Rational Inattention Lecture 1

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Behavioral Economics G6943
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The Story So Far.....

- (Hopefully) convinced you that attention costs are important
- Introduced the concept of consideration sets
    - Along with sequential search and satisficing
- Showed that the model did a reasonable job in some circumstances
- But, there is something restrictive about consideration sets
    - Items are either in the consideration set and fully understood
    - Or outside the consideration set, and nothing is learned
- Seems like a good model for choice over a large number of simple alternatives
- Not for a small number of complex alternatives

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Name	Description
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IRA Contribution Calculator	Estimate how much you may be able to contribute to your IRA this year and how much might be tax-deductible.
Minimum Required Distribution (MRD) Calculator	Determine your required retirement account withdrawals after age 70½.
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Savings Planner	Take a few minutes to tell us your priorities and we'll help you create a plan to save and pay down debt.
Portfolio Review	Identify a target asset mix that aligns with your goals. Then easily implement an investment strategy that will help you stay on track.
myPlan Snapshot®	Answer five easy questions for a quick estimate of how much you may need to save for retirement compared with your projected savings.
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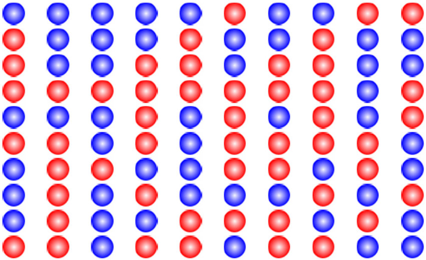


Act	Payoff 47 red dots	Payoff 53 red dots
a	20	0
b	0	10

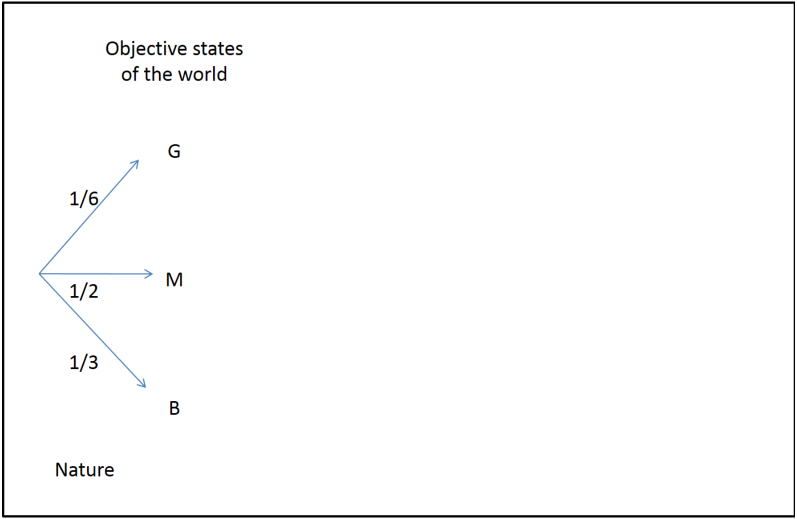
Set Up

- Objective states of the world
  - e.g. Demand could be 'good', 'medium' or 'bad'
- Decision maker chooses an action
  - e.g. Set price to be high, medium, or low
- Gross payoff depends on action and state
  - e.g. Quantity sold depends on price and demand
- Decision maker get to learn something about the state before choosing action
  - e.g. Could do market research, focus groups, etc.
- Can choose what to learn conditional on the problem

