Behavioral Economics

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Homework 1 - Autumn 2018

Due 16th November 2018

Question 2 In this question we will consider models of rational inattention with costs based on Shannon mutual information.

1. In class, I introduced the ILR condition

$$\frac{\gamma^a(\omega)}{z(a,\omega)} = \frac{\gamma^b(\omega)}{z(b,\omega)}$$

where $z(a,\omega) = \exp\left\{\frac{u(a,\omega)}{\lambda}\right\}$. Show using first order conditions that this is a necessary condition for behavior to match the Shannon model.

2. Consider the problem of a consumer who faces a choice of whether or not to buy a good at price p. The good can be either of high quality or low quality. If a high quality good is purchased, then the consumer gets utility $u^* - p$. If they purchase a low quality good then they get utility $u_* - p$. if they do not purchase the good then they get utility 0. The consumer initially places probability β on the good being of high quality, and can gather more information on the quality of good, paying costs based on mutual information.

Consider a strategy in which γ^i_j is the probability of true state j when action i is taken, where $j \in \{h, l\}$ for high or low quality goods and $i \in \{b, n\}$ for buy or not. Verify that the value of such a strategy is

$$P(\gamma^b)[H(\gamma^b) + U(\gamma^b, b)] + P(\gamma^n)[H(\gamma^n) + U(\gamma^n, n)] - H(\beta)$$

where $P(\gamma^i)$ is the unconditional probability of taking act i, $H(\gamma^i)$ is the entropy of distribution γ^i and $U(\gamma^i, j)$ is the expected utility of taking act j under posterior distribution γ^i.

3. Assume that $p = 2$, $u^* = 4$, $u_* = 0, k = 1$ and $\beta = 0.5$. Draw a graph which has as its horizontal axis the probability of the good being of high quality. Add the following lines to that graph (you will need to use excel, Matlab or somesuch)

   (a) The entropy associated with each distribution

   (b) The expected utility of taking act b at each distribution

   (c) The expected utility of taking act n at each distribution

   (d) The expected utility of taking the optimal action at each distribution

   (e) The 'net utility' associated with each distribution (i.e. the expected utility of the optimal act plus entropy)

   Use the result from section two to show that the value of a particular strategy t^b, t^n must lie on the chord between the net utility function evaluated at these two points

4. Show that, by Bayes' rule, once γ^b and γ^n have been chosen, then $P(\gamma^b)$ and $P(\gamma^n)$ are determined. Use this result to show that the value of a strategy is equal to the height of the chord from section 3 as it passes over the prior belief of the agent

5. Solve for the optimal strategy for the case described in section 3. Draw the resulting optimal posteriors $\hat{\gamma}^b$ and $\hat{\gamma}^n$ on your graph. What is the relationship between the chord that connects these two posteriors and the net utility function?

6. What would be the optimal behavior of the consumer if they had a prior belief about the probability of a high quality firm that was different from 0.5 but between $\hat{\gamma}^b$ and $\hat{\gamma}^n$? What about if their prior belief was above $\hat{\gamma}^b$ Or below $\hat{\gamma}^n$?

7. Consider a firm who produces the high quality good with probability β and otherwise a low quality good. Having observed the quality of the good the firm can set a price of either p_H or p_L with the former greater than the latter. Having observed the price (but not the quality of the good) a consumer chooses what information to gather, and on the basis of the resulting signal chooses whether to purchase the product. Show that there is a perfect Bayesian equilibrium of this game in which the firm always sets price p_H for the high quality good, and sets the price p_H for the low quality good with probability

$$\min\left(\frac{\beta}{1-\beta}\frac{(1-\gamma_h^n)(1-\gamma_h^b)}{\gamma_h^n(1-\gamma_h^b)+\frac{p_L}{p_H}\left(\gamma_h^b-\gamma_h^n\right)}, 1\right)$$

   Notice that in an equilibrium it must be the case that (1) The firm is setting prices optimally given the attention and purchasing strategy of the consumer and (2) the attention and purchasing strategy of the consumer must be optimal given the pricing strategy of the firm.

Question 2 (Consideration Sets) Consider the following model of choice from consideration sets. The DM has a one-to-one utility function u on a finite set X (i.e. we are not allowing for indifference). For any choice set $A \subset X$ the DM has a consideration set $\Gamma(A)$, such that

$$C(A) = \max_{a \in \Gamma(A)} u(a)$$

Consideration sets obey the following property: if $x \in S \subset T$ and $x \in \Gamma(T)$ then $x \in \Gamma(S)$. i.e. if an alternative is considered in a larger set , it must also be considered in a smaller set.

1. Show that a model of this type can explain 'too much choice' effects of the type demonstrated by Iyengar and Lepper (and discussed in the first lecture)
2. What sort of behavior reveals that an alternative x is preferred to alternative y in this set up?
3. Assume that you observe choices between all possible choice sets. Use the above observation to construct necessary and sufficient conditions for the data set to be consistent with the above model. Prove your result
4. Did your above proof rely on the fact that you observed choices from all possible choice sets?