# Models of Reference Dependent Preferences 

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Behavioral Economics G6943<br>Autumn 2018

## Modelling Reference Dependence

- Likely that there are many different causes of reference dependence
- As we discussed in the introduction
- Broadly speaking two classes of models
(1) Preference-based reference dependence
- Reference points affect preferences which affect choices
(2) 'Rational' reference dependence
- Reference dependence as a rational response to costs
- Effort costs
- Attention Costs
- Focus on the former, say a little about the latter


## Loss Aversion

- In 1979 Kahneman and Tversky introduced the idea of 'Loss Aversion'
- Basic idea: Losses loom larger than gains
- Utility calculated on changes, not levels
- The magnitude of the utility loss associated with losing $x$ is greater than the utility gain associated with gaining $x$
- Initially applied to risky choice
- Later also applied to riskless choice [Tversky and Kahneman 1991]
- Can explain
- Endowment effect
- Increased risk aversion for lotteries involving gains and losses
- Status quo bias


## A Simple Loss Aversion Model

- World consists of different dimensions
- e.g cash and mugs
- Will be asked to choose between alternatives that provide different amount of each dimension

$$
\binom{x_{c}}{x_{m}}
$$

- Has a reference point for each dimension

$$
\binom{r_{c}}{r_{m}}
$$

- Key Point: Utility depends on changes, not on levels


## A Simple Loss Aversion Model

- Utility of an alternative comes from comparison of output to reference point along each dimension

$$
\binom{x_{c}}{x_{m}},\binom{r_{c}}{r_{m}}
$$

- Utility for gains relative to $r$ given by a utility function $u$

$$
\begin{aligned}
u_{c}\left(x_{c}-r_{c}\right) \text { if } x_{c} & >r_{c} \\
u_{m}\left(x_{m}-r_{m}\right) \text { if } x_{m} & >r_{m}
\end{aligned}
$$

- Utility of losses relative to $r$ given buy $u$ of the equivalent gain multiplied by $-\lambda$ with $\lambda>1$

$$
\begin{aligned}
-\lambda u_{c}\left(r_{c}-x_{c}\right) \text { if } x_{c} & <r_{c} \\
-\lambda u_{m}\left(r_{m}-x_{m}\right) \text { if } x_{m} & <r_{m}
\end{aligned}
$$



- $x$ is a gain of $\$ 1$ and loss of 1 mug relative to $r$
- Utility of $x$

$$
u_{c}(1)-\lambda u_{m}(1)
$$

## Loss Aversion and the Endowment Effect

- How can loss aversion explain the Endowment Effect (i.e. WTP/WTA gap)?
- Willingness to pay:
- Let $\left(r_{c}, r_{m}\right)$ be the reference point with no mug
- How much would they be willing to pay for the mug?
- i.e. what is the $z$ such that

$$
0=U\left(\begin{array}{cc}
r_{c} & r_{c} \\
r_{m} & r_{m}
\end{array}\right)=U\left(\begin{array}{cc}
r_{c}-z & r_{c} \\
r_{m}+1 & r_{m}
\end{array}\right)
$$

- Assume linear utility for money
- Utility of buying a mug given by

$$
U\left(\begin{array}{cc}
r_{c}-z & r_{c} \\
r_{m}+1 & , \\
r_{m}
\end{array}\right)=u_{m}(1)-\lambda z
$$

- Break even buying price given by $z=\frac{u_{m}(1)}{\lambda}$


## A Simple Loss Aversion Model



- Buying is a loss of $\$ \mathrm{z}$ and gain of 1 mug relative to $r$
- Utility of buying

$$
u_{m}(1)-\lambda z
$$

## Loss Aversion and the Endowment Effect

- Willingness to accept:
- Let $\left(r_{c}, r_{m}\right)$ be the reference point with mug
- How much would they be willing to sell your mug for?
- i.e. what is the $y$ such that

$$
0=U\left(\begin{array}{ll}
r_{c} & r_{c} \\
r_{m}, & r_{m}
\end{array}\right)=U\left(\begin{array}{ll}
r_{c}+y & r_{c} \\
r_{m}-1, & r_{m}
\end{array}\right)
$$