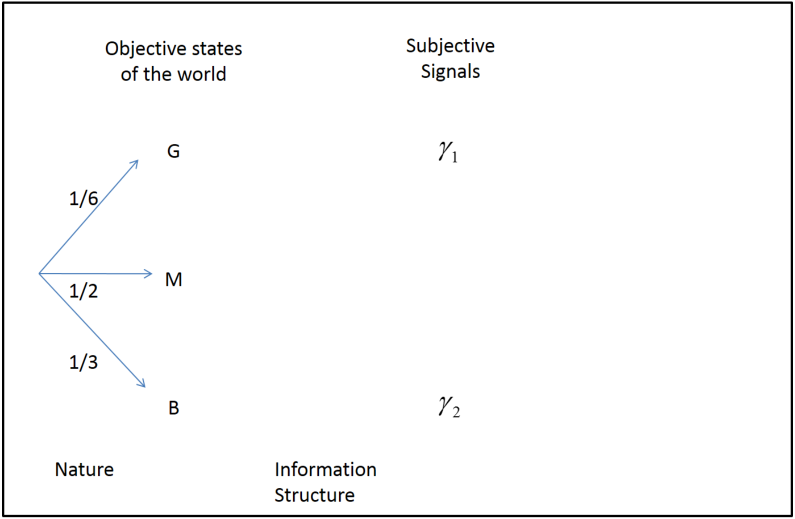
The Choice Problem

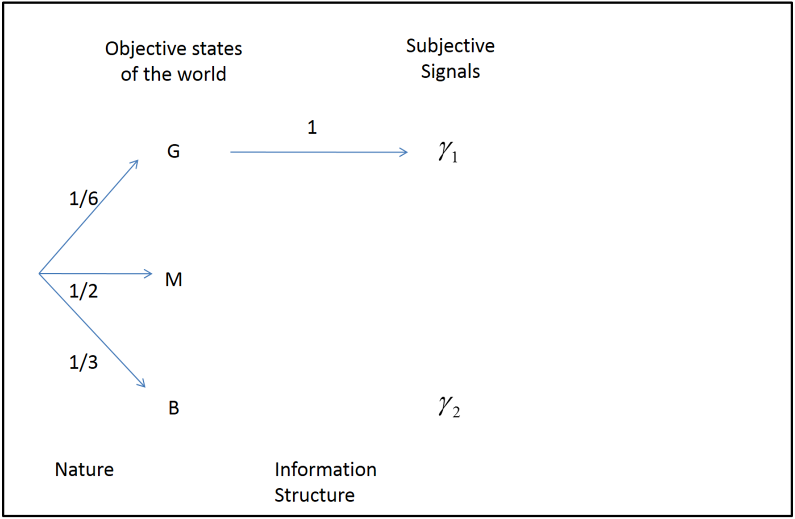
- The specifics of the process of information acquisition may be very complex
- We model the choice of information in an *abstract* way
- The decision maker chooses an *information structure*
  - Set of signals to receive
  - Probability of receiving each signal in each state of the world
- Choose action conditional on signal received
- Value of strategy given by
  - Expected value of actions taken given posterior beliefs
  - Minus cost of information
- Notice that this is an optimizing model with additional constraints
  - Subjects respond to costs and incentives
  - At least an interesting benchmark

The Choice Problem

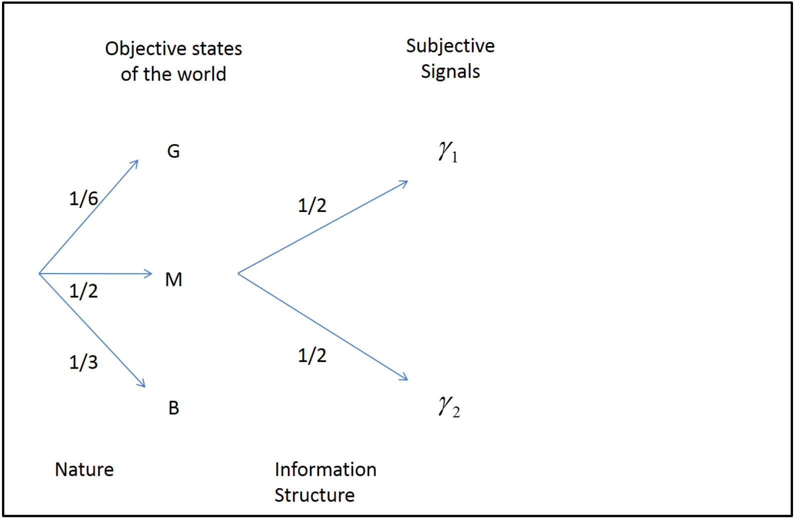


The Choice Problem

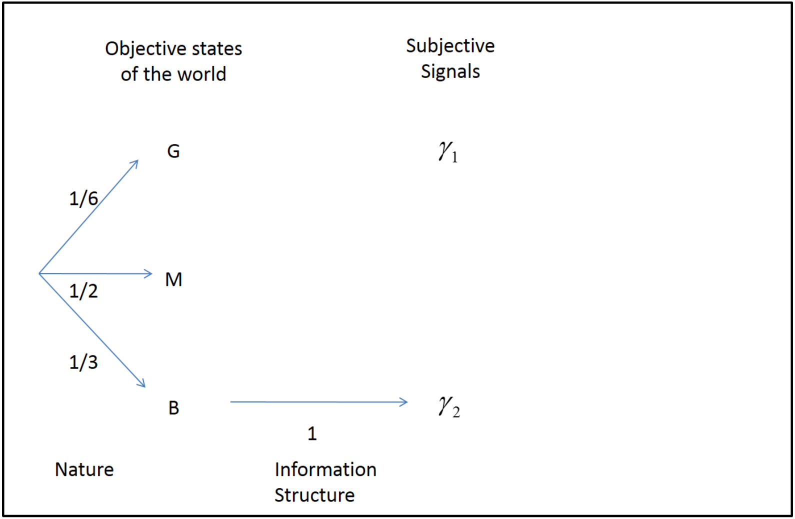


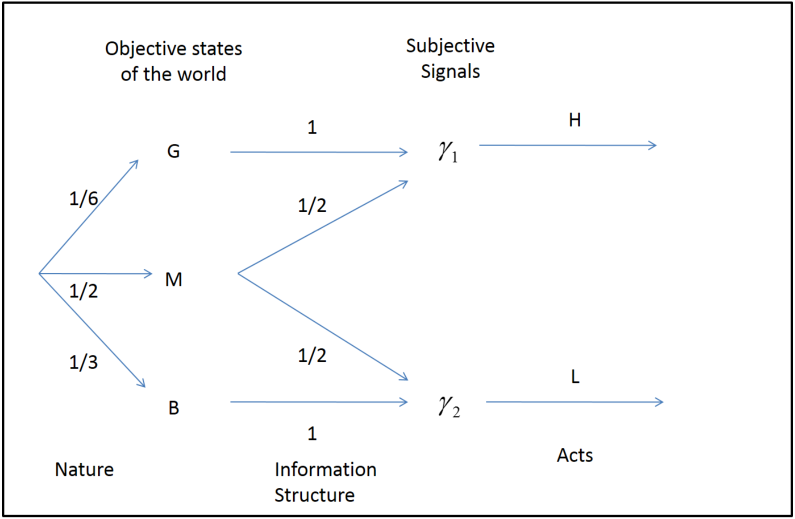

Objective states of the world
Subjective Signals
1
G
γ_1
1/6
M
1/2
1/3
B
γ_2
Nature
Information Structure

