Rational Reference Dependence

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- In the previous lecture we considered models in which the effect of reference points was 'psychological'
- Affected preferences
  - Loss aversion
- Or the choice procedure given preferences
  - SQB model
  - Choice overload
- In neither case was the effect of reference points due to some optimal procedure
- In this lecture we will consider two models in which reference dependence is rational
  - Allows for interesting comparative static predictions
  - Makes welfare analysis easier

- The most obvious cause of reference dependence is transaction costs
    - It costs me an amount c to move away from the status quo option
    - Utility of alternative x is $u(x)$ if it is the status quo, $u(x) - c$ otherwise
- Because there is nothing 'psychological' about the impact of reference points, makes welfare analysis staightforward
    - Want to maximize utility net of transaction costs

- We can think of the design problem of a social planner choosing the default in order to maximize welfare of an agent
- In the case of a single agent whose preferences are known, the problem is trivial
- Set the default equal to the highest utility alternative
- Carrol et al [2009] make the problem more interesting in three ways
  - Several agents, each with potentially different rankings
  - Each agent's ranking is not observable to the social planner
  - Agent has quasi-hyperbolic discount function, but the social planner wants to maximize exponentially discounted utility

The Agent's Problem

- Agent lives for an infinite number of periods
- They start life with a default savings rate d
- They have an optimal savings rate s
- In any period in which they have a savings rate d they suffer a loss

$$L = \kappa(s - d)^2$$

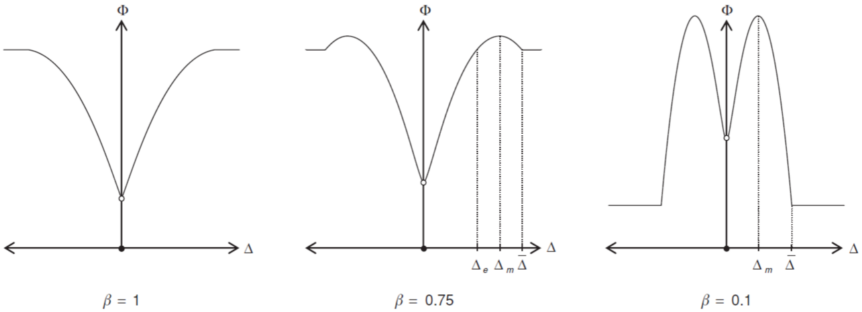
- In any period they can change to their optimal savings rate at cost c
  - Cost drawn in each period drawn from a uniform distribution
- Discounted utility given by quasi-hyperbolic function of expected future losses

The Agent's Problem

- Restrict attention to stationary equilibria
- Agent has a fixed c^*
- Will switch to the optimal savings rate if $c < c^*$
- c^* is
    - Increasing in β
    - Decreasing in $|s - d|$

The Planner's Problem

- Facing a population of agents drawn from a uniform distribution on $[s_*, s^*]$
- Cannot observe s
- Wishes to choose d in order to minimize expected, exponentially discounted loss of the population
- Has to take into account two trade offs
    - A default that is good for one agent may be bad for another
    - A default that is too good may lead present-biased agents to procrastinate

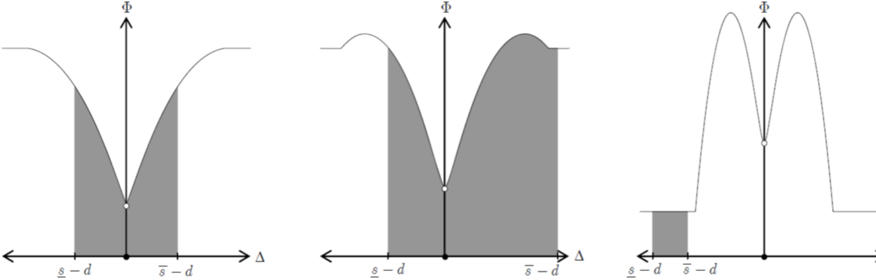


- Expected total loss (from the planner's point of view) based on the distance between default and optimal savings rate
  - If $\beta = 1$ always better to have default closer to optimal
  - if $\beta < 1$ may be better to have default further away to overcome procrastination

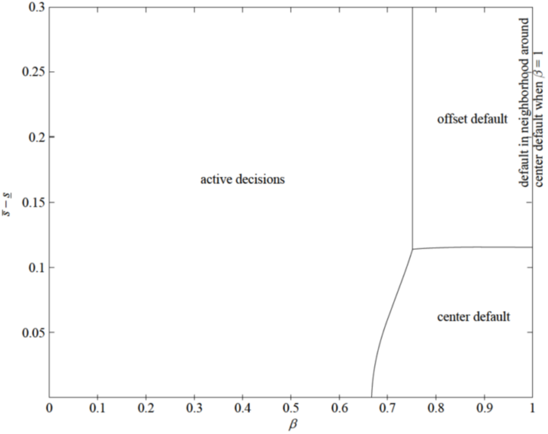
The Planner's Problem

- Leads to three possible optimal policy regimes
  - Center default - minimize the expected distance between s and d
  - Offset default - Encourage the most extreme agents to make active decisions
  - Active decisions - Set a default so bad that all agents to move away from the default.

