# Behavioral Economics

## Midterm, 13th - Suggested Solutions[1]

**Question 1 (40 points)** Consider the following (completely standard) decision maker: They have a utility function $u$ on a set of finite alternatives $X$. Their preferences over sets of these alternatives (which we indicate by $\trianglerighteq$ for weak preferences) are given by the following. For any $A \in 2^X/\emptyset, B \in 2^X/\emptyset$

$$A \trianglerighteq B \Leftrightarrow max_{x \in A} u(x) \geq max_{x \in B} u(x)$$

**Part 1** Show that the binary relation $\trianglerighteq$ is complete and reflexive

**Complete** A preference relation $\succeq$ is *complete* if for all $x, y \in X$, we have that $x \succeq y$, or $y \succeq x$.

In our case, the set $X$ is defined as $2^X/\emptyset$, and therefore the elements of this set are sets $A \in 2^X/\emptyset$, therefore what **we want to show** is that for all $A \in 2^X/\emptyset, B \in 2^X/\emptyset$ either $A \trianglerighteq B$, $B \trianglerighteq A$ or both. That is, we want to show that for all $A \in 2^X/\emptyset, B \in 2^X/\emptyset$ either $max_{x \in A} u(x) \geq max_{x \in B} u(x)$, $max_{x \in B} u(x) \geq max_{x \in A} u(x)$ or both.

Let $u(x_A) \equiv max_{x \in A} u(x)$, $u(x_B) \equiv max_{x \in B} u(x)$, where we use the finiteness of $X$ for the existence of the maximum. Then by the property of the real numbers we have that $u(x_A) \geq u(x_B)$ and/or $u(x_B) \geq u(x_A)$; therefore we have that either $A \trianglerighteq B$, $B \trianglerighteq A$ or both.

**Reflexive** A preference relation $\succeq$ is *reflexive* if for all $x \in X$, we have that $x \succeq x$.

In our case we **want to show** that for all $A \in 2^X/\emptyset$ it is the case that $A \trianglerighteq A$, which in turn implies that $max_{x \in A} u(x) \geq max_{x \in A} u(x)$. As before since $X$ is finite, let $u(x_A) \equiv max_{x \in A} u(x)$, then we have that $u(x_A) = max_{x \in A} u(x) \geq max_{x \in A} u(x) = u(x_A)$, and in particular that $max_{x \in A} u(x) \geq max_{x \in A} u(x)$, which give us the desired result.

**Part 2.** Show that it is transitive

A preference relation $\succeq$ is *transitive* if for all $x, y, z \in X$, such that $x \succeq y$, $y \succeq z$, we have that $x \succeq z$.

In our case **we want to show that** $A, B, C \in 2^X/\emptyset$ such that $A \trianglerighteq B$, $B \trianglerighteq C$ we must have that $A \trianglerighteq C$. That is, we want to show that for all $A, B, C \in 2^X/\emptyset$ such that $max_{x \in A} u(x) \geq max_{x \in B} u(x)$ and $max_{x \in B} u(x) \geq max_{x \in C} u(x)$, we must have that $max_{x \in A} u(x) \geq max_{x \in C} u(x)$.

Let $u(x_A) \equiv max_{x \in A} u(x)$, $u(x_B) \equiv max_{x \in B} u(x)$ and $u(x_C) \equiv max_{x \in C} u(x)$, using again

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the finiteness of $X$ for the existence of the maximum; then by transitivity of the real numbers we have that $u(x_A) \geq u(x_B) \geq u(x_C)$ therefore $u(x_A) \geq u(x_C)$ and then we have that $A \trianglerighteq C$.

**Part 3.** Show that it satisfies the following property: if $A \trianglerighteq B$, then $A \bowtie A \cup B$ (where $\bowtie$ indicates indifference - i.e $A \bowtie B \Leftrightarrow A \trianglerighteq B \& B \trianglerighteq A$)

**We want to show** that if $A \trianglerighteq B$, then $A \bowtie A \cup B$, that is if $max_{x \in A} u(x) \geq max_{x \in B} u(x)$, then $max_{x \in A} u(x) = max_{x \in A \cup B} u(x)$, since $A \bowtie A \cup B \Leftrightarrow A \trianglerighteq A \cup B \& A \cup B \trianglerighteq A$ by the definition of indifference, which in turn implies that $max_{x \in A} u(x) \geq max_{x \in A \cup B} u(x)$ and $max_{x \in A \cup B} u(x) \geq max_{x \in A} u(x)$.

If we have that $A \trianglerighteq B$, that implies that $max_{x \in A} u(x) \geq max_{x \in B} u(x)$ and therefore $max_{x \in A \cup B} u(x) = max_{x \in A} u(x)$ which give us the desired result.