- Assume linear utility for money
- Utility of selling a mug given by

$$
U\left(\begin{array}{cc}
r_{c}+y & r_{c} \\
r_{m}-1, & r_{m}
\end{array}\right)=-\lambda u_{m}(1)+y
$$

- Break even selling price given by $y=\lambda u_{m}(1)$


## A Simple Loss Aversion Model



- Selling is a gain of $\$ y$ and loss of 1 mug relative to $r$
- Utility of selling

$$
-\lambda u_{m}(1)+y
$$

## Loss Aversion and the Endowment Effect

- Willingness to pay

$$
z=\frac{u_{m}(1)}{\lambda}
$$

- Willingness to accept

$$
y=\lambda u_{m}(1)
$$

- WTP/WTA ratio

$$
\frac{z}{y}=\frac{1}{\lambda^{2}}
$$

- Less that 1 for $\lambda>1$


## Axiomatization

- Tversky and Kahneman [1991] provide an axiomatization of a (closely related) model

Axiom 1: Cancellation if, for some reference point

$$
\binom{x_{1}}{z_{2}} \succeq\binom{z_{1}}{y_{2}} \text { and }\binom{z_{1}}{x_{2}} \succeq\binom{y_{1}}{z_{2}}
$$

then

$$
\binom{x_{1}}{x_{2}} \succeq\binom{y_{1}}{y_{2}}
$$

- (guarantees additivity)


## Axiomatization

- Define the 'quadrant' that $x$ is in relative to $r$



## Axiomatization

Axiom 2: Sign Dependence Let options $x$ and $y$ and reference points $s$ and $r$ be such that
(1) $x$ and $y$ are in the same quadrant with respect to $r$ and with respect to $s$
(2) $s$ and $r$ are in the same quadrant with respect to $x$ and with respect to $y$
Then $x \succeq y$ when $r$ is the status quo $\Longleftrightarrow x \succeq y$ when $s$ is the status quo

- Guarantees that only the 'sign' matters


## Axiomatization

Axiom 3: Preference Interlocking Say that, for some reference point $r$, we saw that

$$
\binom{x_{1}}{x_{2}} \sim\binom{w_{1}}{w_{2}} \text { and }\binom{z_{1}}{x_{2}} \sim\binom{y_{1}}{w_{2}}
$$

And, for another reference point $s$ (that puts everything in the same quadrant, but maybe a different quadrant to $r$ )

$$
\begin{aligned}
& \binom{x_{1}}{\bar{x}_{2}} \sim\binom{w_{1}}{\bar{w}_{2}} \Rightarrow \\
& \binom{z_{1}}{\bar{x}_{2}} \sim\binom{y_{1}}{\bar{w}_{2}}
\end{aligned}
$$

- Ensures that the same trade offs that work in the gain domain also work in the loss domain


## Loss Aversion in Risky Choice

- Loss aversion can also lead to increased risk aversion for lotteries that involve gains and losses
- Now there is only 1 dimension (money)
- Lotteries evaluated as gains/losses relative to some reference point
- See also Kosegi and Rabin [2007]
- Again, assume linear utility for money
- Utility of winning $x$ is $x$
- Utility of losing $x$ is $-\lambda x$

Loss Aversion in Risky Choice


## Loss Aversion in Risky Choice

- What is the certainty equivalence of
- $50 \%$ chance of gaining $\$ 10$
- $50 \%$ chance of gaining $\$ 0$
- $x$ such that

$$
\begin{aligned}
u_{c}(x) & =0.5 \times u_{c}(10)+0.5 \times u_{c}(10) \\
x & =0.5 \times 10+0.5 \times 0 \\
& =\$ 5
\end{aligned}
$$

- What is the certainty equivalence of
- $50 \%$ chance of gaining $\$ 5$
- $50 \%$ chance of losing $\$ 5$
- $y$ such that

$$
\begin{aligned}
-\lambda u_{c}(-y) & \left.=0.5 \times u_{c}(5)+0.5 \times(-\lambda)\right) u_{c}(5) \\
-\lambda y & =0.5 \times 5-\lambda 0.5 \times 5 \\
y & =\frac{(1-\lambda)}{\lambda}<0
\end{aligned}
$$