The Planner's Problem

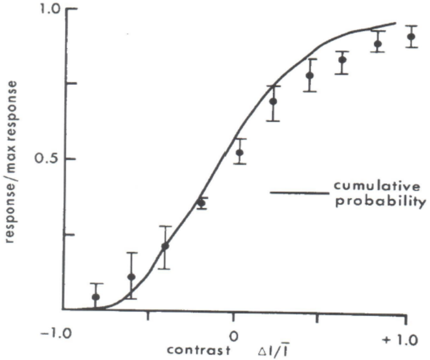


$\beta = 1$, $\overline{s} - \underline{s} = 0.1$ $\beta = 0.75$, $\overline{s} - \underline{s} = 0.25$ $\beta = 0.1$, $\overline{s} - \underline{s} = 0.15$


active decisions
offset default
center default
default in neighborhood around
center default when β = 1
β
s̄ − s̲
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.05
0.1
0.15
0.2
0.25
0.3

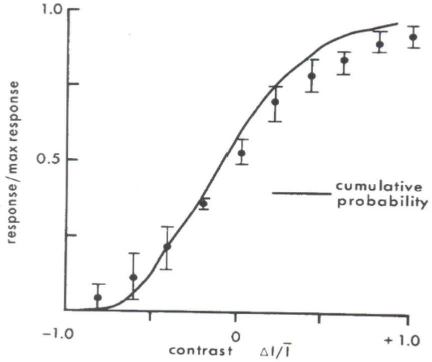
Reference Points and Optimal Coding

- In previous lectures we showed that context effects could change how information is encoded in the brain
- Could this be a rational use of neural resources?
  - Focus attention where it is most useful
- If so, may be a role for reference points affecting valuation and therefore choice
  - Reference points tell us what is most likely to happen
  - and so where it is most likely to be useful to make fine judgements
- This hypothesis is explored in Woodford [2012]



- Shows neural response to contrast differences in light sources (black dots)
- Also CDF of contrast differences in blowfly environment (line)

A Detour Regarding Blowflys



- Sharpest distinction occurs between contrasts which are likely to occur
- i.e slope of line matches the 'slope' of the dots

- Blowflies seem to use neural resources to best differentiate between states that are most likely to occur
- Does this represent 'optimal' use of resources?
- Surprisingly not if costs are based on Shannon mutual information
- Why not?

The Effect of Priors

- Remember Shannon Mutual Information costs can be written as

$$-\left[H(\Gamma) - E(H(\Gamma|\Omega))\right]$$

$$\sum_{\gamma \in \Gamma(\pi)} P(\gamma) \ln P(\gamma) - \sum_{\omega} \mu(\omega) \left(\sum_{\gamma \in \Gamma(\pi)} \pi(\gamma|\omega) \ln \pi(\gamma|\omega) \right)$$

where

$$P(\gamma) = \sum_{\omega \in \Omega} \pi(\gamma|\omega)\mu(\omega)$$

- Changing the precision of a signal in a given state (i.e. $\pi(\gamma|\omega)$) changes info costs by

$$\left(\ln(P(\gamma)) + 1\right) \frac{\partial P(\gamma)}{\partial \pi(\gamma|\omega)} - \mu(\omega) \left(\ln(\pi(\gamma|s) + 1\right)$$

The Effect of Priors

- But $\frac{\partial P(\gamma)}{\partial \pi(\gamma|\omega)} = \mu(\omega)$, so

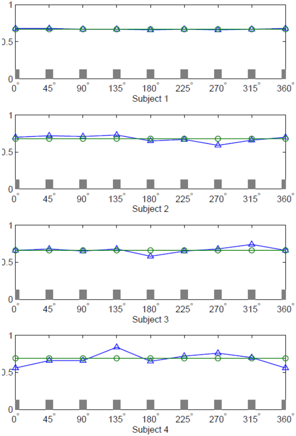
$$\mu(\omega)\left(\ln(P(\gamma)) - \ln(\pi(\gamma|s)\right)$$