**Part 4.** We will now start to prove that if $\trianglerighteq$ satisfies these three properties, then it must be the case that there exists some $u : X \to \mathbb{R}$ such that $A \trianglerighteq B$ if and only if $max_{x \in A} u(x) \geq max_{x \in B} u(x)$ (i.e. these conditions are sufficient as well as necessary). First define the following binary relation on $X$:

$$x \succeq y \Leftrightarrow \{x\} \trianglerighteq \{y\}$$

where $\{x\}$ is the set that contains only $x$. Thus, we will say that the object $x$ is preferred to the object $y$ if and only if the set that contains only $x$ is preferred to the set that contains only $y$. Now show that, if $\trianglerighteq$ satisfies the three properties above, then there is an utility function $u$ such that $x \succeq y$ if and only if $u(x) \geq u(y)$, (hint: what property must $\succeq$ have in order to have an utility representation.)

**We want to show that** if $\trianglerighteq$ satisfies completeness, reflexivity and transitivity, then there is an utility function $u$ such that $x \succeq y$ if and only if $u(x) \geq u(y)$. This follows from the finiteness of $X$ and the three properties as in theorem XX of the lecture notes.

**Part 5.** For any set $A$, prove by induction that there must exist some $x_A$ such that $\{x_A\} \trianglerighteq \{y\}$ for all $y \in A$

Let's prove it by induction on the cardinality of the set $A$, that is we know, by reflexivity that it is true for $|A| = 1$, let assume that is true when $|A| = n - 1$ (inductive hypothesis), and we want to prove that is true when $|A| = n$.

Let consider the set $A$
$x_n$, where $|A| = n$. By the inductive hypothesis we know that there exist some $x_A$ such that $\{x_A\} \trianglerighteq \{y\}$ for all $y \in A$
$x_n$. By completeness of $\trianglerighteq$, we know that either $\{x_n\} \trianglerighteq \{x_A\}$ [1], $\{x_A\} \trianglerighteq \{x_n\}$ [2] or both. [3]

If [1] or [3] by transitivity we will have that $\{x_n\} \trianglerighteq \{x_A\} \trianglerighteq \{y\} \Rightarrow \{x_n\} \trianglerighteq \{y\}$ for all $y \in A$.

If [2] or [3] we would have that $u(x_n) \geq u(x_A) \geq u(y)$ for all $y$ in $\{x_A\} \trianglerighteq \{y\}$ for all $y \in A$, which completes our prove.

**Part 6.** Show that, if the three conditions hold, it must be the case that $A \bowtie \{x_A\}$

We want to show that if completeness, transitivity and reflexivity hold then it is the case that $A \bowtie \{x_A\}$, that is $A \trianglerighteq \{x_A\}$ and $\{x_A\} \trianglerighteq A$. We know that $\{x_A\}$ was defined such that $\{x_A\} \trianglerighteq \{y\}$ for all $y \in A$ [1], but therefore by part 3 we know that if $A \trianglerighteq B$, then $A \bowtie A \cup B$. Therefore since [1] we have that $\{x_A\} \bowtie \cup_{y \in A}\{y\} \equiv A$

**Part 7.** Use this to prove that $A \trianglerighteq B$ if and only if $max_{x \in A}u(x) \geq max_{x \in B}u(x)$

By part 6 we have that $A \bowtie \{x_A\}$ and $B \bowtie \{x_B\}$, therefore $A \trianglerighteq B$ if and only if $\{x_A\} \trianglerighteq \{x_B\}$ (using transitivity of $\trianglerighteq$). By definition of $\{x_A\}$, and the fact that by 4 there exists an utility function that represents the preferences for singletons, we know that we can rewrite the previous statement as $A \trianglerighteq B$ if and only if $argmax_{x \in A}u(x) \trianglerighteq argmax_{x \in B}u(x)$, which by definition of argmax/max it is true if and only if $\max_{x \in A} u(x) \geq \max_{x \in B} u(x)$

**Question 2 (40 Points)** Read the attached description of an experiment and its aim (you need not read the comment section)

**Part 1** Do the results of this experiment indicate a violation of the Weak Axiom of Revealed Preference? Assuming these results were repeated in an infinitely large population, what is the minimum proportion of subjects that exhibit such a violation?

Despite the fact that the experiment has an inter-subject design, these results are not commensurate with a population of subjects that satisfy the Independence of Irrelevant Alternatives. To see this, consider the first result, involving the physicians prescribing medicine. The results in the one medication case indicate that only 28% of doctors prefer no medication to ibuprofen. Such doctors should either choose Ibuprofen or Piroxicam in the two-medicine treatment, yet in this treatment, 47% choose no medication. Thus, at a minimum, 19% of subjects must violate IIA

**Part 2** Do the results support the conclusion that "the introduction of additional options can increase decision difficulty and hence the tendency to choose a distinctive option"? Can you think of any other explanations?

