Models of Reference Dependent Preferences

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Modelling Reference Dependence

- Likely that there are many different causes of reference dependence
  - As we discussed in the introduction
- Broadly speaking two classes of models

1. Preference-based reference dependence
   - Reference points affect preferences which affect choices
2. 'Rational' reference dependence
   - Reference dependence as a rational response to costs
     - Effort costs
     - Attention Costs

- Focus on the former, say a little about the latter

Loss Aversion

- In 1979 Kahneman and Tversky introduced the idea of 'Loss Aversion'
- Basic idea: Losses loom larger than gains
  - Utility calculated on changes, not levels
  - The magnitude of the utility loss associated with losing x is greater than the utility gain associated with gaining x
- Initially applied to risky choice
- Later also applied to riskless choice [Tversky and Kahneman 1991]
- Can explain
  - Endowment effect
  - Increased risk aversion for lotteries involving gains and losses
  - Status quo bias

A Simple Loss Aversion Model

- World consists of different dimensions
  - e.g cash and mugs
- Will be asked to choose between alternatives that provide different amount of each dimension

$$\begin{pmatrix} x_c \\ x_m \end{pmatrix}$$

- Has a reference point for each dimension

$$\begin{pmatrix} r_c \\ r_m \end{pmatrix}$$

- **Key Point: Utility depends on changes, not on levels**

A Simple Loss Aversion Model

- Utility of an alternative comes from comparison of output to reference point along each dimension

$$\begin{pmatrix} x_c \\ x_m \end{pmatrix}, \begin{pmatrix} r_c \\ r_m \end{pmatrix}$$

- Utility for gains relative to r given by a utility function u

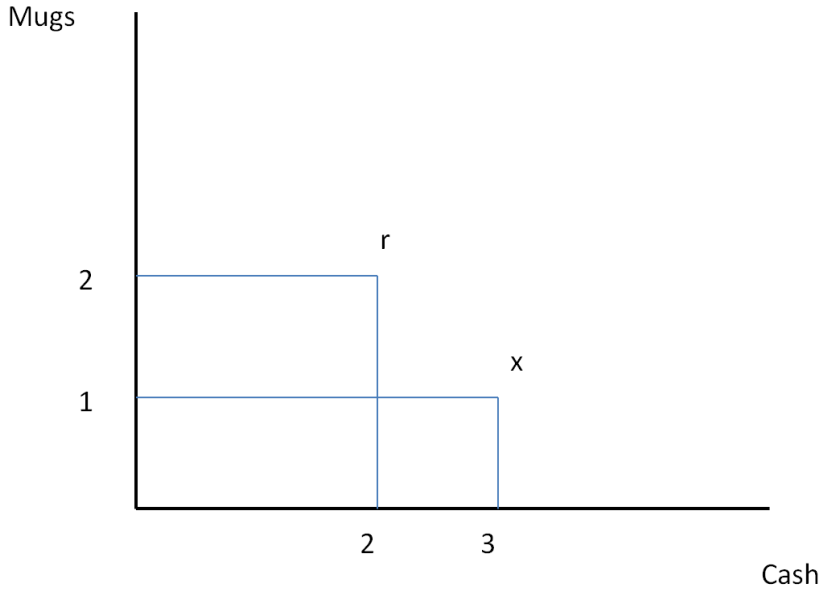
$$\begin{aligned} u_c(x_c) - u_c(r_c) \text{ if } x_c &> r_c \\ u_m(x_m) - u(r_m) \text{ if } x_m &> r_m \end{aligned}$$

- Utility of losses relative to r given buy u of the equivalent gain multiplied by $-\lambda$ with $\lambda > 1$

$$\begin{aligned} \lambda\left(u_c(x_c) - u_c(r_c)\right) \text{ if } x_c &< r_c \\ \lambda\left(u_m(x_m) - u(r_m)\right) \text{ if } x_m &< r_m \end{aligned}$$

- For simplicity assume that utilities are linear: $u_c(x_c) = x_c$, $u_m(x_m) = u_m x_m$

A Simple Loss Aversion Model



- x is a gain of \$1 and loss of 1 mug relative to r
- Utility of x

$$1 - \lambda u_m$$

Loss Aversion and the Endowment Effect

- How can loss aversion explain the Endowment Effect (i.e. WTP/WTA gap)?
- Willingness to pay:
  - Let (r_c, r_m) be the reference point with no mug
  - How much would they be willing to pay for the mug?
  - i.e. what is the z such that

