Econ 1820: Behavioral Economics

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DON'T PANIC

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PLEASE ANSWER EACH QUESTION IN A DIFFERENT BOOK. PLEASE PUT YOUR NAME ON EACH BOOK

Question 1 (30 pts) Imagine you observed the following choices over bundles of guns and healthcare at different prices per unit

Observation	Price G	Price H	Amount G	Amount H
1	2	2	6	1
2	1	3	4	2
3	3	1	1	4

1. Which bundles are revealed directly preferred to which other bundles?

   Answer Consider the following auxiliary table

Prices for observation	Cost of B_1	Cost of B_2	Cost of B_3
1	**14**	12	10
2	9	**10**	13
3	19	14	**7**

   Then by the definition of revealed directly preferred[1], we have that

   (a) $B_1 R^D B_2$

   (b) $B_1 R^D B_3$

   (c) $B_2 R^D B_1$

2. Which bundles are revealed preferred to which other bundles?

   Answer By the definition of revealed preferred to [2] we have that

   (a) $B_1 R^D B_2$

[1] Definition 5 Lecture notes: A commodity bundle x^j is revealed directly preferred to a bundle x^k $[x^j R^D x^k]$ if $p^j x^k \leq p^j x^j$

[2] Again Definition 5 Lecture notes: A commodity bundle x^j is revealed preferred to a bundle x^k $[x^j R x^k]$ if we can find a sequence of bundles $x^1, \ldots, x^n$ such that $x^j R^D x^1 R^D \ldots R^D x^n R^D x^k$

(b) $B_1 R^D B_3$

(c) $B_2 R^D B_1$

(d) From 2c and 2b $B_2 R^D B_3$

3. Which bundles are revealed strictly preferred to which other bundles (assuming local non-satiation)?

Answer By the definition of strictly revealed preferred, assuming local non satiation[3], we have that all the above are strict.

4. Is this data rationalizable by a non-satiated utility function? If not, why not?

Answer It is not, because it violates GARP. In particular we have that $B_1 S B_2$ and $B_2 S B_1$

5. What is the Afriat index for this data set (hint: remember the concept of revealed preferred at level e defined by $ep^i x^i \geq p^i x^j$)?

Answer The Afriat index is the efficiency level (max) e such that the resulting revealed preference relation is acyclic. From previous part, we know we have a cycle between bundle 1 and 2, that once broke, the resulting preference relation is consistent with GARP. Then, we need to find the maximal e such that

$$ep^1 B_1 \geq p^1 B_2$$

or

$$ep^2 B_2 \geq p^2 B_1$$

Using the values from the table we get

$$ep^1 B_1 \geq p^1 B_2 \Leftrightarrow e14 \geq 12 \Leftrightarrow e \geq \frac{6}{7}$$

or

$$ep^2 B_2 \geq p^2 B_1 \Leftrightarrow e10 \geq 9 \Leftrightarrow e \geq \frac{9}{10}$$

So the Afriat Index is $e^* = .9$

Question 2 (40 pts) Consider the following model of choice with limited attention

Definition *We say a set of choice data can be explained as choice with consideration sets if there is (i) a utility function $u : X \to \mathbb{R}$ and (ii) a consideration set correspondence $\Gamma : 2^X/\emptyset \to 2^X/\emptyset$ such that $\Gamma(A) \subseteq A$ and*

$$C(A) = \max_{x \in \Gamma(A)} u(x)$$

[3] Definition 7 lecture notes: A commodity bundle x^j is revealed strictly preferred to a bundle x^k $[x^j S x^k]$ if $p^j x^k < p^j x^j$

In other words, for each set A, $\Gamma(A)$ defines the set of alternatives that the decision maker considers. They then choose the best option from $\Gamma(A)$ according to u.

For simplicity, let's assume that we are dealing with choice functions (not correspondences) and that there is no indifference

1. Show that a model of choice from consideration sets can explain any choice function

   Answer Claim *A model of choice from consideration sets explain any choice function.*

   Proof By assumption we have choice functions then there is an $x^* = C(A)$ for every A and every C. Now, we can explain any choice we observe if we fix $\Gamma(A) = \{x^*\}$. In this case $x^* = argmax_{x\in\{x^*\}}u(x)$ for any function u. ■

   Note that this theory is not very useful. Moreover is not falsifiable. It can never be violated by any observed data.

2. Now add the restriction

$$\Gamma(A) = \Gamma(A/x) \text{ if } x \notin \Gamma(A)$$

   (by A/x I mean the set A with x removed). In other words, If you did not consider x in choice set A, then removing x from the choice set should not affect what you consider

   Is the following set of choices consistent with this model?

$$\begin{aligned} C(\{x,y,z\}) &= x \\ C(\{x,y\}) &= y \end{aligned}$$

   Claim *The choice set above is consistent with the model.*

   Proof

   $\Gamma(\{x,y,z\}) = \{x,z\}$ this is consistent with $x \succ z$

   $\Gamma(\{x,y\}) = \{x,y\}$ this is consistent with $y \succ x$

   Check that this does satisfies the property above.

