}{2\alpha}) + (-\alpha x^2)\mathbb{I}(x < \frac{1}{2\alpha})$
I can divide the problem in two:
(i)
$max_x - \alpha x^2 + \beta x + \gamma(-x + \frac{1}{2\alpha})$

FOC:

$-2\alpha x + \beta - \gamma = 0$; $x = \frac{\beta}{2\alpha} - \frac{\gamma}{2\alpha}$,

In particular if the solution is interior then

$x = \frac{\beta}{2\alpha}$ that fulfills the constraints only if $\beta \leq \frac{1}{2}$.

Otherwise the solution is in the corner or boundary and

$x = \frac{1}{\alpha}$.

It must be the case that

$x^{(i)} = \frac{1}{2\alpha}$ for $\alpha = 1$, $x^{(i)} = x^*(1)$.

(ii)

$max_x - \alpha x^2 + \gamma_1(x - \frac{1}{2\alpha}) + \gamma_2(x)$

$-2\alpha x + \gamma_1 + \gamma_2 = 0$

$0 \leq x \leq \frac{1}{2\alpha}$

The it follows that $x^{(ii)} = 0$.

Finally I want to check whether

$x = \frac{1}{2\alpha}$: $-\alpha x^2 + \beta x \geq 0$ then the worker chooses $-\alpha(\frac{1}{2\alpha})^2 + \beta\frac{1}{2\alpha} \geq 0$ that is $-\frac{1}{4\alpha} + \frac{\beta}{2\alpha} \geq 0$ $\beta \geq \frac{2\alpha}{4\alpha} = \frac{1}{2}$ which is exactly what we want.

$x = \frac{1}{2\alpha}$ when $\beta \geq \frac{1}{2}$ and $x = x^*(1)$ when $\alpha = 1$ and $\beta \geq \frac{1}{2}$ and otherwise $x = 0$.

Time zero, contract decision.

Define the value of the standard contract (flat rate)

$V_0^{flat} = -\alpha(x^+(\alpha,\beta))^2 + x^+(\alpha,\beta)) = -\alpha(\frac{\beta}{2\alpha})^2 + \frac{\beta}{2\alpha} = -\frac{\beta^2}{4\alpha} + \frac{\beta}{2\alpha}$.

$V_0^{flat} = -\frac{9}{64} + \frac{3}{8} = \frac{15}{64}$, $\alpha = 1, \beta = \frac{3}{4}$

The value of the λ contract

$V_0^{\lambda} = -\frac{1}{4\alpha} + \frac{1}{2\alpha}$ for $\lambda = 0$ and $\beta \geq \frac{1}{2}$

$V_0^{\lambda} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

Then $V_0^{\lambda} > V_0^{flat}$.

Part 5

Now again assume that $\beta = \frac{3}{4}$. With probability $1 - \epsilon$, $\alpha = 1$, but with probability ϵ $\alpha = 2$.

a) How much would you choose to work in period 1 if $\alpha = 1$ and $\lambda = 0$?

b) How much would you choose to work in period 1 if $\alpha = 2$ and $\lambda = 0$?

c) How much would you choose to work in period 1 if $\alpha = 1$ and $\lambda = 1$?

d) How much would you choose to work in period 1 if $\alpha = 2$ and $\lambda = 1$?

Answer

a) Using the results above note that $\alpha = 1$ and $\lambda = 0$, I would work $x = \frac{1}{2}$

b) If$\alpha = 2$ and $\lambda = 0$, I would work with $x = 0$ since I would get nothing in return.

c) If $\alpha = 1$ and $\lambda = 1$ then $x = \frac{(3/4)}{2} = \frac{3}{8}$

d) If $\alpha = 2$ and $\lambda = 1$ then $x = \frac{(3/4)}{4} = \frac{3}{16}$

Part 6

Use the above results to derive an expression for $\bar{\epsilon}$ such that, if $\epsilon > \bar{\epsilon}$ then in period 0 you would prefer the work contract with $\lambda = 1$, but if $\epsilon < \bar{\epsilon}$ you would prefer the contract with $\lambda = 0$.

Answer

For $\epsilon = \bar{\epsilon}$ it should be the case that both contracts $\lambda = 0$ and $\lambda = 1$ are indifferent.

$(1 - \bar{\epsilon})V_0^{\lambda=1}(\alpha = 1) + (\bar{\epsilon})V_0^{\lambda=1}(\alpha = 2) = (1 - \bar{\epsilon})V_0^{\lambda=0}(\alpha = 1) + (\bar{\epsilon})V_0^{\lambda=0}(\alpha = 2)$

$(1 - \bar{\epsilon})[-1(\frac{1}{2})^2 + \frac{1}{2}] = (1 - \bar{\epsilon})[-1(\frac{3}{8})^2 + \frac{3}{8}] + \bar{\epsilon}[-2(\frac{3}{16})^2 + \frac{3}{16}]$

Question 4 (30 pts) Imagine that we observe Gertrude choosing between different ways of getting to work.