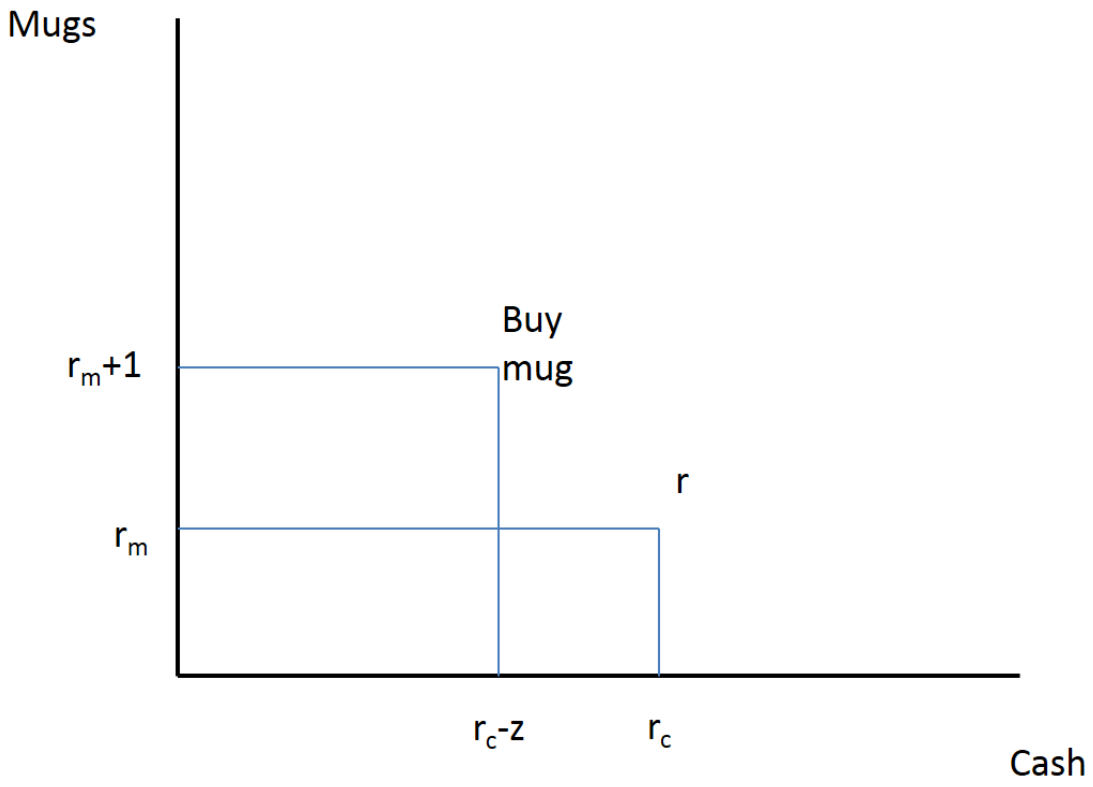
$$0 = U\left(\begin{array}{c} r_c \\ r_m \end{array}, \begin{array}{c} r_c \\ r_m \end{array}\right) = U\left(\begin{array}{c} r_c - z \\ r_m + 1 \end{array}, \begin{array}{c} r_c \\ r_m \end{array}\right)$$

- Utility of buying a mug given by

$$U\left(\begin{array}{c} r_c - z \\ r_m + 1 \end{array}, \begin{array}{c} r_c \\ r_m \end{array}\right) = u_m - \lambda z$$

- Break even buying price given by $z = \frac{u_m}{\lambda}$

A Simple Loss Aversion Model



- Buying is a loss of \$z and gain of 1 mug relative to r
- Utility of buying

$$u_m - \lambda z$$

Loss Aversion and the Endowment Effect

- Willingness to accept:
  - Let (r_c, r_m) be the reference point with mug
  - How much would they be willing to sell your mug for?
  - i.e. what is the y such that