The Choice Problem



The Choice Problem

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1/6
1/2
1/3
1
1/2
1/2
1
γ_1
γ_2
H
L
Nature
Information Structure
Acts

Set Up

- Note that most ‘real world’ information gathering activities can be thought of in terms of as generating information structures
- E.g., say that you have developed a new economics class
- There are two possible states of the world
  - Class is good - $\frac{2}{3}$ of people like it on average
  - Class is bad - $\frac{1}{3}$ of people like it on average
- Each is equally likely
- Release a survey in which all 4 members of the class report if they like the class or not
- This generates an information structure
  - 5 signals: 0,1,2..... people say they like the class
  - Probability of each signal given each state of the world can be calculated

Set Up

- Ω: Objective states of the world (finite)
    - with prior probabilities μ
- a : An action - utility depends on the state
    - $U(a, \omega)$ utility of action a in state ω
    - $\mathcal{A}$: Set of actions:
- $A \subset \mathcal{A}$: Decision problem (finite)

- For each decision problem
  1. Choose information structure (π)
     - Defined by:
       - Set of signals: $\Gamma(\pi)$
       - Probability of receiving each signal γ from each state ω : $\pi(\gamma|\omega)$
  2. Choose action conditional on signal received (C)
     - $C(\gamma)$ probability distribution over actions given signal γ
- In order to maximize
  - Expected value of actions taken given posterior beliefs
  - Minus cost of information K

$$\sum_{\Omega} \mu(\omega) \sum_{\gamma \in \Gamma(\pi)} \pi(\gamma|\omega) \left(\sum_{a \in A} C(a|\gamma) U(a(\omega)) \right) - K(\mu, \pi)$$

The Value of An Information Structure

- What is the value of an information structure?
- In the end you will have to choose an *action*
  - Defined by the outcome it gives in each state of the world
- Assume in previous example, could choose three actions
  - set price H, A or L
- The following table could describe the profits each price gives at each demand level

	Price		
State	H	A	L
G	10	3	1
M	1	2	1
B	-10	-3	-1

The Value of An Information Structure

- What would you choose if you gathered no information?
    - i.e. if you had your prior beliefs

$$\mu(G) = \frac{1}{6}, \mu(M) = \frac{1}{2}, \mu(B) = \frac{1}{3}$$

- Calculate the expected utility for each act

$$\begin{aligned} \frac{1}{6}u(H,G) + \frac{1}{2}u(H.M) + \frac{1}{3}u(H,B) &= \frac{-7}{6} \\ \frac{1}{6}u(A,G) + \frac{1}{2}u(A,M) + \frac{1}{3}u(A,B) &= \frac{1}{2} \\ \frac{1}{6}u(L,G) + \frac{1}{2}u(L,M) + \frac{1}{3}u(L,B) &= \frac{1}{3} \end{aligned}$$

- Choose A
- Get utility $\frac{1}{2}$

The Value of An Information Structure

- What would you choose upon receiving signal γ_1?
- Depends on beliefs conditional on receiving that signal
- Can calculate this using Bayes Rule

$$
\begin{aligned}
P(G|\gamma_1) &= \frac{P(G \cap \gamma_1)}{P(\gamma_1)} \\
&= \frac{\mu(G)\pi(\gamma_1|G)}{\mu(G)\pi(\gamma_1|G) + \mu(M)\pi(\gamma_1|M) + \mu(B)\pi(\gamma_1|B)} \\
&= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{4} + 0} = \frac{2}{5}
\end{aligned}
$$

The Value of An Information Structure

- We can therefore calculate posterior beliefs conditional on signal R

$$\begin{aligned} P(G|\gamma_1) &= \frac{2}{5} = \gamma^1(G) \\ P(M|\gamma_1) &= \frac{3}{5} = \gamma^1(M) \\ P(B|\gamma_1) &= 0 = \gamma^1(B) \end{aligned}$$

- Where we use $\gamma^1(\omega)$ to mean the probability that the state of the world is ω given signal R

The Value of An Information Structure

- And calculate the value of choosing each act given these beliefs

$$\begin{aligned}
\frac{2}{5}u(H,G)+\frac{3}{5}u(H,M) &= \frac{23}{5} \\
\frac{2}{5}u(A,G)+\frac{3}{5}u(A,M) &= \frac{12}{5} \\
\frac{2}{5}u(L,G)+\frac{3}{5}u(L,M) &= \frac{2}{5}
\end{aligned}$$

The Value of An Information Structure

- If received signal γ_1, would choose H and receive $\frac{23}{5}$
- By similar process, can calculate that if received signal γ^2
  - Choose L and receive $-\frac{1}{7}$
- Can calculate the value of the information structure as

$$\begin{aligned} P(\gamma^1)\frac{23}{5} + P(\gamma^2)\frac{-1}{7} &= \\ \frac{5}{12}\frac{23}{5} + \frac{7}{12}\frac{-1}{7} &= \frac{11}{6} \end{aligned}$$

- How much would you pay for this information structure?