Loss Aversion in Risky Choice


## A Unified Theory of Loss Aversion?

- We have claimed that loss aversion can explain
- Increased Risk aversion for 'mixed' lotteries
- Endowment Effect
- Though note somewhat different assumptions re reference points
- Is the same phenomena responsible for both behaviors?
- If so we would expect to find them correlated in the population
- Dean and Ortoleva [2014] estimate
- $\lambda$
- WTP/WTA gap

In the same group of subjects

- Find a correlation of 0.63 (significant $\mathrm{p}=0.001$ )
- See also Gachter et al [2007]
- However do not find such an effect in a recent larger study


## Prospect Theory

- Prospect Theory: Kahneman and Tversky [1979]
- 'Workhorse Model' of choice under risk
- Combines
- Loss Aversion
- Cumulative Probability Weighting
- Diminishing Sensitivity


## Loss Aversion in Risky Choice



- Diminishing sensitivity:
- Differences harder to distinguish as you move away from reference point (similar to perceptual psychology)
- Leads to risk aversion for gains, risk loving for losses
- Looks like many other perceptual phenomena


## Loss Aversion in Risky Choice

- Let $p$ be a lottery with (relative) prizes

$$
x_{1}>x_{2} . . x_{k}>0>x_{k+1}>\ldots>x_{n}
$$

- $p_{i}$ probability of winning prize $x_{i}$
- Utility of lottery $p$ given by

$$
\begin{aligned}
& \pi\left(p_{1}\right) u\left(x_{1}\right) \\
& +\left(\pi\left(p_{2}\right)-\pi\left(p_{1}\right)\right) u\left(x_{1}\right) \\
& +\ldots \\
& +\left(\pi\left(p_{1}+. .+p_{k}\right)-\pi\left(p_{1}+. .+p_{k-1}\right)\right) u\left(x_{k}\right) \\
& -\left(\pi\left(p_{1}+. .+p_{k+1}\right)-\pi\left(p_{1}+. .+p_{k}\right)\right) \lambda u\left(-x_{k+1}\right) \\
& -\ldots \\
& -\left(\pi\left(p_{1}+. .+p_{n}\right)-\pi\left(p_{1}+. .+p_{n-1}\right)\right) \lambda u\left(-x_{n}\right)
\end{aligned}
$$

## A Model of Status Quo Bias

- Kahneman and Tversky start with a model of behavior, and then derive axioms
- Arguably, model is compelling, axioms not so much
- An alternative approach is taken by Masatlioglou and Ok [2005]
- Start with some axioms, and see what model obtains


## Primitives

- $X$ : finite set of alternatives
- $\diamond$ : Placeholder for no status quo
- $\mathcal{D}$ : set of decision problems $\{A, x\}$ where $A \subset X$ and $x \in A \cup \diamond$
- Note the enrichment of the data set
- $C: \mathcal{D} \Rightarrow X:$ choice correspondence


## Axioms

Axiom 1: Status Quo Conditional Consistency For any $x \in X \cup \diamond$, $C(A, x)$ obeys WARP
Axiom 2: Dominance If $y=C(A, x)$ for some $A \subset B$ and $y \in C(B, \diamond)$ then $y \in C(B, x)$
Axiom 3: Status Quo Irrelevance If $y \in C(A, x)$ and for every $\{x\} \neq T \subset A, x \notin C(T, x)$ then $y \in C(A, \diamond)$
Axiom 4: Status Quo Bias If $x \neq y \in C(A, x)$, then $y=C(A, y)$

- These axioms are necessary and sufficient for two representations
- Model 1: There exists
- Preference relation $\succeq$ on $X$
- A completion $\unrhd$ such that

$$
\begin{aligned}
C(A, \diamond) & =\{x \in A \mid x \unrhd y \forall y \in A\} \\
C(A, x) & =x \text { if } \nexists y \in A \text { s.t } y \succ x \\
& =\{y \in A \mid y \unrhd z \forall z \succ x\} \text { otherwise }
\end{aligned}
$$