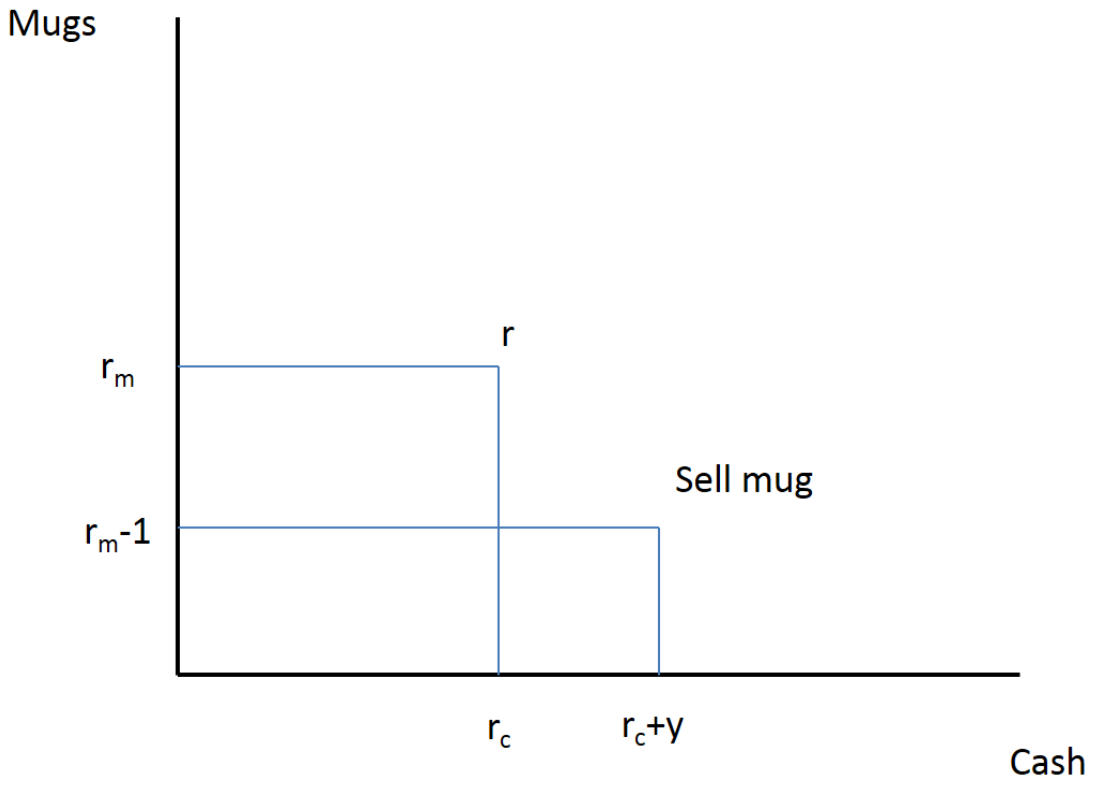
$$0 = U\left(\begin{array}{c} r_c \\ r_m \end{array}, \begin{array}{c} r_c \\ r_m \end{array}\right) = U\left(\begin{array}{c} r_c + y \\ r_m - 1 \end{array}, \begin{array}{c} r_c \\ r_m \end{array}\right)$$

- Utility of selling a mug given by

$$U\left(\begin{array}{c} r_c + y \\ r_m - 1 \end{array}, \begin{array}{c} r_c \\ r_m \end{array}\right) = -\lambda u_m + y$$

