Bounded Rationality I: Consideration Sets

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What is Bounded Rationality?

- Start with a 'standard' economic model
 - e.g. utility maximization

$$C(A) = \max_{x \in A} u(x)$$

- If the model is wrong how can we adjust it?
- Two 'minimal' adjustments we could make
 - Modify objective
 - 2 Modify constraints
- Most of behavioral economics concerned with approach 1
 - Loss aversion
 - Ambiguity aversion
 - etc
- Bounded rationality concerned with approach 2
 - Optimal behavior within some additional costs/constraints

What is Bounded Rationality?

- Costs to acquiring or processing information
 - E.g. Simon [1955], Stigler [1961], Sims [2003]
- Limits on reasoning
 - E.g. Camerer [2004], Crawford [2005]
- Thinking Aversion
 - E.g. Ergin and Sarver [2010], Ortoleva [2013]
- Bounded memory
 - E.g. Wilson [2014]
- Automata
 - E.g. Piccione and Rubinstein [1993]
- Semi-Rational Models
 - E.g. Gabaix et al. [2008], Esponda [2008], Rabin and Vayanos [2010], Gabaix [2013],
- Heuristics
 - Tversky and Kahneman [1974], Gigerenzer [2000]

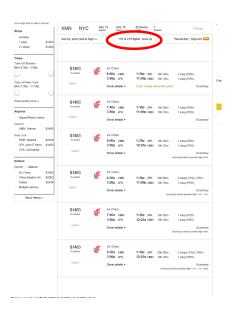
Advantages and Disadvantages of Bounded Rationality

- Advantage:
 - Intuitive plausibility
 - Evolution equipped us to optimize within constraints
 - Can 'microfound' behavioral models
 - Leads to new predictions: how behavioral phenomena can change with the environment
- Disadvantages:
 - May be wrong!
 - What is correct constraint?
 - Regress issue

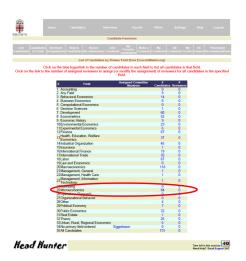
Introduction

- Start with one particular constraint on decision making: Limits on attention
- Attention is a scare resource
- The constraint is binding in economic choice

Choice Problem 1



Choice Problem 2



Consideration Sets

- Choice Problem 1 and 2 are difficult
 - Lots of available alternatives
 - Understanding each available alternative takes time and effort
- Do people really think hard about each available alternative?
- The marketing literature thinks not
- Since the 1960s have made use of the concept of consideration (or evoked) set
 - A subset of the available options from which the consumer makes their choice
 - Alternatives outside the consideration set are ignored
- Some key references
 - Hauser and Wernerfelt [1990]
 - Roberts and Lattin [1991]

Consideration Sets

- What was the evidence that convinced marketers that consideration sets played an important role in choice?
 - Intuitive plausibility
 - Verbal reports (e.g. Brown and Wildt 1992)
 - Lurking around supermarkets and seeing what people look at (e.g. Hoyer 1984)
- What are the implications for choice?
 - i.e. how could we test a model of consideration set formation?
 - What are its implications?

A (Naive) Model of Choice with Consideration Sets

- Let
 - $u: X \to \mathbb{R}$ be a utility function
 - $E: \mathcal{X} \to \mathcal{X}$ describe the evoked set
 - $E(A) \subseteq A$ is the set of considered alternatives from choice problem A
- Choice is given by

$$C(A) = \arg\max_{x \in E(A)} u(x)$$

- What are the testable implications of this model?
- Nothing!
- Any data set can be rationalized by assuming utility is constant and setting E(A) = C(A) for all A

A Testable Model of Choice with Consideration Sets

- In order to be able to test the consideration set model we need to do (at least) one of two things
 - Put more structure on the way consideration sets are formed
 - Enrich the data we use to test the model
- Will start by studying an approach that does a little bit of both.