- Interpretation:
- $\succeq$ represents 'easy' comparisons
- If there is nothing 'obviously' better than the status quo, choose the status quo
- Otherwise think more carefully about all the alternatives which are obviously better than the status quo
- An equivalent representation
- Model 2: there exists
- $u: X \rightarrow \mathbb{R}^{N}$
- A strictly increasing function $f: u(X) \rightarrow \mathbb{R}$ such that

$$
\begin{aligned}
C(A, \diamond) & =\arg \max _{x \in A} f(u(x)) \\
C(A, x) & =x \text { if } U_{u}(A, x) \text { is empty } \\
& =\arg \max _{x \in U_{u}(A, x)} f(u(x)) \text { otherwise }
\end{aligned}
$$

Where $U_{u}(A, x)=\{y \in A \mid u(y)>u(x)\}$

## The Story So Far

- Models of reference dependence discussed so far are preference-based
- A status quo generates a set of preferences:

$$
\succeq_{s} \text { for all } s \in X \cup \diamond
$$

- Decision Maker chooses to maximize these preference

$$
C(A, s)=\left\{z \in A \mid z \succeq_{s} y \text { for all } y \in A\right\}
$$

## Behavioral Implications of Preference-Based Models

- For a fixed status quo, DM maximizes a fixed set of preferences
- Looks like a 'standard' decision maker
- Status Quo Conditional Consistency (SQCC):
- For any $(A, s),(B, s)$
- Independence of Irrelevant Alternatives: If $x \in A \subset B$ and $x \in C(B, s)$ then $x \in C(A, s)$


## The Problem with Preference-Based Models

- This cannot capture too much choice effects
- e.g. lyengar and Lepper
- People switch to choosing the status quo in larger choice sets
- Violates Independence of Irrelevant Alternatives for a fixed status quo
- Status quo chosen in bigger choice set
- Still available in smaller choice set
- Yet not chosen in smaller choice set


## Example 2



## Decision Avoidance

- One possible solution: models of decision avoidance
- Try to avoid hard choices
- 'Easy’ choice:
- Make an active decision to select an alternative
- May move away from the status quo
- 'Difficult' choice
- May avoid thinking about the decision
- End up with the status quo
- May cause switching to the SQ in larger choice sets
- If this leads to more difficult choices


## Models of Decision Avoidance

- What makes choice difficult?
- Conflict model
- Difficulty in comparing two alternatives
- Information overload model
- Ability to compare objects reduces with the size of the choice set


## The Conflict Model

- DM endowed with a possibly incomplete preference ordering
- In any given choice set
- If one alternative is preferred to all others, the DM chooses it
- If not, may avoid decision by choosing the status quo
- If no suitable status quo, uses other decision making mechanism
- 'Think harder' about the problem
- Complete their preference ordering


## The Conflict Decision Avoidance Model

- Formal Representation:
(1) Choice is defined for any $\{Z, s\}$ by
(1) $C(Z, s)=\{x \in Z \mid x \succeq y \forall y \in Z\}$ if such set is non-empty
(2) otherwise $C(Z, s)=s$ if $s \in Z / T(Z)$
(3) otherwise $C(Z, s)=\{x \in Z \mid x \unrhd y \forall y \in Z\}$


## A Multi-Utility Representation

- Incomplete preference ordering $\succeq$ can be represented by a vector-valued utility function:

$$
u(z)=\left(\begin{array}{c}
u_{1}(z) \\
\vdots \\
u_{n}(z)
\end{array}\right)
$$

- Such that

$$
\begin{aligned}
z & \succeq w \\
\text { if and only if } u_{i}(z) & \geq u_{i}(w) \forall i \in 1 . . n
\end{aligned}
$$

## A Multi-Utility Representation



- Choose $y$ as $y$ is best object along all dimensions


## A Multi-Utility Representation



- Choose status quo to avoid having to decide between $z$ and $y$


## Information Overload

- Alternative hypothesis: Information Overload
- Large choice sets are inherently more difficult than small choice sets
- Iyengar and Lepper [2000]
- DM can compare all available options on a bilateral basis,
- May still find large choice set difficult