- Break even selling price given by $y = \lambda u_m(1)$

A Simple Loss Aversion Model



- Selling is a gain of \$y and loss of 1 mug relative to r
- Utility of selling

$$-\lambda u_m + y$$

Loss Aversion and the Endowment Effect

- Willingness to pay

$$z = \frac{u_m}{\lambda}$$

- Willingness to accept

$$y = \lambda u_m$$

- WTP/WTA ratio

$$\frac{z}{y} = \frac{1}{\lambda^2}$$

- Less that 1 for $\lambda > 1$

Axiomatization

- Tversky and Kahneman [1991] provide an axiomatization of this model

Axiom 1: Cancellation if, for some reference point

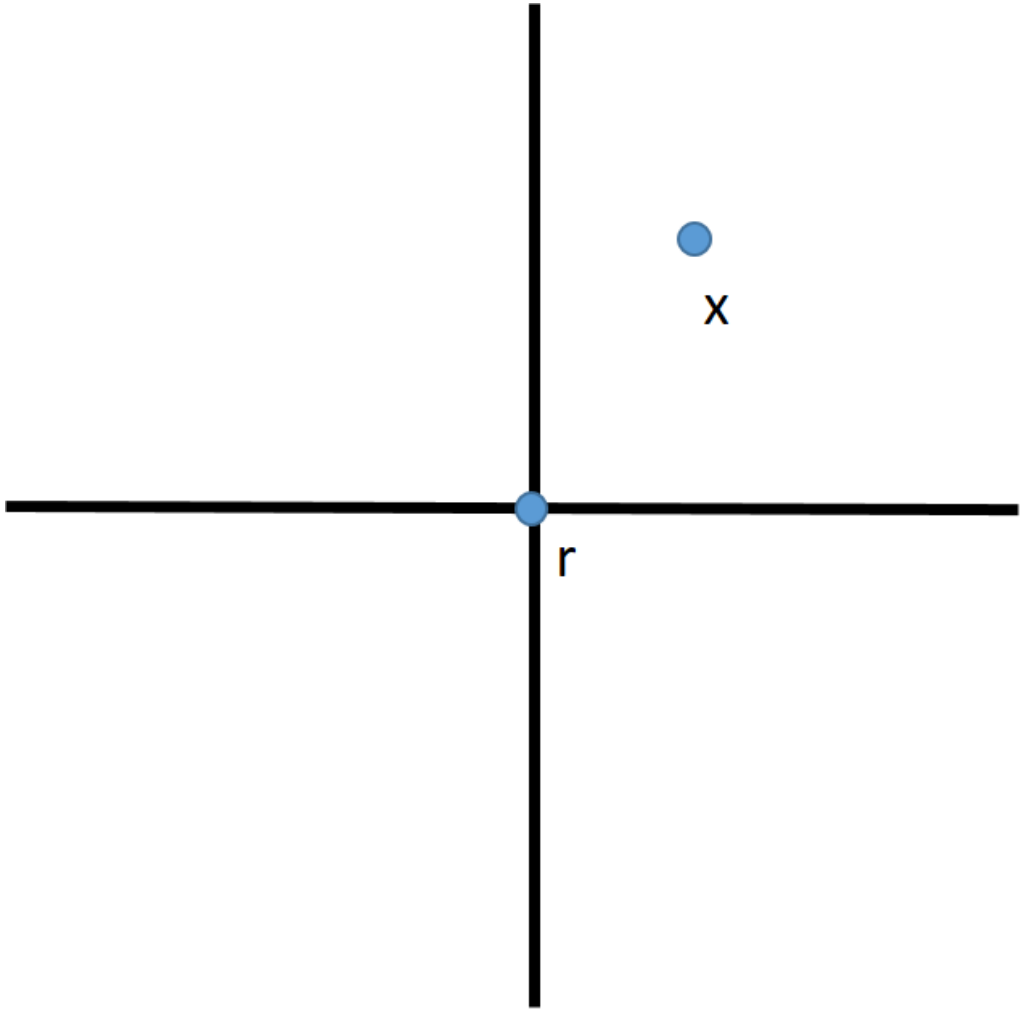
$$\begin{pmatrix} x_1 \\ z_2 \end{pmatrix} \succeq \begin{pmatrix} z_1 \\ y_2 \end{pmatrix} \text{ and } \begin{pmatrix} z_1 \\ x_2 \end{pmatrix} \succeq \begin{pmatrix} y_1 \\ z_2 \end{pmatrix}$$

then

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \succeq \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

- (guarantees additivity)

- Define the 'quadrant' that x is in relative to r



Axiom 2: Sign Dependence Let options x and y and reference points s and r be such that

1. x and y are in the same quadrant with respect to r and with respect to s
2. s and r are in the same quadrant with respect to x and with respect to y

Then $x \succeq y$ when r is the status quo $\iff x \succeq y$ when s is the status quo

- Guarantees that only the 'sign' matters

Axiom 3: Preference Interlocking Say that, for some reference point r, we saw that

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \text{ and } \begin{pmatrix} z_1 \\ x_2 \end{pmatrix} \sim \begin{pmatrix} y_1 \\ w_2 \end{pmatrix}$$

And, for another reference point s (that puts everything in the same quadrant, but maybe a different quadrant to r)

