Time Preferences

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Behavioral Economics Spring 2017

Two Standard Ways

- Before spring break we suggested two possible ways of spotting temptation
  1. Preference for Commitment
  2. Time inconsistency
- Previously we covered Preference for Commitment
- Now, time preferences!

Time Inconsistency

- Imagine you are asked to make a choice involving immediate consumption
  1. Salad or burger for lunch
  2. 2 squirts of juice immediately or 3 squirts of juice in 10 mins
- And a choice involving future consumption
  1. Salad or burger for lunch next Tuesday
  2. 2 squirts of juice in 10 mins or 3 squirts of juice in 20 mins
- Choice {burger,salad} or {2,3} is a 'preference reversal'
- Interpretation: you are tempted by the burger, but would 'prefer' to choose the salad
- In terms of previous model
  - burger maximizes $u + v$
  - salad maximizes u

Time Inconsistency

- Are preference reversals evidence for temptation?
- Not necessarily - could be changing tastes
  - Maybe just had a salad, so fancied a burger today but salad next week
  - Maybe you think you are particularly thirsty now, but won't be in 10 mins time
- Such changes should be distributed randomly
- But in many cases choices vary *consistently*
- Thirsty subjects
  - Juice now (60%) or twice amount in 5 minutes (40%)
  - Juice in 20 minutes (30%) or twice amount in 25 minutes (70%)
- Hard to explain with changing tastes

Time Inconsistency

- This behavior is inconsistent with standard intertemporal choice theory
- Utility given by

$$u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + ... = \sum_{t=0}^{T} \delta^t u(c_t)$$

  - δ is the discount rate
  - c_t is consumption in period t
  - u is stable utility function
- Often called **exponential discounting**

- What does this model say about salad versus burger?
- There should not be a preference reversal
  - If $u(s) > u(b)$ then salad should be chosen over burger both today and next Tuesday
  - If $u(s) < u(b)$ then burger should be chosen over salad both today and next Tuesday

- What about the juice?
- Assume there are only three periods - now, 10 mins time and 20 mins time
- What has to be true for you to prefer 2 squirts of juice immediately over 3 squits of juice in 10 mins?

$$
\begin{aligned}
& u(2)+\delta u(0)+\delta^2 u(0) \\
\geq \quad & u(0)+\delta u(3)+\delta^2 u(0) \\
\Rightarrow \quad & \delta \leq \frac{u(2)-u(0)}{u(3)-u(0)}
\end{aligned}
$$

- What about 2 squirts of juice in 10 mins over 3 squirts of juice in 20 mins?

$$\begin{aligned} & u(0)+\delta u(2)+\delta^2 u(0) \\ \geq \quad & u(0)+\delta u(0)+\delta^2 u(3) \\ \Rightarrow \quad & \delta \leq \frac{u(2)-u(0)}{u(3)-u(0)} \end{aligned}$$

- Same condition!
- Either always prefer the earlier juice or always prefer the later juice

Time Inconsistency

- Exponential discounting cannot explain preference reversals:
- Fundamental property: if you prefer

  x immediately to y in s periods

  Then you must prefer

  x in t periods to y in $t+s$ periods

- This is sometimes called **time invariance**
- We need a new model

Quasi-Hyperbolic Discounting

- One popular approach is **quasi-hyperbolic discounting**
- Relatively minor change to exponential discounting, but with big effects

$$U(c_1, c_2, c_3) = u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3)$$

- Called **present biased** if $\beta < 1$

- Can this model explain preference reversals?
- Prefer 2 squirts today over 3 squirts 10 mins if

$$\delta\beta \leq \frac{u(2) - u(0)}{u(3) - u(0)}$$

- Prefer two squirts in 10 mins over three squirts in 20 mins if

$$\delta \leq \frac{u(2) - u(0)}{u(3) - u(0)}$$

- Present bias possible if

$$\delta\beta < \frac{u(2) - u(0)}{u(3) - u(0)} < \delta$$

- So we now have two approaches to temptation and self control
  1. Preference for commitment
  2. Non-exponential discounting
- Is there any link between the two?
- In order to answer that question, we will think about a standard **consumption and savings problem**

Consumption and Savings

- Set up
  - Decision maker lives for 3 periods
  - Gets income y in each period
  - in each period can consume an amount c_t
  - Can borrow and save, but cannot die in debt
  - For simplicity we will assume that the interest rate is equal to zero
- Question: How much with the DM consume in each period?
- Let's think of two ways to solve this problem
  1. At time 1, DM chooses consumption for all three periods
  2. In each period DM chooses how much to consume