## Nested Preferences

- Modify Conflict model to allow for information overload
- Preferences may become less complete in large choice sets
- Replace fixed preference relation of Conflict model with nested preference relation
- Nested Preferences:
- For every $Z$ a preference relation $\succeq_{Z}$
- Such that, for every $W \subset Z$

$$
x \succeq z y \Rightarrow x \succeq w y
$$

- but not

$$
x \succeq z y \Longleftarrow x \succeq w y
$$

## The Information Overload Model

- Modifies the Conflict Decision Avoidance Model....
(1) Choice is defined for any $\{Z, s\}$ by
(1) $C(Z, s)=\{x \in Z \mid x \succeq z y \forall y \in Z\}$ if such set is non-empty
(2) otherwise $C(Z, s)=s$ if $s \in Z / T(Z)$
(3) otherwise $C(Z, s)=\{x \in Z \mid x \unrhd y \forall y \in Z\}$


## Behavioral Implications of Decision Avoidance Models

- Information overload model and conflict model:
- A1: Limited status quo dependence
- A2: Weak status quo conditional consistency
- Conflict model only
- A3: Expansion


## Limited Status Quo Dependence

- Choice can only depend on status quo in a limited way
- Making an object $x$ the status quo can lead people to switch their choices to $x \ldots$
- ...but cannot lead them to choose another alternative $y$
- A1: LSQD: In any choice set, choice must be either
- The status quo
- What is chosen when there is no status quo
- Note - not implied by preference-based models


## Weak Status Quo Conditional Consistency

- Decision avoidance models allow for violations of SQCC, but only of a specific type
- People may switch to choosing the status quo in larger choice sets
- A2: Weak SQCC: For a fixed status quo
- if $x$ is chosen in a larger choice set
- must also be chosen in a subset
- unless $x$ is the status quo


## Expansion

- A3: Expansion: Adding dominated options cannot lead people to switch to the status quo
- Say $x$ is chosen in a choice set $Z$ when it is not the status quo
- Add option $y$ to the choice set that is dominated by some $w \in Z$
- $w$ is chosen over $y$ even when $y$ is the status quo
- $x$ must still be chosen from the larger choice set


## Expansion

- Conflict model implies expansion
- Adding dominated options does not make choice any more 'difficult'
- Information overload model does not imply expansion
- DM may 'know' their preferred option in smaller choice set
- Adding dominated options to the choice set degrades preferences
- Can no longer identify preferred option in the larger choice set

An Experimental Test of Expansion


## Transaction Costs and Optimal Defaults

Optimal Defaults and Active Decisions [Carrol et al 2009]

- The most obvious cause of reference dependence is transaction costs
- It costs me an amount $c$ to move away from the status quo option
- Utility of alternative $x$ is $u(x)$ if it is the status quo, $u(x)-c$ otherwise
- Because there is nothing 'psychological' about the impact of reference points, makes welfare analysis staightforward
- Want to maximize utility net of transaction costs


## Transaction Costs and Optimal Defaults

Optimal Defaults and Active Decisions [Carrol et al 2009]

- We can think of the design problem of a social planner choosing the default in order to maximize welfare of an agent
- In the case of a single agent whose preferences are known, the problem is trivial
- Set the default equal to the highest utility alternative
- Carrol et al [2009] make the problem more interesting in three ways
- Several agents, each with potentially different rankings
- Each agent's ranking is not observable to the social planner
- Agent has quasi-hyperbolic discount function, but the social planner wants to maximize exponentially discounted utility


## The Agent's Problem

- Agent lives for an infinite number of periods
- They start life with a default savings rate $d$
- They have an optimal savings rate $s$
- In any period in which they have a savings rate $d$ they suffer a loss

$$
L=\kappa(s-d)^{2}
$$

- In any period they can change to their optimal savings rate at cost $c$
- Cost drawn in each period drawn from a uniform distribution
- Discounted utility given by quasi-hyperbolic function of expected future losses


## The Agent's Problem

- Restrict attention to stationary equilibria
- Agent has a fixed $c^{*}$
- Will switch to the optimal savings rate if $c<c^{*}$
- $c^{*}$ is
- Increasing in $\beta$
- Decreasing in $|s-d|$


## The Planner's Problem

- Facing a population of agents drawn from a uniform distribution on $\left[s_{*}, s^{*}\right]$
- Cannot observe $s$
- Wishes to choose $d$ in order to minimize expected, exponentially discounted loss of the population
- Has to take into account two trade offs
- A default that is good for one agent may be bad for another
- A default that is too good may lead present-biased agents to procrastinate


## The Planner's Problem


$\beta=1$

$\beta=0.75$

$\beta=0.1$

- Expected total loss (from the planner's point of view) based on the distance between default and optimal savings rate
- If $\beta=1$ always better to have default closer to optimal
- if $\beta<1$ may be better to have default further away to overcome procrastination


## The Planner's Problem

- Leads to three possible optimal policy regimes
- Center default - minimize the expected distance between $s$ and d
- Offset default - Encourage the most extreme agents to make active decisions
- Active decisions - Set a default so bad that all agents to move away from the default.