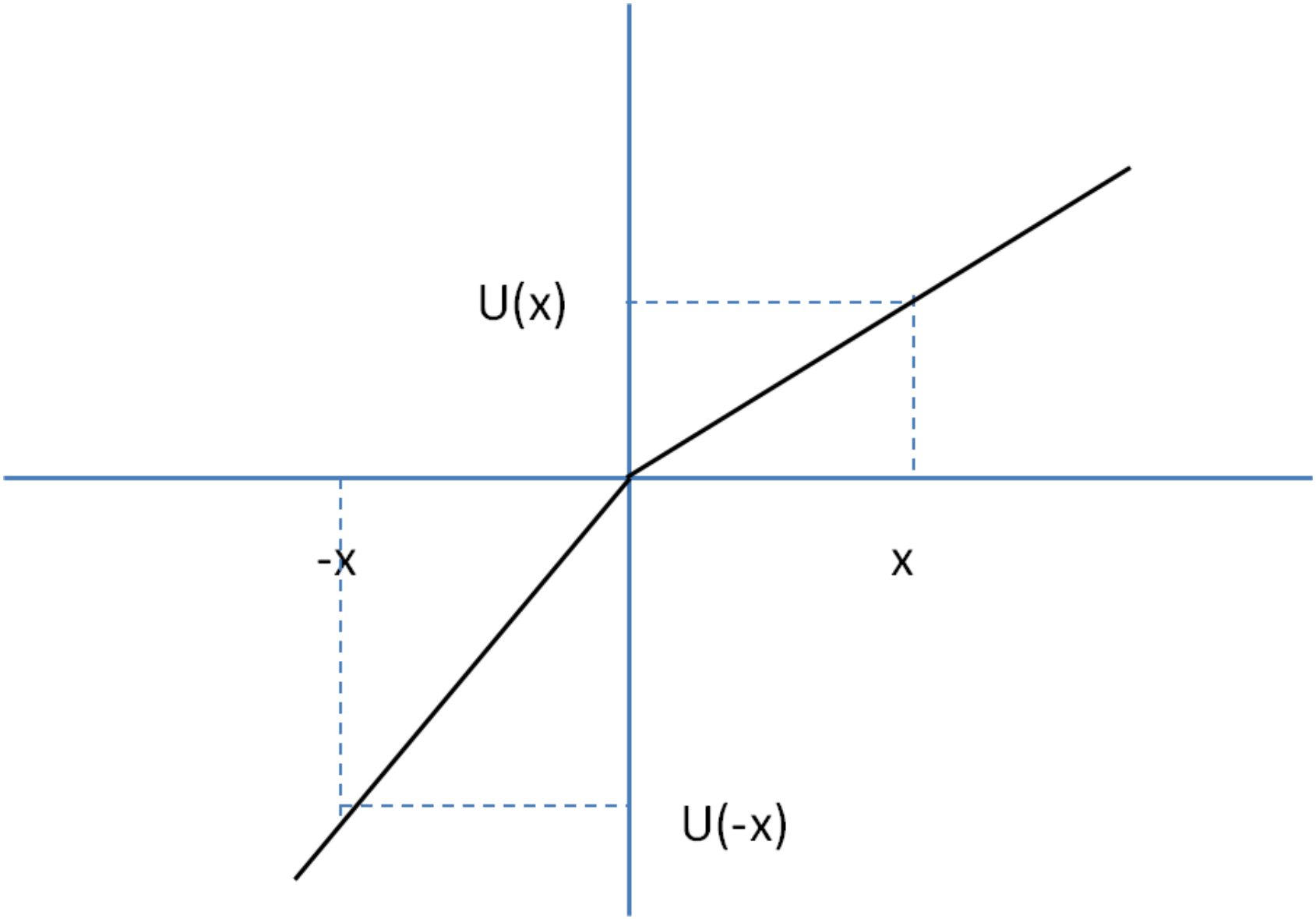
$$\begin{pmatrix} x_1 \\ \bar{x}_2 \end{pmatrix} \sim \begin{pmatrix} w_1 \\ \bar{w}_2 \end{pmatrix} \Rightarrow$$
$$\begin{pmatrix} z_1 \\ \bar{x}_2 \end{pmatrix} \sim \begin{pmatrix} y_1 \\ \bar{w}_2 \end{pmatrix}$$

- Ensures that the same trade offs that work in the gain domain also work in the loss domain

Loss Aversion in Risky Choice

- Loss aversion can also lead to increased risk aversion for lotteries that involve gains and losses
- Now there is only 1 dimension (money)
- Lotteries evaluated as gains/losses relative to some reference point
- See also Kosegi and Rabin [2007]
- Again, assume linear utility for money
  - Utility of winning x is x
  - Utility of losing x is -λx