Satisficing as Optimal Stopping

- Satisficing model (Simon 1955) was an early model of consideration set formation
- Very simple model:
 - Decision maker faced with a set of alternatives A
 - Searches through this set one by one
 - If they find alternative that is better than some threshold, stop search and choose that alternative
 - If all objects are searched, choose best alternative
- Proved extremely influential in economics, psychology and ecology

Satisficing as Optimal Stopping

- Usually presented as a compelling description of a 'choice procedure'
- Can also be derived as optimal behavior as a simple sequential search model with search costs
- Primitives
 - A set A containing M items from a set X
 - A utility function $u: X \to \mathbb{R}$
 - A probability distribution f: decision maker's beliefs about the value of each option
 - A per object search cost k

The Stopping Problem

- At any point DM has two options
- Stop searching, and choose the best alternative so far seen (search with recall)
- 2 Search another item and pay the cost k
- Familiar problem from labor economics

- Can solve for the optimal strategy by backwards induction
- \bullet Choice when there is 1 more object to search and current best alternative has utility \bar{u}
- 1 Stop searching: $\bar{u} (M-1)k$
- 2 Search the final item:

$$\int_{-\infty}^{\bar{u}} \bar{u}f(u)du + \int_{\bar{u}}^{\infty} uf(u)du - Mk$$

• Stop searching if

$$ar{u} - (M-1)k \le \int_{-\infty}^{ar{u}} ar{u}f(u)du + \int_{ar{u}}^{\infty} uf(u)du - Mk$$

Implying

$$k \le \int_{\bar{u}}^{\infty} (u - \bar{u}) f(u) du$$

- Value of RHS decreasing in ū
- Implies cutoff strategy: search continues if $\bar{u}>u^*$ solving

$$k = \int_{u^*}^{\infty} (u - u^*) f(u) du$$

- Now consider behavior when there are 2 items remaining
- $\bar{u} < u^*$ Search will continue
 - Search optimal if one object remaining
 - Can always operate continuation strategy of stopping after searching only one more option
- $\bar{u} > u^*$ search will stop
 - · Not optimal to search one more item only
 - Search will stop next period, as $\bar{u} > u^*$

- Optimal stopping strategy is satisficing!
- Find u* that solves

$$k = \int_{u^*}^{\infty} (u - u^*) f(u) du$$

- Continue searching until find an object with $u > u^*$, then stop
- Model of underlying constrains allow us to make predictions about how reservation level changes with environment
 - u* decreasing in k
 - increasing in variance of f (for well behaved distributions)
 - Unaffected by the size of the choice set
- · Comes from optimization, not reduced form satisficing model

Optimal Stopping - Extensions and Notes

- Satisficing as Framing
 - Imagine you are provided with some ranking of alternatives
 - You believe that this ranking is correlated (arbitrarily weakly) with your preferences
 - This is the only thing you know ex ante about each alternative.
 (e.g. Google searches)
 - What should your search order be?
 - Should search in the same order as the ranking
 - If list is long and correlation is low
 - Ex ante difference in quality between the first and last alternative is very low
 - But you will never pick the last alternative!
- Satsificing is a knife edge case
 - If one changes the problem
 - Learning
 - Varying information costs
 - Then reservation level will change over time
 - Testable prediction about the 'satisficing' model

Optimal Stopping - Extensions and Notes

- Solubility
 - The fact that we can solve this search problem depends on its simple structure
 - Things can get hairy very quickly
 - Explore/exploit
 - Multiple attributes
 - There are some mathematical tools that can help
 - Gittens indicies
 - But often have to rely on arprroximate solutions
 - e.g. Gabaix et al [2006]

Testing Satisficing: The Problem

- Satisficing models difficult to test using choice data alone
- If search order is fixed, behavior is indistinguishable from preference maximization
 - Define the binary relation \supseteq as $x \supseteq y$ if
 - x, y above satisficing level and x is searched before y
 - x is above the satisficing level and y below it
 - x, y both satisficing level and $u(x) \ge u(y)$
 - Easy to show that
 \(\subseteq \) is a complete preorder, and consumer chooses as if to maximize
 \(\subseteq \)
- If search order changes between choice sets, then any behavior can be rationalized
 - Assume that all alternatives are above satisficing level
 - Chosen alternative is then assumed to be the first alternative searched.