## The Planner's Problem



$\beta=0.75, \bar{s}-\underline{s}=0.25$

$\beta=0.1, \bar{s}-\underline{s}=0.15$

## The Planner's Problem



## Reference Points and Optimal Coding

- One possible interpretation of reference point effects is that they focus attention on particular parts of the problem
- Could this be a rational use of neural resources?
- Focus attention where it is most useful
- If so, may be a role for reference points affecting valuation and therefore choice
- Reference points tell us what is most likely to happen
- and so where it is most likely to be useful to make fine judgements
- This hypothesis is explored in Woodford [2012]


## A Detour Regarding Blowflys



- Shows neural response to contrast differences in light sources (black dots)
- Also CDF of contrast differences in blowfly environment (line)


## A Detour Regarding Blowflys



- Sharpest distinction occurs between contrasts which are likely to occur
- i.e slope of line matches the 'slope' of the dots


## Rational Coding

- Blowflies seem to use neural resources to best differentiate between states that are most likely to occur
- Does this represent 'optimal' use of resources?
- Surprisingly not if costs are based on Shannon mutual information
- Why not?


## The Effect of Priors

- Remember Shannon Mutual Information costs can be written as

$$
\begin{aligned}
& -[H(\Gamma)-E(H(\Gamma \mid \Omega))]= \\
& \sum_{\gamma \in \Gamma(\pi)} P(\gamma) \ln P(\gamma)-\sum_{\omega} \mu(\omega)\left(\sum_{\gamma \in \Gamma(\pi)} \pi(\gamma \mid \omega) \ln \pi(\gamma \mid \omega)\right)
\end{aligned}
$$

where

$$
P(\gamma)=\sum_{\omega \in \Omega} \pi(\gamma \mid \omega) \mu(\omega)
$$

- Changing the precision of a signal in a given state (i.e. $\pi(\gamma \mid \omega))$ changes info costs by

$$
(\ln (P(\gamma))+1) \frac{\partial P(\gamma)}{\partial \pi(\gamma \mid \omega)}-\mu(\omega)(\ln (\pi(\gamma \mid s)+1)
$$

## The Effect of Priors

- But $\frac{\partial P(\gamma)}{\partial \pi(\gamma \mid \omega)}=\mu(\omega)$, so

$$
\mu(\omega)(\ln (P(\gamma))-\ln (\pi(\gamma \mid s))
$$

- It is cheaper to get information about states that are less likely to occur
- Intuition: you only pay the expected cost of information
- Expected cost information about states that are unlikely to occur is low
- This offsets the lower value of gathering information about such states
- Prior probability of state should not matter for optimal coding


## The Effect of Priors

- Does this hold up in practice?
- Experiment: Shaw and Shaw [1977]
- Subjects had to report which of three letters had flashed onto a screen
- Letter could appear at one of 8 locations (points on a circle)
- Two treatments
- All positions equally likely
- 0 and 180 degrees more likely
- Shannon prediction: behavior the same in both cases


## Shaw and Shaw [1977]: Treatment 1



## Shaw and Shaw [1977]: Treatment 2



## Shannon Capacity

- This observation lead Woodford [2012] to consider an alternative cost function
- Shannon Capacity
- Let

$$
I_{\mu}(\Gamma, \Omega)
$$

be the Mutual Information between signal and state under prior beliefs $\mu$

- Shannon Capacity is given by

$$
\max _{\mu \in \Delta(\Omega)} I_{\mu}(\Gamma, \Omega)
$$

- i.e. the maximal mutual information across all possible prior beliefs
- True priors no longer affect costs
- Signals on less likely states no cheaper than signals on more likely states


## Shannon Capacity



- Optimal behavior when objective is linear in squared error
- Upper panel prior is $N(2,1)$, lower panel prior is $N(-2,1)$