Loss Aversion in Risky Choice



Loss Aversion in Risky Choice

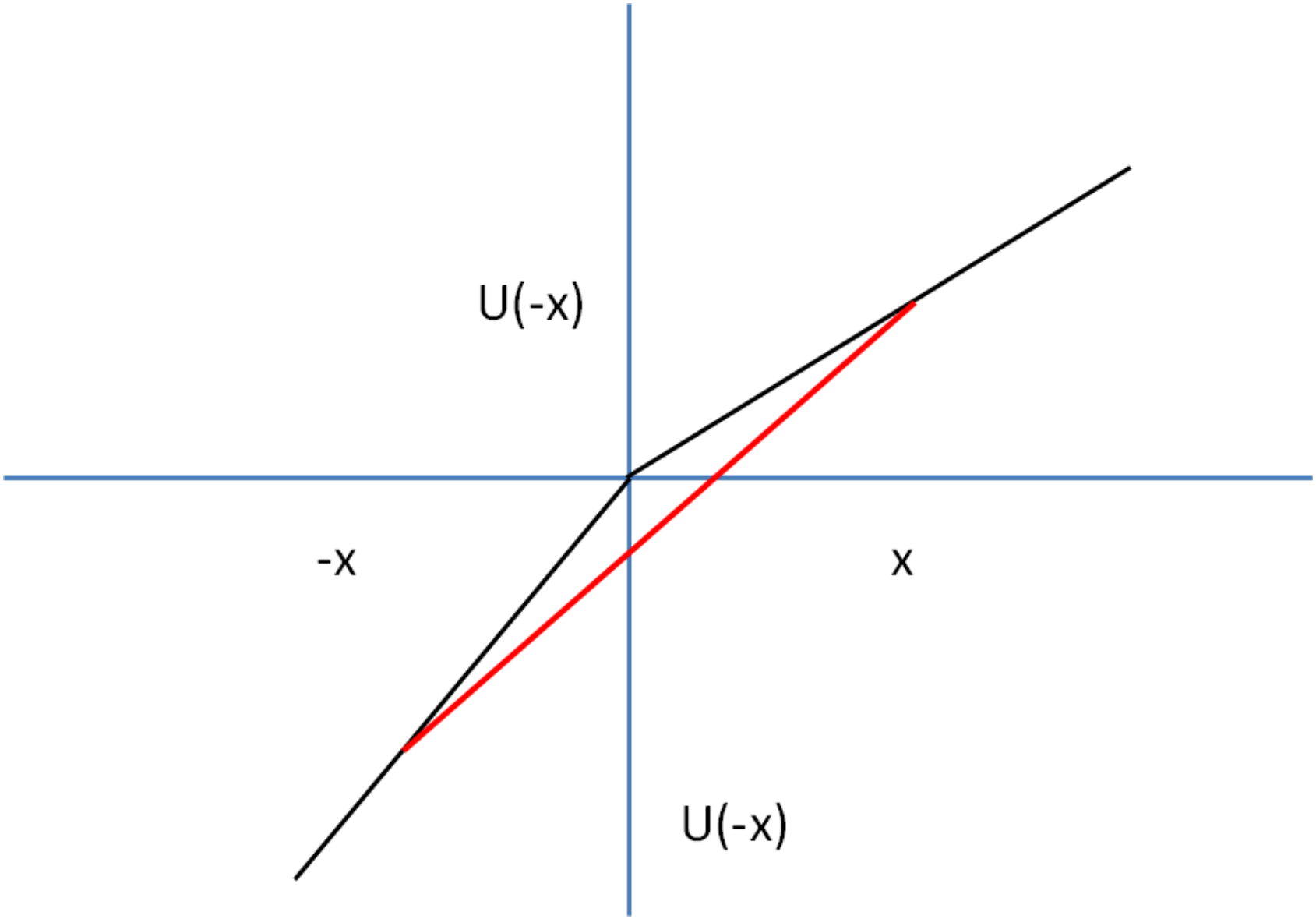
- What is the certainty equivalence of
  - 50% chance of gaining \$10
  - 50% chance of gaining \$0
- x such that

$$\begin{aligned} u_c(x) &= 0.5 \times u_c(10) + 0.5 \times u_c(10) \\ x &= 0.5 \times 10 + 0.5 \times 0 \\ &= \$5 \end{aligned}$$

- What is the certainty equivalence of
  - 50% chance of gaining \$5
  - 50% chance of losing \$5
- y such that

$$\begin{aligned} -\lambda u_c(-y) &= 0.5 \times u_c(5) + 0.5 \times (-\lambda))\, u_c(5) \\ -\lambda y &= 0.5 \times 5 - \lambda 0.5 \times 5 \\ y &= \frac{(1-\lambda)}{\lambda} < 0 \end{aligned}$$

Loss Aversion in Risky Choice



A Unified Theory of Loss Aversion?