Choice Process Data

- Need to either
 - Add more assumptions
 - Enrich the data
- Examples
 - Search order observed from internet data [De los Santos, Hortacsu, and Wildenbeast 2012]
 - Stochastic choice data [Aguiar, Boccardi and Dean 2016]

Choice Process Data

- We will start by considering one possible data enrichment: 'choice process' data
- Records how choice changes with contemplation time
 - C(A): Standard choice data choice from set A
 - C_A(t): Choice process data choice made from set A after contemplation time t
- Easy to collect such data in the lab
 - Possible outside the lab using the internet?
- Has been used to
 - Test satisficing model [Caplin, Dean, Martin 2012]
 - Understand play in beauty contest game [Agranov, Caplin and Tergiman 2015]
 - Understand fast and slow processes in generosity [Kessler, Kivimaki and Niederle 2016]

Notation

- How can we use choice process data to test the satisficing model?
- First, introduce some notation:
 - X : Finite grand choice set
 - \mathcal{X} : Non-empty subsets of X
 - $Z \in \{Z_t\}_t^{\infty}$: Sequences of elements of \mathcal{X}
 - \mathcal{Z} set of sequences Z
 - $\mathcal{Z}_A \subset \mathcal{Z}$: set of sequences s.t. $Z_t \subset A \in \mathcal{X}$

A Definition of Choice Process

Definition

A Choice Process Data Set (X, C) comprises of:

- finite set X
- choice function $C: \mathcal{X} \to \mathcal{Z}$

such that $C(A) \in \mathcal{Z}_A \ \forall \ A \in \mathcal{X}$

• $C_A(t)$: choice made from set A after contemplation time t

Characterizing the Satisficing Model

- Two main assumptions of the satisficing model of consideration set formation
- 1 Search is alternative-based
 - DM searches through items in choice set sequentially
 - Completely understands each item before moving on to the next
- 2 Stopping is due to a fixed reservation rule
 - Subjects have a fixed reservation utility level
 - Stop searching if and only if find an item with utility above that level
 - First think about testing (1), then add (2)

Alternative-Based Search (ABS)

- DM has a fixed utility function
- Searches sequentially through the available options,
- Always chooses the best alternative of those searched
- May not search the entire choice set

Alternative-Based Search

• DM is equipped with a utility function

$$\mu: X \to \mathbb{R}$$

• and a search correspondence

$$S: \mathcal{X} \to \mathcal{Z}$$

with
$$S_A(t) \subseteq S_A(t+s)$$

Such that the DM always chooses best option of those searched

$$C_A(t) = \arg\max_{x \in S_A(t)} u(x)$$

Revealed Preference

- Key to testing the model is understanding what revealed preference means in this setting
- This is true for many models of incomplete consideration
 - Identify what behavior implies strict and weak revealed preference
 - Insist that these behaviors satisfy GARP
 - Use this to construct utility orders and consideration sets
- Possible general theorem?

Revealed Preference and ABS

- What type of behavior reveals preference in the ABS model?
- Finally choosing x over y does not imply (strict) revealed preference
 - DM may not know that y was available
- Replacing y with x does imply (strict) revealed preference
 - DM must know that y is available, as previously chose it
 - Now chooses x, so must prefer x over y
- Choosing x and y at the same time reveals indifference
- Use \succ^{ABS} to indicate ABS strict revealed preference
- Use \sim^{ABS} to indicate revealed indifference

Characterizing ABS

• Choice process data will have an ABS representation if and only if \succ^{ABS} and \sim^{ABS} can be represented by a utility function u

$$x \succ {}^{ABS}y \Rightarrow u(x) > u(y)$$

 $x \sim {}^{ABS}y \Rightarrow u(x) = u(y)$

- Necessary and sufficient conditions for utility representation GARP
 - Let $\succeq^{ABS} = \succ^{ABS} \cup \sim^{ABS}$
 - $xT(\succeq^{ABS})y$ implies not $y \succeq^{ABS} x$

Theorem 1

Theorem

Choice process data admits an ABS representation if and only if \succ^{ABS} and \sim^{ABS} satisfy GARP

Proof.