## Coding Values

- One can apply this model to economic choice
- Assume that DM have to encode the value of a given alternative
- Assume alternative is characterized along different dimensions
- Has a limited capacity to encode value along each dimension
- Chooses optimal encoding given costs, prior beliefs and the task at hand


## Reference Dependence

- This model can explain diminishing sensitivity
- But not, in an obvious way, loss aversion
- Remember, diminishing sensitivity predicts
- Risk aversion for gains
- Risk seeking for losses
- E.g.
- Choice 1: start with 1000 , choose between a gain of 500 for sure or a $50 \%$ chance of a gain of 1000
- Choice 2: start with 2000, choose between a loss of 500 for sure or a $50 \%$ chance of a loss of 1000


## Reference Dependence

- Assume that the change in the reference point changes the prior distribution over final outcomes
- Choice 2 has a mean which is 1000 higher than choice 1
- Assume that prior is normal
- In Choice 11000 most likely, then 1500, then 2000
- 1000 most precisely encoded, then 1500 then 2000
- More 'sensitive' to the change between 1000 and 1500 than between 1500 and 2000
- Leads to risk aversion
- In Choice 22000 most likely, then 1500, then 1000
- 2000 most precisely encoded, then 1500 then 1000
- More 'sensitive' to the change between 2000 and 1500 than between 1500 and 1000
- Leads to risk loving


## Reference Dependence



- Plot of Mean Squared Normalized Value under the two different coding schemes


## Framing and Perception

- This is part of a developing literatature looking at behavioral biases from a perceptual standpoint
- Khaw, Mel Win, Ziang Li, and Michael Woodford. Risk aversion as a perceptual bias. No. w23294. National Bureau of Economic Research, 2017.
- Gabaix, Xavier, and David Laibson. Myopia and discounting. No. w23254. National bureau of economic research, 2017.
- Adriani, Fabrizio, and Silvia Sonderegger. Optimal similarity judgements in intertemporal choice, 2015.


## Where do Reference Points Come From?

- Up until now, we have assumed that we get to observe what reference points are observable
- Where do they come from?
- What you are currently getting?
- What happens if you do nothing?
- What you expect to happen in the future?
- Often (but not always) these things may be highly correlated


## Where do Reference Points Come From?

- There is some experimental work trying to differentiate these different effects
- e.g. Ritov and Baron [1992], Schweitzer [1994]
- Try to separate between
- Pure status quo bias (Preference for the current state of affairs)
- Omission bias (preference for inaction)
- Former study found only omission bias, latter found both


## Where do Reference Points Come From?

- Koszegi and Rabin [2006, 2007] made two innovations
(1) Allowed for reference points to be stochastic
- If your reference point is a lottery you treat it as a lottery
(2) Allowed for 'rational expectations'
- There is a problem if we think that the reference point should be what we expect
- What we expect should depend on our actions!
- Introduce the concept of 'personal equilibrium'


## Personal Equilibrium

- Consider an option $x$
- What would I choose if $x$ was my reference point?
- If it is $x$, then I will call $x$ a personal equilibrium
- If I expect to buy $x$ then it should be my reference point
- If it is my reference point then I should actually buy it


## Example

- Consider shopping for a pair of earmuffs
- The utility of the earmuffs is 1
- Prices is $p$
- Again, assume that utility is linear in money
- What would you do if reference point was to buy the earmuffs?
- Utility from buying earmuffs is 0
- Utility from not buying earmuffs is $p-\lambda$
- Buy earmuffs if $p<\lambda$
- What would you do if reference point was to not buy the earmuffs?
- Utility from not buying the earmuffs is 0
- Utility from buying earmuffs is $1-\lambda p$
- Would buy the earmuffs if $p<\frac{1}{\lambda}$


## Example



## Evidence

- Endowments as Expectations (Ericson and Fuster [2011])
- Endowments and expectations often move together
- Which determines the reference point?
- Experiment in which subjects were endowed with a mug
- Would be allowed to trade for a pen with some probability
- Higher probability of being forced to keep the mug $\Rightarrow$ lower probability of trade if allowed
- Heffetz and List [2013] find exactly the opposite!
- Reference effects driven by assignment
- Not obvious what drives the differences
- For a nice review see
- Marzilli Ericson, Keith M., and Andreas Fuster. "The Endowment Effect." Annu. Rev. Econ. 6.1 (2014): 555-579.