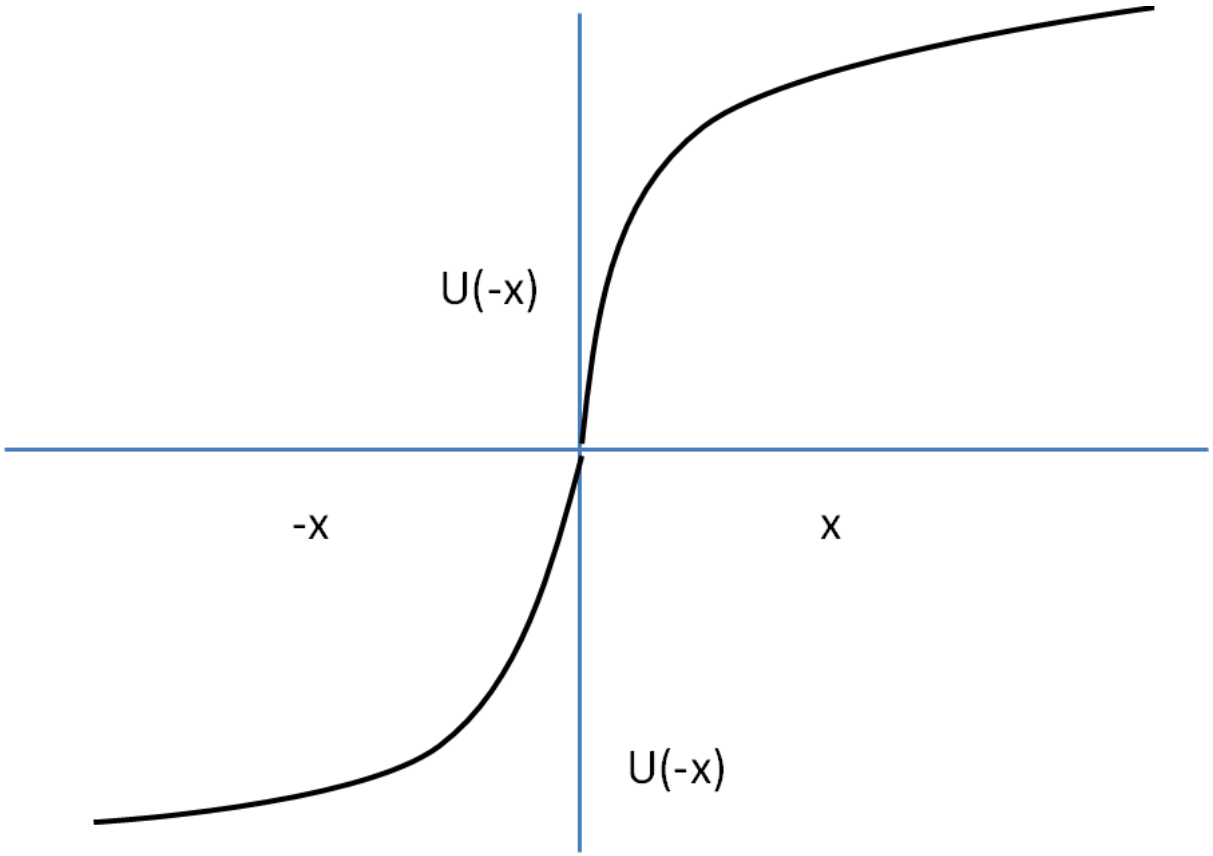
- We have claimed that loss aversion can explain
  - Increased Risk aversion for 'mixed' lotteries
  - Endowment Effect
- Though note somewhat different assumptions re reference points
- Is the same phenomena responsible for both behaviors?
- If so we would expect to find them correlated in the population
- Dean and Ortoleva [2014] estimate
  - λ
  - WTP/WTA gap