1. There are three possible routes, A, B and C, and as the economist we get to observe how Gertrude chooses from (A, B), (B, C), (A, C) and (A, B, C). For each of the following choice procedures, prove whether or not the resulting choices will satisfy property α (i..e. if x is chosen from a set S and $x \in T \subset S$ then x is chosen from T),
   (a) She calculates the average time taken on each available route and chooses the quickest (assume there are no ties)
   ANSWER: Choices will satisfy α : let u_A, u_B and u_C be the average times taken for each route. Then for any set S, if x is chosen, then $u_x > u_y$ for all y in S. This implies that $u_x > u_y$ for all y in T and so x is chosen from T
   (b) She calculates the average time taken on each available route and chooses the slowest (assume there are no ties)
   ANSWER: Choices will satisfy α : let u_A, u_B and u_C be the average times taken for each route. Then for any set S, if x is chosen, then $u_x < u_y$ for all y in S. This implies that $u_x < u_y$ for all y in T and so x is chosen from T
   (c) She calculates the average time taken on each available route and chooses the second quickest of all available routes (assume there are no ties)
   ANSWER: Choices will not satisfy α Let $u_A = 1$, $u_B = 2$, $u_C = 3$. Then the choice from $\{B, C\}$ would be C but the choice from $\{A, B, C\}$ would be B
2. Now imagine that Gertrude uses the following procedure: She prefers shorter routes to longer routes, and routes with less traffic to more. C is shorter than A and A is shorter than B. B has less traffic than route C, but she does not know how much traffic there is on route A. In any given choice set, Gertrude first rules out any routes such that there is an available route which definitely has less traffic. Then of the remaining routes she chooses the shortest
   (a) Under this procedure, figure out what Gertrude will choose from each choice set
   ANSWER: $C(\{A, B\}) = A$, $C(\{A, C\}) = C$, $C(\{B, C\}) = B$, $C(\{A, B, C\}) = A$

(b) Do the resulting choices satisfy condition α?

ANSWER: No, these choices do not satisfy condition α, which can be seen by comparing choices from $\{A, C\}$ and $\{A, B, C\}$

3. Here is a more general way of describing the procedure above: The decision maker has two rationales P_1 and P_2 (for example traffic and length). P_2 is antisymmetric (if xP_2y and yP_2x then $y = x$), transitive (xP_2y, yP_2z implies xP_2z) and complete (xP_2y or yP_2x for all x, y). P_1 is antisymmetric, but not necessarily complete and transitive. When choosing from a set S the decision maker first identifies all the elements that are not dominated by some other element according to P_1 - i.e. the set S_1 such that

$$S_1 = \{x \in S| \text{ there is no } y \in S \text{ such that } y \neq x \text{ and } yP_1x\}$$

Then chooses the best thing from this set according to P_2, so if $C(S)$ is the choice function

$$C(S) = \{x \in S_1|\text{there is no } y \in S_1 \text{ such that } y \neq x \text{ and } yP_2x\}$$

We call this the rational shortlist method. Show that the procedure described in 2 is a rational shortlist method

ANSWER: The procedure above is a rational shortlist method with P_1 (traffic) defined as BP_2C and P_2 (length) defined as

$$\begin{gathered} AP_2A \\ BP_2B \\ CP_2C \\ CP_2A \\ CP_2B \\ AP_2B \end{gathered}$$

It is easy to verify that P_1 is antisymmetric and P_2 is complete, transitive and antisymmetric

4. Imagine that we observed Gertrude make the following choices:

$$\begin{aligned} C(\{A, B\}) &= B \\ C(\{A, C\}) &= A \\ C(\{B, C\}) &= B \\ C(\{A, B, C\}) &= A \end{aligned}$$

Show that this behavior is not consistent with the rational shortlist method (i.e. there is no P_1, P_2 that would rationalize these choices according to the rational shortlist method)

ANSWER: As $C(\{B, C\}) = B$ it cannot be the case that CP_1B. Thus, the fact that $C(\{A, B, C\}) = A$ means that either AP_1B or Not BP_1A and AP_2B. In either case this should imply that $C(\{A, B\}) = A$

5. Here are two properties of choices

- **WEAK WARP:** If x is chosen from $\{x, y\}$ and is also chosen from $\{x, y, z_1, ..., z_n\}$, then y is not chosen from any set consisting of x, y and some subset of $\{z_1, .., z_n\}$. i.e.

$$\begin{aligned}\{x, y\} &\subset S \subset T \\ C(\{x, y\}) &= C(T) = x \\ &\Rightarrow y \neq C(S)\end{aligned}$$

- **EXPANSION:** if x is chosen from each of two sets is also chosen from their union. i.e.

$$\begin{aligned}x &= C(S) = C(T) \\ &\Rightarrow x = C(S \cup T)\end{aligned}$$

Show that

(a) If choices satisfy property α they will also satisfy expansion and weak WARP
ANSWER: α directly implies weak WARP, as if $C(T) = x$ then $C(S) = x$ and so $C(S) \neq y$ Furthermore, assume that expansion does not hold, so $x \neq y = C(S \cup T)$. But $y \in S$ or $y \in T$, so the fact that $x = C(S) = C(T)$ then leads directly to a violation of WARP

(b) If choices are made according to the rational shortlist method then they will satisfy expansion and weak WARP.
ANSWER.The fact that $x \in C(T)$ means that for no $z \in S$ is it the case that zP_1x, so $x \in S_1$. The fact that $C(\{x, y\}) = x$ implies that either xP_1y or xP_2y. In either case, this rules out $y \in C(S)$, thus proving weak WARP
The fact that $x \in C(S)$ and $x \in C(T)$ means that there is no $y \in S \cup T$ such that yP_1x, and so $x \in (S \cup T)_1$. Moreover, we know that xP_2y for all $y \in S_1 \cup T_1$, and that $(S \cup T)_1 \subset S_1 \cup T_1$ (as if (for example) $y \in S/S_1$, there is some $z \in S$ such that zP_1y, and so $y \notin (S \cup T)_1$). Thus xP_2y for all $y \in (S \cup T)_1$ and so $x = C(S \cup T)$