## Narrow Bracketing

- In applications, loss aversion is often combined with Narrow Bracketing
- Decision makers keep different decisions separate
- Evaluate each of those decisions in isolation
- For example, evaluate a particular investment on its own, rather than part of a portfolio
- Evaluate it every year, rather than as part of lifetime earnings


## Applications: Loss Aversion and Narrow Bracketing

- Equity Premium Puzzle [Benartzi and Thaler 1997]
- Average return on stocks much higher than that on bonds
- Stocks much riskier than bonds - can be explained by risk aversion?
- Not really - calibration exercise suggests that the required risk aversion would imply

$$
\begin{aligned}
& 50 \% \$ 100,000+50 \% \$ 50,000 \\
\sim & 100 \% \$ 51,329
\end{aligned}
$$

- What about loss aversion?
- In any given year, equities more likely to lose money than bonds
- Benartzi and Thaler [1997] calibrate a model with loss aversion and narrow bracketing
- Find loss aversion coefficient of 2.25 - similar to some experimental findings
- See also
- Barberis, Nicholas, and Ming Huang. The loss aversion/narrow framing approach to the equity premium puzzle. [2007].


## Applications: Diminishing Sensitivity

- Disposition Effect [Odean 1998]
- People are more likely to hold on to stocks which have lost money
- More likely to sell stocks that have made money
- Losing stocks held a median of 124 days, winners a median of 104 days
- Is this rational?
- Hard to explain, as winners subsequently did better
- Losers returned $5 \%$ on average in the following year
- Winners returned $11.6 \%$ in subsequent year
- Buying price shouldn't enter into selling decision for rational consumer
- But will do for a consumer with reference dependent preferences
- Diminishing sensitivity


## Applications: Loss Aversion and Narrow Bracketing

- Taxi driver labor supply [Camerer, Babcock, Loewenstein and Thaler 1997]
- Taxi drivers rent taxis one day at a time
- Significant difference in hourly earnings from day to day (weather, subway closures etc)
- Do drivers work more on good days or bad days?
- Standard model predicts drivers should work more on good days, when rate of return is higher
- In fact, work more on bad days
- Can be explained by a model in which drivers have a reference point for daily earnings and are loss averse


## Applications: Reported Tax Balance Due [Rees-Jones 2014]



## Loss Aversion and Information Aversion

- Loss aversion can also lead to information aversion
- Imagine that you have linear utility with $\lambda=2.5$
- Say you are offered a $50 \%$ chance of 200 and a $50 \%$ chance of -100 repeated twice
- Two treatments:
- The result reported after each lottery
- The result reported only after both lotteries have been run.
- What would choices be?
- In the first case

$$
\begin{aligned}
& \frac{1}{4}(200+200)+\frac{1}{2}(200-\lambda 100)+\frac{1}{4}(-\lambda 100-\lambda 100) \\
= & -200
\end{aligned}
$$

- In the second case

$$
\frac{1}{4}(400)+\frac{1}{2}(100)+\frac{1}{4}(-\lambda 200)
$$

$$
=25
$$

## Loss Aversion and Information Aversion

- With loss aversion and narrow bracketing, risk aversion depends on evaluation period
- The longer period, the less risk averse
- This also provides an 'information cost'
- A similar argument shows that if you owned the above lottery, you would prefer only to check it after two flips rather than every flip
- May explain why people check their portfolios less in more turbulent times
- See Andries and Haddad [2015] and Pagel [2017]
- In general, strong link between non-expected utility and preference for one shot resolution
- Dillenberger [2011]


## Reference Dependent Preferences

- Strong evidence that people evaluate options relative to some reference point
- Change in reference point can change preferences
- Endowment Effect
- Risk aversion
- One robust finding is loss aversion
- Losses loom larger than gains
- Can explain the endowment effect and increased risk aversion for mixed choice
- One open question is where reference points come from
- Prospect theory is a workhorse model of choice under risk
- Loss Aversion
- Probability Weighting
- Diminishing Sensitivity
- Has been used to explain many 'real world' phenomena
- Choice of financial asset
- Labor supply