Consumption and Savings - Example

- First, the 'commitment' case
- The problem is

$$
\begin{aligned}
& \max_{c_1, c_2, c_3} u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3) \\
\text{subject to } c_1 + c_2 + c_3 &= 3y
\end{aligned}
$$

Consumption and Savings - Example

- We can solve this using Lagrangians

$$L(c_1, c_2, c_3) = u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3)$$
$$-\lambda(c_1 + c_2 + c_3 - 3y)$$

- Giving first order conditions

$$\begin{aligned} u'(c_1) &= \lambda \\ \beta\delta u'(c_2) &= \lambda \\ \beta\delta^2 u'(c_3) &= \lambda \end{aligned}$$

Consumption and Savings - Example

- We will assume that utillity is constant relative risk aversion

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

- And so the FOC give us

$$c_1^{-\sigma} = \beta\delta c_2^{-\sigma} = \beta\delta^2 c_3^{-\sigma}$$

Consumption and Savings - Example

- Plugging into the budget constrain gives

$$c_1 + (\beta\delta)^{\frac{1}{\sigma}} c_1 + \left(\beta\delta^2\right)^{\frac{1}{\sigma}} c_1 = 3y$$

- or

$$\begin{aligned} c_1 &= \frac{3y}{1 + (\beta\delta)^{\frac{1}{\sigma}} + \left(\beta\delta^2\right)^{\frac{1}{\sigma}}} \\ c_2 &= (\beta\delta)^{\frac{1}{\sigma}} c_1 \\ c_3 &= \left(\beta\delta^2\right)^{\frac{1}{\sigma}} c_1 \end{aligned}$$

- So

$$c_3 = \delta^{\frac{1}{\sigma}} c_2$$

Consumption and Savings - Example

- What happens if we assume that the DM decides what to do one period at a time?
- Model as a 'game' between three people
  - In period 1 DM decides on c_1 and how much wealth to leave to the next period (w_2)
  - In period 2 DM decides on c_2 and how much wealth to leave to the next period (w_3)
- Utility for period 1 player is

$$u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3)$$

for period 2

$$u(c_2) + \beta\delta u(c_3)$$

and for period 3

$$u(c_3)$$

Consumption and Savings - Example

- How can we solve this game?
- Work backwards!
- What will the player in the last period do?
- Eat everything they are left!
- So

$$c_3 = w_3 + y$$

Consumption and Savings - Example

- What about the penultimate period?
- Their optimization problem is to choose c_2 in order to maximize

$$u(c_2) + \beta\delta u(c_3)$$

subject to

$$\begin{aligned} c_3 &= w_3 + y = (w_2 + y - c_2) + y \\ &= w_2 + 2y - c_2 \end{aligned}$$

Consumption and Savings - Example

- First order conditions give

$$u'(c_2) = \beta\delta u'(c_3)$$

- Some handy algebra gives us that

$$\begin{aligned} c_2 &= \frac{w_2 + 2y}{1 + (\beta\delta)^{\frac{1}{\sigma}}} \\ c_3 &= (\beta\delta)^{\frac{1}{\sigma}} \frac{(w_2 + 2y)}{1 + (\beta\delta)^{\frac{1}{\sigma}}} \end{aligned}$$

- Whcih implies that

$$c_3 = (\beta\delta)^{\frac{1}{\sigma}} c_2$$

Consumption and Savings - Example

- Let's compare the two solutions:
- From the case where period 1 person chooses everything:

$$c_3 = \delta^{\frac{1}{\sigma}} c_2$$

- And the case where each person chooses as they go along

$$c_3 = (\beta\delta)^{\frac{1}{\sigma}} c_2$$

- What do you notice if $\beta = 1$?
- Magic!

Consumption and Savings - Example

- This is a very important feature of exponential discounting
- It leads to **time consistent** decisions
  - DM at time $t+1$ wants to stick to plans made at time t
  - In this case, the DM at time 2 agrees with the DM at time 1 about the split between consumption between period 2 and period 3
- Notice that this is **only** true for the case where $\beta = 1$
  - If $\beta < 1$ then the DM at time 1 will want higher c_3 relative to c_2 than will the DM at time 2
- So when is commitment valuable?