- It is cheaper to get information about states that are less likely to occur
  - Intuition: you only pay the expected cost of information
  - Expected cost information about states that are unlikely to occur is low
- This offsets the lower value of gathering information about such states
  - Prior probability of state should not matter for optimal coding

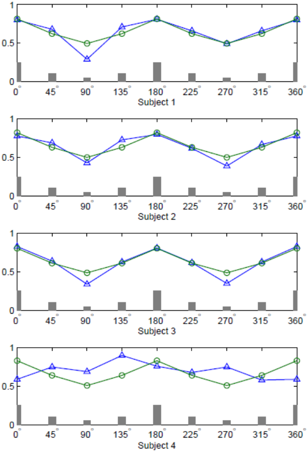
The Effect of Priors

- Does this hold up in practice?
- Experiment: Shaw and Shaw [1977]
  - Subjects had to report which of three letters had flashed onto a screen
  - Letter could appear at one of 8 locations (points on a circle)
- Two treatments
  - All positions equally likely
  - 0 and 180 degrees more likely
- Shannon prediction: behavior the same in both cases

Shaw and Shaw [1977]: Treatment 1



Shaw and Shaw [1977]: Treatment 2



- This observation lead Woodford [2012] to consider an alternative cost function
  - Shannon Capacity
- Let

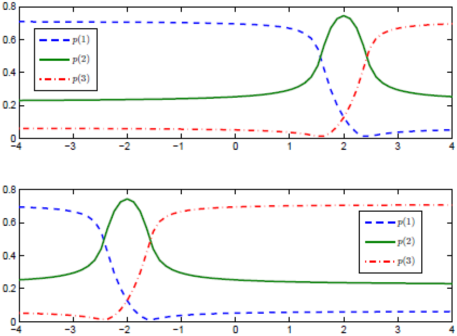
$$I_\mu(\Gamma, \Omega)$$

  be the Mutual Information between signal and state under prior beliefs μ
- Shannon Capacity is given by

$$\max_{\mu \in \Delta(\Omega)} I_\mu(\Gamma, \Omega)$$

- i.e. the maximal mutual information across all possible prior beliefs
- True priors no longer affect costs
- Signals on less likely states no cheaper than signals on more likely states

Shannon Capacity



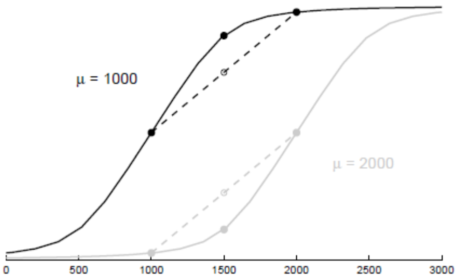
- Optimal behavior when objective is linear in squared error
- Upper panel prior is $N(2, 1)$, lower panel prior is $N(-2, 1)$

Coding Values

- One can apply this model to economic choice
- Assume that DM have to encode the value of a given alternative
- As in previous lectures can assume alternative is characterized along different dimensions
- Has a limited capacity to encode value along each dimension
- Chooses optimal encoding given costs, prior beliefs and the task at hand

- This model can explain diminishing sensitivity
  - But not, in an obvious way, loss aversion
- Remember, diminishing sensitivity predicts
  - Risk aversion for gains
  - Risk seeking for losses
- E.g.
  - Choice 1: start with 1000, choose between a gain of 500 for sure or a 50% chance of a gain of 1000
  - Choice 2: start with 2000, choose between a loss of 500 for sure or a 50% chance of a loss of 1000

- Assume that the change in the reference point changes the prior distribution over final outcomes
  - Choice 2 has a mean which is 1000 higher than choice 1
  - Assume that prior is normal
- In Choice 1 1000 most likely, then 1500, then 2000
  - 1000 most precisely encoded, then 1500 then 2000
  - More 'sensitive' to the change between 1000 and 1500 than between 1500 and 2000
  - Leads to risk aversion
- In Choice 2 2000 most likely, then 1500, then 1000
  - 2000 most precisely encoded, then 1500 then 1000
  - More 'sensitive' to the change between 2000 and 1500 than between 1500 and 1000
  - Leads to risk loving



- Plot of Mean Squared Normalized Value under the two different coding schemes

Summary

- Treating reference dependence as optimal can have benefits
  - Allows for welfare analysis
  - Provides new comparative static pedictions
- Examples
  - Optimal defaults
  - Adaptive coding
- Does, of course, depend on whether reference dependence is optimal