The Value of An Information Structure

- Value of this information structure is $\frac{11}{6}$
- Value of being uninformed is $\frac{1}{2}$
- Would prefer this information structure to being uninformed if cost is below $\frac{8}{6}$
- Note that the value of an information structure depends on the acts available

$$
\begin{aligned}
G(\pi, A) &= \sum_{\gamma \in \Gamma(\pi)} P(\gamma) g(\gamma, A) \\
g(\gamma, A) &= \max_{a \in A} \sum_{\omega \in \Omega} \gamma(\omega) u(a, \omega)
\end{aligned}
$$

- $g(\gamma, A)$ value of receiving signal γ if available actions are A
    - Highest utility achievable given the resulting posterior beliefs

- Easy to calculate the *value* of an information structure

$$G(A, \pi) = \max_{C:\Gamma(\pi)\rightarrow\Delta(A)} \sum_{\Omega} \mu(\omega) \sum_{\gamma\in\Gamma(\pi)} \pi(\gamma|\omega) \left(\sum_{a\in A} C(a|\gamma) U(a, \omega) \right)$$

  - Assuming you know utility
- But what is the correct information processing technology?
  - Choose variance of normal signal (e.g. Verrecchia 1982)?
  - Shannon mutual information costs (e.g. Sims 1998)?
  - Choose from set of available partitions (e.g. Ellis 2012)?
  - Sequential search (e.g. McCall 1970)?
- As usual, have two possible approaches
  1. Make further assumptions
  2. Ask if there is *any* cost function that can explain the data
- Today we take approach 2
- Next week we will follow approach 1

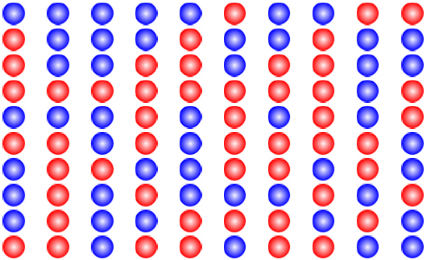
A Caveat

- We **will** assume throughout that costs are additively separable from utilities
- Is this assumption restrictive?
- Yes - see Chambers, Christopher P., Ce Liu, and John Rehbeck. "Nonseparable Costly Information Acquisition and Revealed Preference."
- Can you think of cases in which non-separability might be an important feature?

Data

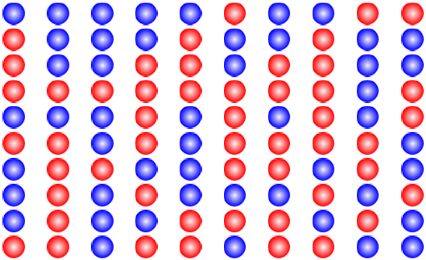
- Let D be a collection of decision problems
- What could we observe?
- Standard choice data
  - $C(A)$: what is chosen from A
- Stochastic choice data
  - $P_A(a)$: probability of choosing alternative a
- **State dependent stochastic choice data** P_A
  - $P_A(a|\omega)$ probability of choosing action a conditional on state ω
- Also assume we observe:
  - Prior probabilities μ
  - Utilities U
- Do **not** observe
  - Information structures π_A
  - Subjective signals γ
  - Information costs K

An Experimental Example



- Subjects presented with 100 balls
- State is determined by the number of red balls
- Prior distribution of red balls known to subject

An Experimental Example



Action	Payoff 49 red balls	Payoff 51 red balls
a	10	0
b	0	10

- No time limit: trade off between effort and financial rewards

An Experimental Example

- Data: State dependant stochastic choice
  - Probability of choosing each action in each objective state of the world

Action	**State = 49 red balls**	**State = 51 red balls**
Prob choose a	$P(a\|49)$	$P(a\|51)$
Prob choose b	$P(b\|49)$	$P(b\|51)$

- Observe subject making same choice 50 times
- Can use this to estimate P_A
  - But we will not be able to observe P_A perfectly
  - Will only be able to make probabilistic statements
- Can collect this type of data in the lab
  - What about outside?

Question

- What type of stochastic choice data $\{D, P\}$ is consistent with optimal information acquisition?
- i.e. there exists a cost function K
- For each decision problem $A \in D$ an information structure π_A and choice function C_A s.t.
  - C_A is optimal for each γ
  - π_A is optimal given K
  - C_A and π_A are consistent with P_A

$$P_A(a|\omega) = \sum_{\gamma \in \Gamma(\pi_A)} \pi_A(\gamma|\omega) C_A(a|\gamma).$$

- What 'mistakes' are consistent with optimal behavior in the face of information costs?