(Sketch of Sufficiency)

- **1** Generate U that represents \succeq^{ABS}
- **2** Set $S_A(t) = \bigcup_{s=1}^t C_A(s)$

Satisficing

- Choice process data admits an satisficing representation if we can find
 - An ABS representation (u, S)
 - A reservation level ρ
- Such that search stops if and only if an above reservation object is found
 - If the highest utility object in $S_A(t)$ is above ρ , search stops
 - If it is below ρ , then search continues
- Implies complete search of sets comprising only of below-reservation objects

Revealed Preference and Satisficing

- Final choice can now contain revealed preference information
 - If final choice is **below-reservation** utility
- How do we know if an object is below reservation?
- If they are non-terminal: Search continues after that object has been chosen

Directly and Indirectly Non-Terminal Sets

- Directly Non-Terminal: $x \in X^N$ if
 - $x \in C_A(t)$
 - $C_A(t) \neq C_A(t+s)$
- Indirectly Non Terminal: $x \in X^I$ if
 - for some $y \in X^N$
 - $x, y \in A$ and $y \in \lim_{t \to \infty} C_A(t)$
- Let $X^{IN} = X^I \cup X^N$

Add New Revealed Preference Information

- If
- one of $x, y \in A$ is in X^{IN}
- x is finally chosen from some set A when y is not,
- then, $x \succ^S y$
 - If x is is in X^{IN} , then A must have been fully searched, and so x must be preferred to y
 - If y is in X^{IN} , then either x is below reservation level, in which case the set is fully searched, or x is above reservation utility
- Let $\succ = \succ^S \cup \succ^{ABS}$

Theorem 2

Theorem

Choice process data admits an satisficing representation if and only if \succ and \sim^{ABS} satisfy GARP

Experiments and Bounded Rationality

- The experimental lab is often a good place to test models of bounded rationality
- Pros
 - Easy to identify choice mistakes
 - Can collect precisely the type of data you need
 - Can control the parameters of the problem
- Cons
 - Lack of external validity?
- A good approach (and good dissertation!) is to combine
 - Theory
 - Lab experiments
 - Field experiments/non experimental data

Experimental Design

- Experimental design has two aims
 - · Identify choice 'mistakes'
 - Test satisficing model as an explanation for these mistakes
- Two design challenges
 - Find a set of choice objects for which 'choice quality' is obvious but subjects do not always choose best option
 - · Find a way of eliciting 'choice process data'
- We first test for 'mistakes' in a standard choice task...
- ... then add choice process data in same environment
- Make life easier for ourselves by making preferences directly observable

Choice Objects

Subjects choose between 'sums'

four plus eight minus four

- Value of option is the value of the sum
- 'Full information' ranking obvious, but uncovering value takes effort
- 6 treatments
 - 2 x complexity (3 and 7 operations)
 - 3 x choice set size (10, 20 and 40 options)
- No time limit

Size 20, Complexity 7

0	zero
0	seven minus four minus two minus four minus two plus eleven minus four
0	six plus five minus eight plus two minus nine plus one plus four
0	seven minus two minus four plus three plus four minus three minus three
0	seven plus five minus two minus two minus three plus zero minus two
0	six plus seven plus six minus two minus six minus eight plus four
0	six plus two plus five minus four minus two minus seven plus three
0	six minus four minus one minus one plus five plus three minus six
0	two plus six plus seven minus two minus four minus two plus zero
0	two minus three minus five plus nine minus one plus five minus three
0	three plus zero plus two plus zero plus one minus three minus one
0	four plus three plus zero minus two plus three plus four minus ten
0	seven plus two plus seven minus seven plus three minus two minus two
0	three plus three minus two plus zero plus zero minus four plus five
0	two minus two plus zero plus nine minus two minus one minus one
0	three plus four minus three plus three minus four plus three minus four
0	three plus five plus seven plus five minus two minus seven minus ten
0	three plus six minus eight plus one plus two minus two plus zero
0	three plus five plus zero plus four plus three minus four minus two
0	eight minus one plus one minus four minus four minus five plus six
0	four minus five plus four minus one minus four plus zero plus four

Results

Failure rates (%) (22 subjects, 657 choices)

Failure rate					
Complex	Complexity				
3	7				
7% 24	24%				
22% 56	66%				
29% 65	55%				
29% 65	55%				

Results Average Loss (\$)