Consumption and Savings - Exponential Discounting and Commitment

- Assuming that the DM is sophisticatied, we have

$$\begin{aligned} & \text{Present bias} \\ \Longleftrightarrow \quad & \text{non-exponential discounting} \\ \Longleftrightarrow \quad & \text{Time Inconsistency} \\ \Longleftrightarrow \quad & \text{Preference for commitment} \end{aligned}$$

Consumption and Savings with Quasi Hyperbolic Discounting

- What about period 1 behavior in the 'game between three people' case?
- Depends on whether we assume that the DM is
  - Naive: Thinks that in period 2 they will follow the plan they made in period 1
  - Sophisticated: Understand that this will not happen

Consumption and Savings with Quasi Hyperbolic Discounting

- Naive case: In period 1 act as if they are in the commitment case, and so choose

$$c_1 = \frac{3y}{1 + (\beta\delta)^{\frac{1}{\sigma}} + (\beta\delta^2)^{\frac{1}{\sigma}}}$$

- But we know that in period 2 they will choose

$$c_3 = (\beta\delta)^{\frac{1}{\sigma}} c_2$$

rather than

$$c_3 = \delta^{\frac{1}{\sigma}} c_2$$

- So, compared to the commitment case, the Naive person will
  - Consume the same amount in period 1
  - Consume more in period 2
  - Consume less in period 3
  - No preference for commitment

Consumption and Savings with Quasi Hyperbolic Discounting

- Sophisticated case: Here, the DM realizes that in period 2 their savings will not be used as they would like
  - More consumption in period 2 relative to period 3
- Their problem is to choose c_1 in order to maximize

$$u(c_1) + \beta\delta u\left(\frac{w_2 + 2y}{1 + (\beta\delta)^{\frac{1}{\sigma}}}\right) + \beta\delta^2 u\left((\beta\delta)^{\frac{1}{\sigma}}\frac{(w_2 + 2y)}{1 + (\beta\delta)^{\frac{1}{\sigma}}}\right)$$

subject to

$$w_2 = y - c_1$$

- Note that $c_2 = \frac{w_2 + 2y}{1 + (\beta\delta)^{\frac{1}{\sigma}}}$ comes from the solution to period 2 person's optimization problem

Consumption and Savings with Quasi Hyperbolic Discounting

- Solving this problem gives

$$c_1 = \left[1 + \left(\frac{\beta\delta}{\left(1 + (\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}} + \frac{\delta\,(\beta\delta)^{\frac{1}{\sigma}}}{\left(1 + (\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}} \right)^{\frac{1}{\sigma}} \right]^{-1} 3y$$

- (you can probably derive this yourself, but I won't ask you to in an exam)
- Compare to the commitment solution

$$c_1 = \left(1 + (\beta\delta)^{\frac{1}{\sigma}} + \left(\beta\delta^2\right)^{\frac{1}{\sigma}} \right)^{-1} 3y$$

- Which is higher?

Consumption and Savings with Quasi Hyperbolic Discounting

- Period 1 consumption from the sophisticated agent will generally be different to that of the commitment person
- But can be higher or lower
  - Two offsetting forces
  - Money saved is 'wasted' by person in period 2
  - Period 3 agent gets completely screwed
  - Which one wins depends on the value of σ
- So for the sophisticated consumer
  - First period consumption will generally be different to the commitment case, but can be higher or lower
  - The ratio of second period to third period consumption will be higher than in the commitment case.
  - There is a preference for commitment

Other Explanations for Preference Reversals

- Is temptation the only explanation for preference reversals?
- No
  - Imagine you are an exponential discounter
  - You are asked to make choices between payments at different points in time
  - But you don't trust the experimenter.
  - Anything that is paid today you are sure you will get
  - Anything to get paid in the future, you think the experimenter will forget to pay you with probability ε

Other Explanations for Preference Reversals

- Imagine you are offered a payment of 1 today or 1+R tomorrow
- Will choose the earlier payment if

$$\begin{aligned} u(1) &> (1-\varepsilon)(\delta u(1+R)) \\ &\Rightarrow \frac{u(1)}{u(1+R)} > \delta(1-\varepsilon) \end{aligned}$$

- Whereas if offered 1 tomorrow or 1+R in two days time will choose the earlier payment if

$$\begin{aligned} (1-\varepsilon)\delta u(1) &> (1-\varepsilon)(\delta^2 u(1+R)) \\ &\Rightarrow \frac{u(1)}{u(1+R)} > \delta \end{aligned}$$

- Can cause preference reversals but no preference for commitment

Summary

- Systematic preference reversals present a challenge to the standard model of time separable, exponential discounting
  - A violation of stationarity
- There is a strong theoretical link between preference reversals, non-exponential discounting and preference for commitment
- Quasi-hyperbolic discounting model a popular alternative used to explain the data
  - Treats today as special