Notes

- This approach is very flexible
  - No in principle restriction on information structures
  - No restrictions on costs
- Nests other models of information acquisition
  - e.g. Shannon Mutual Information set costs to

$$K(\pi) = \lambda E\left(\log \frac{\mu(\omega)\pi(\gamma|\omega)}{\mu(\omega)\pi(\gamma)}\right)$$

- Can mimic a hard constraints
  - e.g. a model in which subjects choose the variance of a normal signal, set the cost of all other information structures to ∞

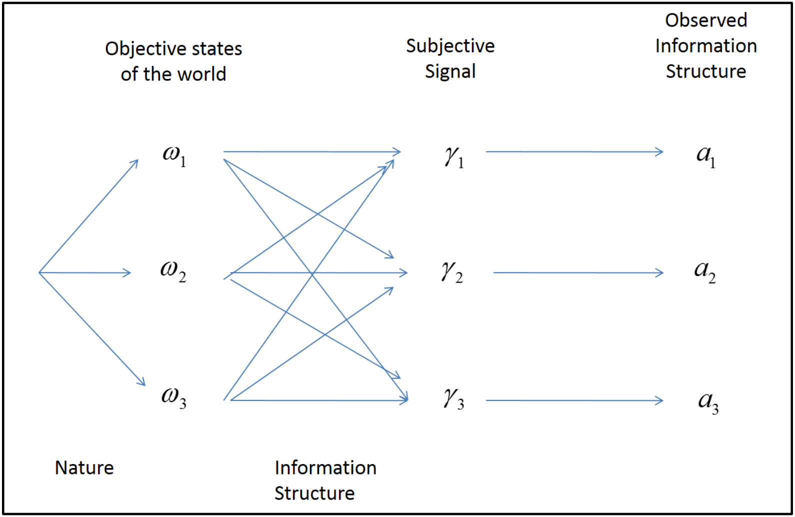
Observing Information Structures

- Key observation: State dependent stochastic choice data tells us a lot about the information structure a decision maker has used
- Assume that decision maker is 'well behaved'
  - Chooses each action in response to at most one signal
  - No mixed strategies - one action per signal
- Information structure can be observed directly from state dependent stochastic choice
  - For each chosen action a there is an associated signal $\bar{\gamma}^a$
  - Probability of signal $\bar{\gamma}^a$ in state ω is the same as the probability of choosing a in ω

$$\bar{\pi}(\bar{\gamma}^a|\omega) = P(a|\omega)$$

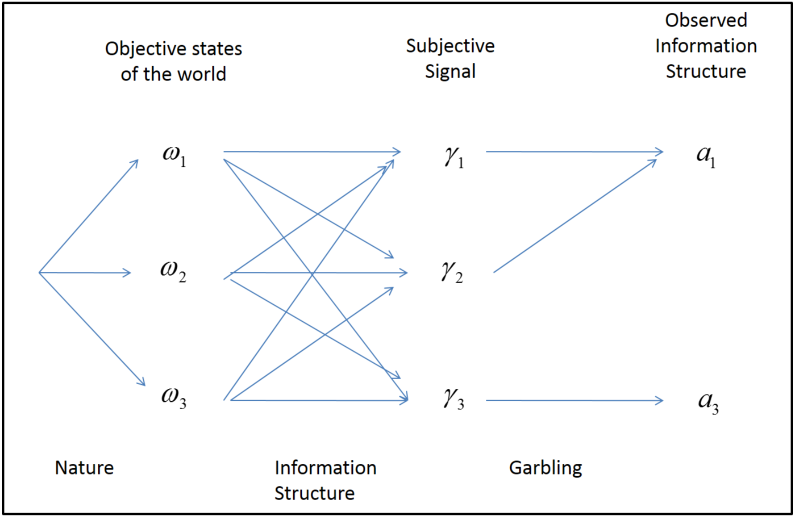
- Call $\bar{\pi}$ the 'revealed information structure'

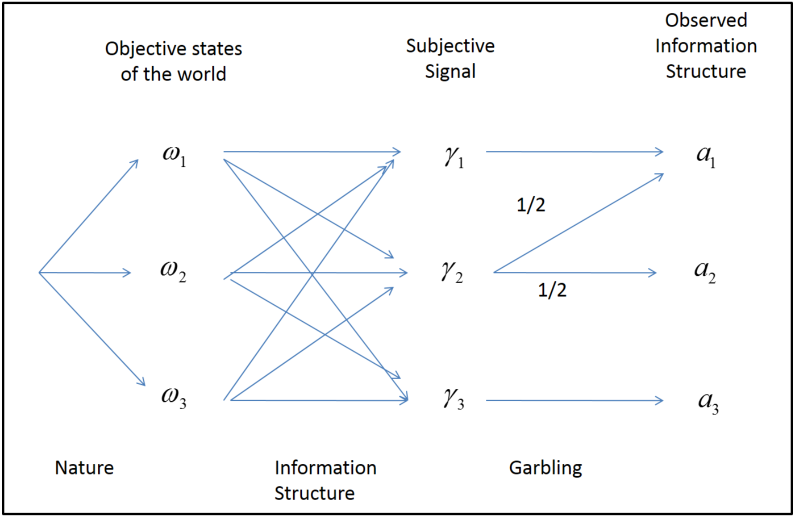
Recovering Attention Strategy



Observing Attentional Strategies

- What if decision maker is not well behaved?
  - Chooses some act in more than one subjective state
  - Mixed strategies - more than one act in an subjective state