Average Loss (\$)					
	Complexity				
Set size	3	7			
10	0.41	1.69			
20	1.10	4.00			
40	2.30	7.12			

Eliciting Choice Process Data

- 1 Allow subjects to select any alternative at any time
 - Can change selection as often as they like
- Choice will be recorded at a random time between 0 and 120 seconds unknown to subject
 - Incentivizes subjects to always keep selected current best alternative
 - Treat the sequence of selections as choice process data
- 3 Round can end in two ways
 - After 120 seconds has elapsed
 - When subject presses the 'finish' button
 - · We discard any rounds in which subjects do not press 'finish'

Stage 1: Selection

Finished

Round	Current selection:	
2 of 30	four plus eight minus four	
Choose one:		
0	zero	
0	three plus five minus seven	
0	four plus two plus zero	
0	four plus three minus six	
P _d	four plus eight minus four	
O'II	three minus three plus one	
0	five plus one minus one	
0	eight plus two minus five	
0	three plus six minus five	
0	four minus two minus one	Î
0	five plus five minus one	

Stage 2: Choice Recorded



Choice Recorded

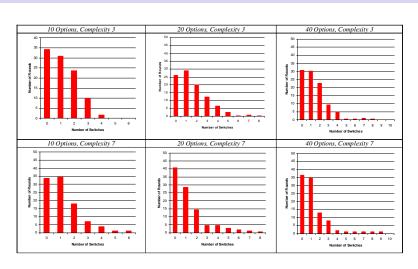
In this round, your choice was recorded after 9 seconds. At that time, you had selected:

four plus four minus six

Next

Do We Get Richer Data from Choice Process Methodology?

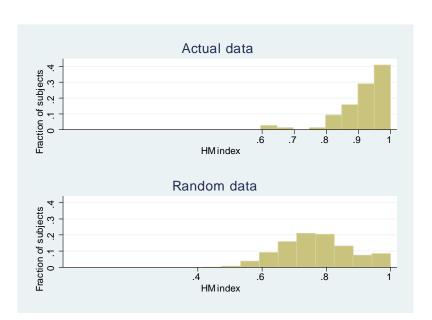
978 Rounds, 76 Subjects



Testing ABS

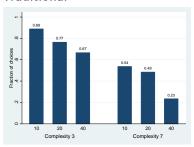
- Assume that more money is preferred to less
- Implies subjects must always switch to higher-valued objects (Condition 1)
- Calculate Houtman-Maks index for Condition 1
 - Largest subset of choice data that is consistent with condition

Houtman-Maks Measure for ABS

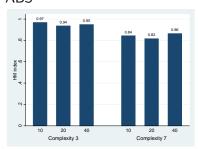


Traditional vs ABS Revealed Preference

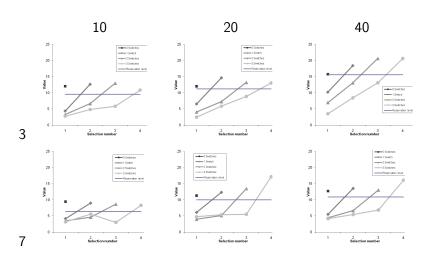
Traditional



ABS



Satisficing Behavior



Estimating Reservation Levels

- · Choice process data allows observation of subjects
 - Stopping search
 - Continuing to search
- Allows us to estimate reservation levels
- Assume that reservation level is calculated with some noise at each switch
- Can estimate reservation levels for each treatment using maximum likelihood

Estimated Reservation Levels

	Complexity				
Set size	3		et size 3 7		7
10	9.54	(0.20)	6.36	(0.13)	
20	11.18	(0.12)	9.95	(0.10)	
40	15.54	(0.11)	10.84	(0.10)	

Estimating Reservation Levels

- Increase with 'Cost of Search'
 - In line with model predictions
- Increase with size of choice set
 - In violation of model predictions
- See Brown, Flinn and Schotter [2011] for further insights

- Model choice with consideration sets using standard choice data
- Add an additional assumption to make consideration set model testable

$$E(S/x) = E(S)$$
 if $x \notin E(S)$

- Removing an item that is not in the consideration set does not affect the consideration set
- Allows the researcher to identify objects that were in the consideration set, and preferences

$$x \neq y = C(S) \neq C(S/x)$$

implies

- x was in E(S)
- y is strictly preferred to x
- Leads to testable predictions

De los Santos et al [2012]