   If $x \notin \Gamma(A)$ implies $\Gamma(A) = \Gamma(A/x)$ this is equivalent (to the contrapositive) If$\Gamma(A) \neq \Gamma(A/x)$ then $x \in \Gamma(A)$

Check the property:

$\Gamma(\{x,y,z\}/\{z\}) = \{x,y\}$ is not equal to $\Gamma(\{x,y,z\}) = \{x,z\}$ then it must be the case that $z \in \Gamma(\{x,y,z))$. Indeed this is the case so we verified it. ■

3. Show that, if we observe that $C(A) \neq C(A/x)$ (i.e. removing x from A changes the choice from A), it must be the case $x \in \Gamma(A)$

Claim *If* $C(A) \neq C(A/x)$ *then* $x \in \Gamma(A)$.

Proof We can use the contrapositive. ($p \Rightarrow q \iff \neg q \Rightarrow \neg p$)

We have to prove:

If $x \notin \Gamma(A)$ then $C(A) = C(A/x)$.

This statement is almost trivial to prove since if $x \notin \Gamma(A)$ then $\Gamma(A) = \Gamma(A/x)$ (Property *)

By definition $C(A) = argmax_{x \in \Gamma(A)} u(x)$ and $C(A/x) = argmax_{x \in \Gamma(A/x)} u(x)$.

Using the property (*) above. $C(A) = C(A/x)$ since the consideration set is the same for both case.

Then we can conclude (by the contrapositive equivalence) that: If $C(A) \neq C(A/x)$ then $x \in \Gamma(A)$. ■

4. Show that the model implies the following property (hint, let x^* be the object in the set S with the highest utility)

$$\begin{aligned} \text{For any non-empty set } S \text{, there exists } x^* &\in S\text{, such that, for any set } T \text{ including } x^* \\ C(T) &= x^* \text{ whenever} \\ (i)\ C(T) &\in S \text{ and} \\ (ii)\ C(T) &\neq C(T/x^*) \end{aligned}$$

Property ** : For any non-empty set S, there exists $x^* \in S$, such that, for any set T including x^*.

$C(T) = x^*$ whenever

(i) $C(T) \in S$ and

(ii) $C(T) \neq C(T/x^*)$.

Claim *Choice under consideration sets satisfies property **.*

Proof Let $y = C(T)$ by (i) $y \in S$.

By (ii) $C(T) \neq C(T/x^*)$ implies that $x^* \in \Gamma(T)$ by property (*).

Then we know that both $y, x^* \in \Gamma(T)$ and furthermore $y, x^* \in S$. Since $u(x^*) > u(z)$ for all $z \in S$ by definition it must be the case that $u(y) = u(x^*)$. By the assumption of no indifference this implies that $y = x^*$.

Then $C(T) = x^*$. ■

5. Show that, if $x = C(A)$ and $y \in A$, then it is not necessarily the case that $u(x) > u(y)$, but if $C(A) = x \neq C(A \backslash y)$, then it must be the case that $u(x) > u(y)$

   Answer First, just let $y \notin \Gamma(A)$, then $u(y) > u(x)$ but $u(x) > u(z)$ for all $z \in \Gamma(A)$, such that $x = C(A)$.

   If $C(A) = x \neq C(A/y)$ then $y \in \Gamma(A)$ by (3) and we conclude that $u(x) > u(y)$.

6. (DO THIS PART ONLY IF YOU HAVE COMPLETED THE REST OF THE EXAM) Come up with a pattern of choices that cannot be explained by this model

   Answer Let

   $$C(\{x, y, a, b\}) = x \text{ and } C(\{x, a, b\}) = a$$

   This means that $y \in \Gamma(\{x, y, a, b\})$ and it is obvious that $x \in \Gamma(\{x, y, a, b\})$ since it has to be considered to be chosen.

   $$C(\{x, y, z, w\}) = y \text{ and } C(\{y, z, w\}) = w$$

   This says that $x \in \Gamma(\{x, y, z, w\})$ and also $y \in \Gamma(\{x, y, z, w\})$

   But by the first pair of choices we have $u(x) > u(y)$ and by the second pair of choices we have $u(y) > u(x)$. This is a contradiction. Then we have a data set that cannot be explained by the model.

Question 3 (30 pts) Consider the following two player game

	Column Player	
Row Player	In	Out
In	a, b	$0, 0$
Out	$0, 0$	$-1, -1$

The row player can choose to play either 'In' or 'Out', as can the column player. The table tells you the outcome for each player (row player first) depending on the strategy of both (so for example, if both row player and column player play 'Out', then both receive -1). a and b represent real numbers.