Objective states of the world
Subjective Signal
Observed Information Structure
ω_1
ω_2
ω_3
γ_1
γ_2
γ_3
a_1
a_3
Nature
Information Structure
Garbling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a_2
a_3
1/2
1/2
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Information Structure
Garbling

Observing Information Structures

- Can still recover revealed information structure $\bar{\pi}$
- Not necessarily the same as true information structure π
- But will be a garbling of the true information structure
  - i.e. π is statistically sufficient for $\bar{\pi}$
- There exists a stochastic $|\Gamma(\pi)| \times |\Gamma(\bar{\pi})|$ matrix B such that if we
  - Apply π
  - For each state γ^i move to state $\bar{\gamma}^j$ with probability B^{ij}
  - We obtain $\bar{\pi}$
- i.e.

$$\begin{aligned} \sum_j B^{ij} &= 1 \ \forall\ j \\ \bar{\pi}(\bar{\gamma}^j|\omega) &= \sum_i B^{ij}\pi(\gamma^i|\omega) \ \forall\ j \end{aligned}$$

- Intuition: SDSC data cannot be more informative than the signal that created it

An Aside: Blackwell's Theorem

- Recall $G(A, \pi)$ is the *gross value* of using information structure π in decision problem A

$$G(A, \pi)$$
$$= \max_{C:\Gamma(\pi)\to\Delta(A)} \sum_{\Omega} \mu(\omega) \sum_{\gamma\in\Gamma(\pi)} \pi(\gamma|\omega) \left(\sum_{a\in A} C(a|\gamma) U(a(\omega)) \right)$$

- An information structure π is sufficient for information structure π' if and only if

$$G(A, \pi) \geq G(A, \pi') \ \forall \ A$$

- $\bar{\pi}$ may not be the agent's true information structure
  - But the true information structure π must be sufficient for $\bar{\pi}$
  - π will be at least as valuable as $\bar{\pi}$ in any decision problem
- Turns out that this is all we need

Characterizing Rational Inattention

- Choice of act optimal given attentional strategy
- Choice of attention strategy optimal

- Choice of act optimal given attentional strategy
- Choice of attention strategy optimal

Optimal Choice of Action

- We need to ensure that the DM is making optimal choices **conditional on the information the recieved**
- Note that this is a property required of many models outside the RI class as well

Action	**Payoff 49 red balls**	**Payoff 51 red balls**
$\mathbf{a}^1$	20	0
$\mathbf{b}^1$	0	10

Prior: $\{0.5, 0.5\}$

Action	**State = 49 red balls**	**State = 51 red balls**
Prob choose *a*	$\frac{1}{2}$	$\frac{1}{3}$
Prob choose *b*	$\frac{1}{2}$	$\frac{2}{3}$

- Posterior probability of 49 red balls when action b was chosen

$$\begin{aligned}\Pr(\omega &= 49 | b \text{ chosen}) = \frac{\Pr(\omega = 49, b \text{ chosen})}{\Pr(b \text{ chosen})} \\ &= \frac{\frac{1}{4}}{\frac{1}{4} + \frac{2}{6}} = \frac{3}{7}\end{aligned}$$

- But for this posterior

$$\begin{aligned}\frac{3}{7}U(a(49)) + \frac{4}{7}U(a(51)) &= \frac{3}{7}20 + \frac{4}{7}0 = 8.6 \\ \frac{3}{7}U(b(49)) + \frac{4}{7}U(b(51)) &= \frac{3}{7}0 + \frac{4}{7}10 = 5.7\end{aligned}$$

- To avoid such cases requires

$$a \in \arg\max_{a \in A} \sum_{\Omega} \Pr(\omega|a) U(a(\omega))$$

- Which implies

Condition 1 (No Improving Action Switches) For every chosen action a

$$\sum \mu(\omega) P_A(a|\omega) \left[u(a(\omega)) - u(b(\omega))\right] \geq 0.$$

for all $b \in A$

- If $\bar{\pi}$ not true information structure, condition still holds
  - a optimal at all posteriors in which it is chosen
  - Must also be optimal at convex combination of these posteriors

- Choice of act optimal given attentional strategy
- Choice of attention strategy optimal

Action	**Payoff 49 red balls**	**Payoff 51 red balls**
$\mathbf{a}^1$	10	0
$\mathbf{b}^1$	0	10

Prior: $\{0.5, 0.5\}$

Action	**State = 49 red balls**	**State = 51 red balls**
Prob choose a	$\frac{3}{4}$	$\frac{1}{4}$
Prob choose b	$\frac{1}{4}$	$\frac{3}{4}$

Action	**Payoff 49 red balls**	**Payoff 51 red balls**
$\mathbf{a}^2$	20	0
$\mathbf{b}^2$	0	20

Prior: $\{0.5, 0.5\}$

Action	**State = 49 red balls**	**State = 51 red balls**
Prob choose a	$\frac{2}{3}$	$\frac{1}{3}$
Prob choose b	$\frac{1}{3}$	$\frac{2}{3}$

- $G(A,\pi)$ is the gross value of using information structure π in decision problem A

G	$\bar{\pi}^1$	$\bar{\pi}^2$
$\{a^1,b^1\}$	$7\frac{1}{2}$	$6\frac{2}{3}$
$\{a^2,b^2\}$	15	$13\frac{1}{3}$