- Use data from internet search engines on book purchases
- Makes visible what was searched not just what was chosen
 - People often do not search all available sellers
- · Use this to derive testable predictions of the satisficing model
 - Chosen item should be the last item searched, unless search is complete
 - Search should be more likely to stop after a high value (low price) alternative
- Find evidence against the satisficing model
 - Favor a model in which size of consideration set is fixed in advance

- Model choice with consideration sets using stochastic choice data
 - p(a, A): probability of alternative a chosen from set A
- Assume that every alternative has a fixed, strictly positive probability that it will be included in the consideration set
 - There is a default alternative which is always considered
- As usual, chosen item is the highest utility alternative in the consideration set.
- Allows preferences to be identified

$$\frac{p(a, A/b)}{p(a, A)} > 1 \Leftrightarrow u(b) > u(a)$$

• Provides testable predictions: e.g.

$$\frac{p(a, A/b)}{p(a, A)} > 1 \Rightarrow \frac{p(b, A/a)}{p(b, A)} = 1$$

Abaluck and Adams [2017]

- There is also a significant literature on this in consumer choice/IO
- Recent example is Abaluck and Adams [2017]
 - Also surveys previous literature
- Consideration sets can lead to violations of Slutsky Symmetry
 - · Absent income effects the following should be equal
 - The impact of a price change in good j on demand for good i
 - ullet The impact of a price change of good i on demand for good j

Abaluck and Adams [2017]

- Simple example:
 - Two products, 0 and 1
 - x_i price of good j
 - 0 is default always observed
 - 1 is alternative whether it is looked at depends on the price of 0
 - $\mu(x_0)$ probability that good 1 will be looked at give x_0

- s_i*(x₀, x₁) probability of buying good i given prices if both are observed
 - Derived from maximizing a quasilinear utility function
 - · Probabalistic due to some random utility component
- $s_i(x_0, x_1)$ probability that good i is chosen:

$$s_0(x_0, x_1) = (1 - \mu(x_0)) + \mu(x_0)s_0^*(x_0, x_1)$$

$$s_1(x_0, x_1) = \mu(x_0)s_1^*(x_0, x_1)$$

Claim: with quasi-linear utility and no outside option

$$\frac{\partial s_0^*(x_0, x_1)}{\partial x_1} = \frac{\partial s_1^*(x_0, x_1)}{\partial x_0}$$

• What if consideration is imperfect?

$$\frac{\partial s_0(x_0, x_1)}{\partial x_1} = \mu(x_0) \frac{\partial s_0^*(x_0, x_1)}{\partial x_1}
\frac{\partial s_1(x_0, x_1)}{\partial x_0} = \frac{\partial \mu(x_0)}{\partial x_0} s_1^*(x_0, x_1) + \mu(x_0) \frac{\partial s_1^*(x_0, x_1)}{\partial x_0}$$

impying

$$\frac{\partial s_1(x_0, x_1)}{\partial x_0} - \frac{\partial s_0(x_0, x_1)}{\partial x_1} = \frac{\partial \mu(x_0)}{\partial x_0} s_1^* = \frac{\partial \ln \mu(x_0)}{\partial x_0} s_1$$

$$\frac{\partial \ln \mu(x_0)}{\partial x_0} = \frac{1}{s_1} \left[\frac{\partial s_1(x_0, x_1)}{\partial x_0} - \frac{\partial s_0(x_0, x_1)}{\partial x_1} \right]$$

- Attention changes with prices if and only if Slutsky symmetry is violated
- Level of attention can be identified by integrating this expression

Summary

- There is good evidence that people do not look at all the available alternatives when making a choice
 - Lab experiments
 - Internet search
 - Verbal reports
 - · Direct observation of search
- Pure consideration set models cannot be tested on choice data alone
- Need either more data or more assumptions
- A variety of both approaches have been applied in the literature
 - Choice process
 - Internet search
 - Stochastic choice
- As yet, no real consensus on what is the correct model of consideration set formation
 - Though we do have some hints.