Behavioral Economics

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Homework 2 - Autumn 2018

Due Friday 30th November

Question 1 (Quasi-Hyperbolic Discounting) Consider an infinitely lived quasi-hyperbolic decision maker who is trying to lose weight. Their preferences in each period are given by

$$u(\gamma_t, w_t, w_{t-1}, x_t) = \gamma_t w_t - (w_t - w_{t-1})^2 - n w_t^2 - m x_t$$

Where w_t is weight in period t and γ_t is a preference parameter drawn randomly from some distribution f (for the sake of argument lets assume it is $N(0, \sigma^2)$) and x_t is any amount of money spent in period t. The interpretation is that the first term is the utility from consumption (which varies randomly), the second term is an adjustment cost, the third term is the cost of excess weight while the fourth term is the utility from any money spent (initially we will assume that this is zero)

1. Model the behavior of the agent as a game played between different 'agents' in each period. Guess and verify that the game has a solution of the form

$$w_t = a w_{t-1} + b\gamma_t + c$$

Solve for a, b and c.

2. Imagine that you observed the weight path of this agent. Which of the parameters of the original model could you recover?

3. Now assume that in some time period t, the agent faces a 'commitment contract' such that they have to pay an amount x_t if their weight is above a threshold y_t. Characterize the behavior of the agent in this period.

4. Now assume that, after setting their weight in period $t-1$, the agent is (unexpectedly) given the option of setting a target weight for period t (assume that the amount they forfeit x is fixed, but they can choose y). What will their optimal target be? How will it vary with the parameters of the problem?

Question 2 (An Alternative Data Set) Consider a data set consisting of choices from sets of alternatives of the form (a, t), where $a \in A$ (with A finite) is some task that can be completed and t is a time between 0 and some upper bound T. Choosing (a, t) therefore means completing a task at time t. From any given choice set only one completion option can be chosen. Let $\mathbf{\Gamma}$ be the set of all such (a, t). Say we observe a choice function on subsets of Γ.

Consider a model of decision making consisting of a series of one-to-one utility functions for each time t such that $U_t : A_{\geq t} \to \mathbb{R}$ where $A_{\geq t} = \{(a, s) \,|a \in A \text{ and } s \geq t\}$.A **sophisticated** strategy for any $B \in \Gamma$ is a mapping q from $\tau = \{t \in 0, ...T|(a, t) \in B\}$ to $A \cup \{\text{wait}\}$ such that

1. If $s = \max_{t \in \tau} t$ then $q(s) = \arg\max_{a \in A} \{U_s(a, s)|(a, s) \in B\}$
2. Otherwise, if $\max_{a \in A} \{U_s(a, s)|(a, s) \in B\} \geq U_s(\hat{a}_s, \hat{t}_s)$ then $q(s) = \arg\max_{a \in A} \{U_s(a, s)|(a, s) \in B\}$
3. Otherwise $q(s) = \{wait\}$

   where $\hat{t}_s = \min\{s' > s|q(s) \neq \{wait\}\}$ and $\hat{a}_s = q(\hat{t}_s)$

The interpretation of this strategy is that the DM at each stage solves the game using backwards induction. In any period s the DM figures out what will happen if they do not choose a completion option available to them at that time. They will choose to wait if and only if the utility of the completion option that will occur is better (according to period s utility) than what they could currently obtain.

1. Define an equivalent notion of a **naive** strategy
2. We say a choice function can be represented by the sophisticated model if we can find a set of utility functions such that, for every $B \in \Gamma$ $c(B) = \{\{a, t\} = \{q(s), s\} \,|s = \min s \in \tau|q(s) \neq wait\}$. It is time consistent if it can be represented by a sophisticated model in which the utility function is the same at every s. Show that c is time consistent if and only if it satisfies the independence of irrelevant alternatives

3. A reversal occurs if for $B \in \Gamma$, $c(B) = (a_1, t_1)$ yet $C(B \cup (a_2, t_2)) = (a_3, t_3)$. A reversal is called a doing it later reversal if $t_1 < t_3$ and either $t_2 \leq t_3$ or $(a_1, t_1) = c(\{(a_1, t_1), (a_3, t_3)\}$ or both. A doing it earlier reversal is one in which $t_3 < t_1$. Explain why these are appropriate terms for this type of reversal.

4. Show that the naive model you defined in stage 1 does not exhibit any doing-it-earlier reversals

5. Show that the sophisticated model does not exhibit any doing-it-later reversals