- Cost function must satisfy

$$\begin{aligned} G(\{a^1,b^1\},\pi^1)-K(\pi^1) &\geq G(\{a^1,b^1\},\pi^2)-K(\pi^2) \\ G(\{a^2,b^2\},\pi^2)-K(\pi^2) &\geq G(\{a^2,b^2\},\pi^1)-K(\pi^1) \end{aligned}$$

- Which implies

$$\frac{5}{6}=G(\{a^1,b^1\},\pi^1)-G(\{a^1,b^1\},\pi^2)\geq$$
$$K(\pi^1)-K(\pi^2)\geq$$
$$G(\{a^2,b^2\},\pi^1)-G(\{a^2,b^2\},\pi^2)=1\frac{2}{3}$$

Optimal Choice of Attention Strategy

- Surplus must be maximized by correct assignments

$$G(\{a^1, b^1\}, \pi^1) + G(\{a^2, b^2\}, \pi^2)$$
$$\geq G(\{a^1, b^1\}, \pi^2) + G(\{a^2, b^2\}, \pi^1)$$

- What if $\bar{\pi} \neq \pi$?
- We know that revealed and true information structure must give same value in DP it was observed

$$G(A^i, \bar{\pi}^i) = G(A^i, \pi^i)$$

- Also, as π weakly Blackwell dominates $\bar{\pi}$

$$G(A^i, \bar{\pi}^j) \leq G(A^i, \pi^j)$$

Optimal Choice of Attention Strategy

- Surplus must be maximized by correct assignments

$$G(\{a^1, b^1\}, \bar{\pi}^1) + G(\{a^2, b^2\}, \bar{\pi}^2)$$
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Optimal Choice of Attention Strategy

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Condition 2

- To guarantee the existence of a cost function requires a stronger condition

Condition 2 (No Improving Attention Cycles) For an observed sequence of decision problems $A^1 ... A^K$ and associated revealed information structures $\bar{\pi}^1 ... \bar{\pi}^K$

$$
\begin{aligned}
& G(A^1, \bar{\pi}^1) - G(A^1, \bar{\pi}^2) \\
& + G(A^2, \bar{\pi}^2) - G(A^2, \bar{\pi}^3) \\
& + ... \\
& + G(A^K, \bar{\pi}^K) - G(A^K, \bar{\pi}^1) \\
\geq \; & 0
\end{aligned}
$$

- Note that this condition relies only on observable objects

Theorem

For any data set $\{D, P\}$ the following two statements are equivalent

1. $\{D, P\}$ *satisfy NIAS and NIAC*
2. *There exists a* $K : \Pi \rightarrow \mathbb{R}$, $\{\pi^A\}_{A \in D}$ *and* $\{C^A\}_{A \in D}$ *such that* π^A *and* $C^A : \Gamma(\pi^A) \rightarrow A$ *are optimal and generate* P^A *for every* $A \in D$

Proof.

$2 \rightarrow 1$ *Trivial*

$1 \rightarrow 2$ *Rochet [1987] (literature on implementation)* □

Proof

- This problem is familiar from the implementation literature
- Say there were a set of environments $X_1....X_N$ and actions $B_1....B_M$ such that the utility of each environment and each state is given by

$$u(X_i, B_j)$$

- Say we want to implement a mechanism such that action $Y(X_i)$ is taken at in each environment.
- We need to find a taxation scheme $\tau : B_1....B_M \to \mathbb{R}$ such that

$$\begin{aligned} u(X_i, Y(X_i)) - \tau(Y(X_i)) \quad &\geq \quad u(X_i, B) - \tau(B) \\ &\quad\ \forall\ B_1....B_M \end{aligned}$$

- This is the same as our problem.

Proof

- Our problem is equivalent to finding $\theta : D \rightarrow \mathbb{R}$, such that, for all A_i, $A_j \in D$

$$G(A_i, \pi^i) - \theta(A_i) \geq G(A_i, \pi^j) - \theta(A_j)$$

  - Just define $K(\pi) = \theta(A_i)$ if $\pi = \pi^i$ for some i, or $= \infty$ otherwise
- We can apply a proof from Rockerfellar [1970] to show that NIAC gives us this condition

Proof

- Pick some arbitrary A_0 and define

$$T(A) = \sup_{\textit{all chains } A_0 \textit{ to } A=A_M} \sum_{n=0}^{M-1} G(A_{i+1}, \pi^i) - G(A_i, \pi^i)$$

- NIAC implies that $T(A_0) = 0$
- Also note that

$$T(A_0) \geq T(A_i) + G(A_0, \pi^i) - G(A_i, \pi^i)$$

- So $T(A_i)$ is bounded

Proof

- Furthermore, for any A_i A_j we have

$$T(A_i) \geq T(A_j) + G(A_i, \pi^j) - G(A_j, \pi^j)$$

- So, setting $\theta(A_j) = G(A_j, \pi^j) - T(A_j)$, we get

$$G(A_i, \pi^i) - \theta(A_i) \geq G(A_i, \pi^j) - \theta(A_j)$$

- So far we have been completely agnostic about the cost function
- Perhaps we want to impose some more structure
    - e.g. information structure that are more (Blackwell) Informative are (weakly) more expensive
- Turns out we get this ‘for free’
- Say we observe π^A in A and π^B in B such that π^A is sufficient for π^B
- It must be the case that

$$\begin{aligned} G(B,\pi^B) - K(\pi^B) &\geq G(B,\pi^A) - K(\pi^A) \Rightarrow \\ K(\pi^A) - K(\pi^B) &\geq G(B,\pi^A) - G(B,\pi^B) \end{aligned}$$