Consider the level k model we discussed in class. Assume that level 0 players play 'In' 50% of the time and 'Out' 50% of the time. Remember that level 1 players best respond to level 0 players and level 2 players best respond to level 1 players.

1. Find values for a and b such a level 1 column player will play 'In'

   Answer: A level 1 column player assumes that the row player will play out half the time and in half the time. Playing In will therefore give them $0.5 \times b + 0.5 \times 0 = 0.5b$, while playing Out will give them $0.5 \times 0 + 0.5 \times -1 = -0.5$. They will therefore definitely play In if $b > -1$, Out if $b < -1$, and will be indifferent between In and Out if $b = -1$

2. Find values for a and b such a level 1 row player will play 'In'

   Answer: Using the same logic above, the level 1 row player will play In if $a > -1$, Out if $a < -1$ and will be indifferent if $a = -1$

3. Find values for a and b such a level 2 column player will play 'In'

   Answer: A level 2 column player best responds to a level 1 row player. From the above, we know that if $a > -1$ then the the level 1 row player will play IN. In this case, it is best for the column player to play IN if $b > 0$. If $a < -1$ then the row player will play Out, so the best response of the column player is to play IN. Thus the conditions in which the level two row player will definitely play in are

   (a) $a < -1$

   (b) $a \geq -1$, $b > 0$

   If $b = 0$ and $a \geq -1$ then it is indeterminate whether the level 2 column player will play In or Out

4. Find values for a and b such a level 2 row player will play 'In'

   Answer: Using the same logic as above we have the following cases

   (a) $b < -1$

   (b) $b \geq -1$, $a > 0$

5. Find values for a and b such that a level 2 row player playing against a level 2 column player will play strategies that also form a Nash Equilibrium

   Answer: First let's figure out the Nash Equilibrium of the game as a function of a and b. If $a \geq 0$ then In is a weakly dominant strategy for the row player. Similarly, if $b \geq 0$ then In is a weakly dominant strategy for the column player. If $a < 0$ then the row player wants to play In if the column player plays Out, and Out if the column player plays in. Similarly, if $b < 0$ then the column player wants to play In if the row player plays Out, and Out if the row player plays In. This gives us the following cases:

   (a) $a > 0$ $b > 0$: $\{In, In\}$ (i.e. row player plays In, column player plays In) is the only Nash Equilibrium
   (b) $a = 0$, $b = 0$: $\{In, In\}$, $\{Out, In\}$, $\{In, Out\}$ are all Nash Equilibria
   (c) $a \geq 0$, $b < 0$ $\{In, Out\}$ is the only Nash Equilibrium
   (d) $a < 0$, $b \geq 0$ $\{Out, In\}$ is the only Nash Equilibrium
   (e) $a < 0$, $b < 0$ $\{Out, In\}$ and $\{In, Out\}$ are both Nash Equilibrium

   We are looking for values for which a level 2 row player and a level 2 column player playing against each other will constitute a Nash equilibrium. From the answers to 3 and 4 (for simplicity we will ignore the cases where a and b are equal to 0 or minus 1)

   (a) if $a > 0$ and $b > 0$ $\{In, In\}$: Nash Equilibrium
   (b) if $a > 0$ and $b < 0$ $\{In, Out\}$: Nash Equilibrium
   (c) if $a < 0$ and $b > 0$ $\{Out, In\}$: Nash Equilibrium
   (d) if $a \in (-1, 0)$ and $b \in (-1, 0)$: $\{Out, Out\}$: NOT a Nash Equilibrium
   (e) if $a < -1$ and $b \in (-1, 0)$: $\{Out, In\}$: Nash Equilibrium
   (f) if $a \in (-1, 0)$ and $b < -1$: $\{In, Out\}$: Nash Equilibrium
   (g) if $a < -1$ and $b < -1$: $\{In, In\}$: Not a Nash Equilibrium

6. Find values for a and b such that a level 2 row player playing against a level 1 column player will play strategies that also form a Nash Equilibrium

   Answer: Using the anwers from 1 and 4 we see that the following cases

   (a) if $a > 0$ and $b > 0$ $\{In, In\}$: Nash Equilibrium
   (b) if $a > 0$, and $b \in (-1, 0)$: $\{In, In\}$: Not a Nash Equilibrium
   (c) if $a > 0$, and $b < -1$: $\{In, Out\}$: Nash Equilibrium
   (d) if $a < 0$, and $b > -1$: $\{Out, In\}$: Nash Equilibrium
   (e) if $a < 0$, and $b < -1$: $\{In, Out\}$: Nash Equilibrium