The article does a good job of illustrating a failure of IIA. However, it is not clear that these failures can be attributed either to the tenancy to choose a distinctive option, or to maintain the status quo. The main problem is that there are only a small number of examples used, and the options in these examples differ in a number of ways - for example - doing nothing or not doing nothing, choosing the course of action that had already been decided on etc. Thus it is not clear what facets of these options are driving the results. Moreover, it could be that (for example) subjects tend to randomize in larger choice sets, which would also explain this data. A more convincing test of this hypothesis would occur if the experiment included many different type of question where the options varied on all possible dimensions apart form the one under test (so, for example, the option sets varied in all sorts of ways, apart from the fact that one was always the status quo option). The fact that all other properties were randomized would strengthen the conclusion that people really are switching to the status quo.

**Part 3** How well do you think these results will generalize? How well do they stand up to the criticisms of Levitt and List?

(3) Levitt and List list (broadly speaking) 5 things that we should think about when forming an opinion about the generalizability of experimental results

*Scrutiny:* One issue is whether there is a higher degree of scrutiny of subjects in the experiment than there would be in many real life experiments. It is hard to see that this is a concern here. While the subjects are under scrutiny in this experiment, it seems likely that they would be if anything under far more scrutiny if this were a 'real life' decision - especially if something went wrong (or in the case of the politician). Moreover, if subjects were worried about scrutiny, then one would expect them to put more effort into making their decisions than they would do otherwise, potentially reducing any decision making bias. Thus, the concern in this experiment is that a lack of scrutiny, linked with low incentives, may lead the subjects to not deliberate enough.

*Subject pool:* Here, the basic pool of subjects is more relevant that the standard lab experiment, and is, in fact, the group the behavior of which this study wishes to describe. There is a selection effect, because not all subjects respond to the survey. As response rates are similar across experimental groups, it is difficult to construct a plausible story about how these selection effects are driving these results: if one group of 'decision averse' subjects that were answering the simple treatment but not the the expanded treatment, there would have to be another group of potential subjects that had the opposite bias.

*Stakes:* This is arguably the biggest flaw with this study: As the questions are hypothetical, the respondents may be answering in a quick, slapdash way, and therefore exhibiting biases that they would not if these were real decisions involving patients

*Experience:* Again, this is much less of a concern here, as the subjects of this experiment are making the type of decision with which they are very familiar with. Thus, unlike a typical lab experiment with inexperienced student subjects, a lack of experience with the decision making environment is not really an issue.

*Context/Framing*: Here there is something of an issue because the experimenters are not clear about the hypothesis that they want to test. The choices are clearly and deliberately framed. For example, in the first experiment, one option is framed as the status quo. However, it is also the option that is 'different' from the other two. If the experimenters were clear about they hypothesis that they were testing, then they could orthogonalize these frames.

**Question 3** Consider the following model of choice in the presence of a status quo. A decision maker has two "preference relations" on a set X

- $\succeq$ which are their "normal" preferences and are complete, transitive and reflexive
- $\rhd$ which represents "strongly preferred". These are transitive, but not necessarily complete (i.e. it is not true that, for some $x, y$ either $x \rhd y$ or $y \rhd x$). Moreover, if $x \rhd y$ it must be the case that $x \succ y$, but if $x \succ y$ it is not necessarily the case that $x \rhd y$

The decision maker makes choices in the following way

- If there is no status quo, they choose the item in the set that is best according to the preference relation $\succeq$
- If there is status quo $x$ in the set, they identify the set of objects that are strongly preferred to $x$ (i.e. all the $y's$ such that $y \rhd x$), then choose the best of these objects. If there are no objects in $y$ in the set such that $y \rhd x$, then they choose the status quo.

For convenience, you can write $C(A, x)$ to indicate the choice that the decision maker makes from $A$ when the status quo is $x$.

**Part 1** For a set of objects $x, y, z$ find a set of preferences (i.e. some $\succeq$ and $\rhd$) such that there is status quo bias - i.e. such that $C(\{x, y, z\}, x) = x$ and $C(\{x, y, z\}, y) = y$. Show that, in order to get status quo bias, it must be the case that $\rhd$ is not complete.

Consider for example $x \succeq y \succeq z$, with $x \succeq x$, $y \succeq y$ and $z \succeq z$, that are complete, transitive and reflexive. And then consider $\rhd$ as an empty relation, therefore we have that $y, z$ are not such that $y \rhd x$ or $z \rhd x$, as well no $x, z$ such that $x \rhd y$ or $z \rhd y$, therefore they choose the status quo.

It cannot be complete because if it were we must have that for some $x, y$ either $x \rhd y$ [1] or $y \rhd x$ [2], which contradicts $C(\{x, y, z\}, x) = x$ from [2] and $C(\{x, y, z\}, y) = y$ from [1].

**Part 2** Can this model explain "too much choice"?. Remember, too much choice occurs when people switch to choosing the status quo when the choice set gets too large - so for example $C(\{x, y\}, x) = y$ but $C(\{x, y, z\}, x) = x$

It cannot explain "too much choice", that is this model cannot explain choices like $C(\{x, y\}, x) = y$ [1] but $C(\{x, y, z\}, x) = x$ [2], since by [1] we must have that $y \rhd x$, and by [2] we must have that is not the case that $y \rhd x$ nor $z \rhd x$, therefore we have a contradiction.