- But by Blackwell's theorem

$$G(B,\pi^A) \geq G(B,\pi^B)$$

Restrictions on the Cost Function

- Any behavior that can be rationalized can be rationalized with a cost function that
  - Is weakly monotonic with respect to Blackwell?
  - Allows mixing
  - Positive with free inattention
- Reminiscent of Afriat's theorem
- Can also extend to 'sequential rational inattention'

Recovering Costs

- Say $\bar{\pi}^A$ is the revealed attn. strategy in decision problem A.
- Assuming weak monotonicity, it must be that

$$K(\bar{\pi}^A) - K(\pi) \leq G(A, \bar{\pi}^A) - G(A, \pi)$$

- If $\bar{\pi}^B$ is used in decision problem B then we can bound relative costs

$$G(B, \bar{\pi}^A) - G(B, \bar{\pi}^B) \leq K(\bar{\pi}^A) - K(\bar{\pi}^B) \leq G(A, \bar{\pi}^A) - G(A, \bar{\pi}^B)$$

- Tighter bounds can be obtained using chains of observations

$$\begin{aligned} & \max_{\{A^1 \ldots A^n \in D | A^1 = B, A^n = A\}} \sum \left[G(A^i, \bar{\pi}^{A^i}) - G(A^i, \bar{\pi}^{A^{i+1}}) \right] \\ \leq \quad & K(\bar{\pi}^A) - K(\bar{\pi}^B) \\ \leq \quad & \min_{\{A^1 \ldots A^n \in D | A^1 = A, A^n = B\}} \sum \left[G(A^i, \bar{\pi}^{A^i}) - G(A^i, \bar{\pi}^{A^{i+1}}) \right] \end{aligned}$$

What If Utility and Priors Are Unobservable?

- Can add 'there exists' to the statement of the NIAS and NIAC conditions
- Data has an optimal costly attention representation if **there exists** $\mu \in \Delta(\Omega)$ and $U : X \rightarrow \mathbb{R}$ such that
  - NIAS is satisfied
  - NIAC is satisfied
- If μ is known but U is unknown, conditions are linear and (relatively) easy to check
- If μ and U are unknown, conditions are harder to check
  - Still not vacuous
- Alternatively, can enrich data so that these objects can be recovered

Rational Inattention vs Random Utility

- Alternative model of random choice: Random Utility
  1. Agent receives some information about the state of the world
  2. Draws a utility function from some set
  3. Chooses in order to maximize utility given information
- Key differences between Random Utility and Rational Inattention
  1. Random Utility allows for multiple utility functions
  2. Rational Inattention allows attention to vary with choice problem
- How can we differentiate between the two?

- Random Utility implies monotonicity
  - In fact, fully characterized by Block Marschak monotonicity
- For any two decision problems $\{A, A \cup b\}$, $a \in A$ and $b \notin A$

$$P_A(a|\omega) \geq P_{A\cup b}(a|\omega)$$

- Rational Inattention can lead to violations of monotonicity (Ergin, Matejka and McKay)

Act	**Payoff 49 red dots**	**Payoff 51 red dots**
a	23	23
b	20	25
c	40	0

- Adding act c to $\{a, b\}$ can increase the probability of choosing b in state 51

Other Approaches

- There are lots of other papers testing the rational inattention hypothesis for specific cost functions:
  - Shannon mutual information (e.g. Sims 2003)
  - Shannon capacity (e.g. Woodford 2012)
  - Choice of optimal partitions (Ellis 2012)
  - All or nothing (Reis 2006)
- We will talk (in particular) about mutual information next week.

de Oliveira et al [2017]

- One other paper considers optimal information acquisition without making any assumption about the cost functions
- Rather than state dependant stochastic choice data, uses preferences over menus
  - i.e would you prefer to make a choice for menu A or menu B
- Timeline is as follows
  - Choose between menu
  - State resolves itself
  - Choose what information processing to do
  - Choose an alternative based on signal

de Oliveira et al [2017]

- Two key conditions for rational inattention

1. Preference for Flexibility
   - $A \cup \{a\} \succeq A$
   - Always prefer to have more options
   - Note relation to 'too much choice'
2. Preference for Early Resolution of Uncertainty
   - Define $\frac{1}{2}$ mixture of A and B as

$$\left\{ c = \frac{1}{2}a + \frac{1}{2}b | a \in A, b \in B \right\}$$

   - Choosing from $\frac{1}{2}A + \frac{1}{2}B$ is like choosing from A, choosing from B then flipping a coin to see which choice you get
   - This is costly from an informational standpoint

$$\begin{aligned} A &\sim B \Rightarrow \\ A &\succeq \frac{1}{2}A + \frac{1}{2}B \end{aligned}$$