  In the same group of subjects
- Find a correlation of 0.63 (significant p=0.001)
  - See also Gachter et al [2007]
- However do not find such an effect in a recent larger study

Prospect Theory

- Prospect Theory: Kahneman and Tversky [1979]
- 'Workhorse Model' of choice under risk
- Combines
  - Loss Aversion
  - Cumulative Probability Weighting
  - Diminishing Sensitivity

Loss Aversion in Risky Choice



- Diminishing sensitivity:
  - Differences harder to distinguish as you move away from reference point (similar to perceptual psychology)
  - Leads to risk aversion for gains, risk loving for losses
- Looks like many other perceptual phenomena

Loss Aversion in Risky Choice

- Let p be a lottery with (relative) prizes

$$x_1 > x_2..x_k > 0 > x_{k+1} > .. > x_n$$

- p_i probability of winning prize x_i
- Utility of lottery p given by

$$\begin{aligned}
&\pi(p_1)u(x_1) \\
&+ (\pi(p_2) - \pi(p_1))\, u(x_1) \\
&+ ... \\
&+ (\pi(p_1 + .. + p_k) - \pi(p_1 + .. + p_{k-1}))\, u(x_k) \\
&- (\pi(p_1 + .. + p_{k+1}) - \pi(p_1 + .. + p_k))\, \lambda u(-x_{k+1}) \\
&- ... \\
&- (\pi(p_1 + .. + p_n) - \pi(p_1 + .. + p_{n-1}))\, \lambda u(-x_n)
\end{aligned}$$

A Model of Status Quo Bias

- Kahneman and Tversky start with a model of behavior, and then derive axioms
- Arguably, model is compelling, axioms not so much
- An alternative approach is taken by Masatlioglou and Ok [2005]
- Start with some axioms, and see what model obtains

Primitives

- X: finite set of alternatives
- $\diamond$: Placeholder for no status quo
- $\mathcal{D}$: set of decision problems $\{A, x\}$ where $A \subset X$ and $x \in A \cup \diamond$
  - Note the enrichment of the data set
- $C : \mathcal{D} \Rightarrow X$: choice correspondence

Axioms

Axiom 1: Status Quo Conditional Consistency For any $x \in X \cup \diamond$, $C(A, x)$ obeys WARP

Axiom 2: Dominance If $y = C(A, x)$ for some $A \subset B$ and $y \in C(B, \diamond)$ then $y \in C(B, x)$

Axiom 3: Status Quo Irrelevance If $y \in C(A, x)$ and for every $\{x\} \neq T \subset A, x \notin C(T, x)$ then $y \in C(A, \diamond)$

Axiom 4: Status Quo Bias If $x \neq y \in C(A, x)$, then $y = C(A, y)$

Model

- These axioms are necessary and sufficient for two representations
- Model 1: There exists
  - Preference relation $\succeq$ on X
  - A completion $\trianglerighteq$
    such that

$$
\begin{aligned}
C(A, \diamond) &= \{x \in A | x \trianglerighteq y \ \forall \ y \in A\} \\
C(A, x) &= x \text{ if } \nexists \ y \in A \text{ s.t } y \succ x \\
&= \{y \in A | y \trianglerighteq z \ \forall \ z \succ x\} \text{ otherwise}
\end{aligned}
$$

- Interpretation:
  - $\succeq$ represents 'easy' comparisons
  - If there is nothing 'obviously' better than the status quo, choose the status quo
  - Otherwise think more carefully about all the alternatives which are obviously better than the status quo

Model

- An equivalent representation
- Model 2: there exists
  - $u : X \to \mathbb{R}^N$
  - A strictly increasing function $f : u(X) \to \mathbb{R}$ such that

$$
\begin{aligned}
C(A, \diamond) &= \arg\max_{x \in A} f(u(x)) \\
C(A, x) &= x \text{ if } U_u(A, x) \text{ is empty} \\
&= \arg\max_{x \in U_u(A,x)} f(u(x)) \text{ otherwise}
\end{aligned}
$$

Where $U_u(A, x) = \{y \in A | u(y) > u(x)\}$

The Story So Far

- Models of reference dependence discussed so far are preference-based
- A status quo generates a set of preferences:

$$\succeq_s \text{ for all } s \in X \cup \Diamond$$

- Decision Maker chooses to maximize these preference

$$C(A, s) = \{z \in A | z \succeq_s y \text{ for all } y \in A\}$$

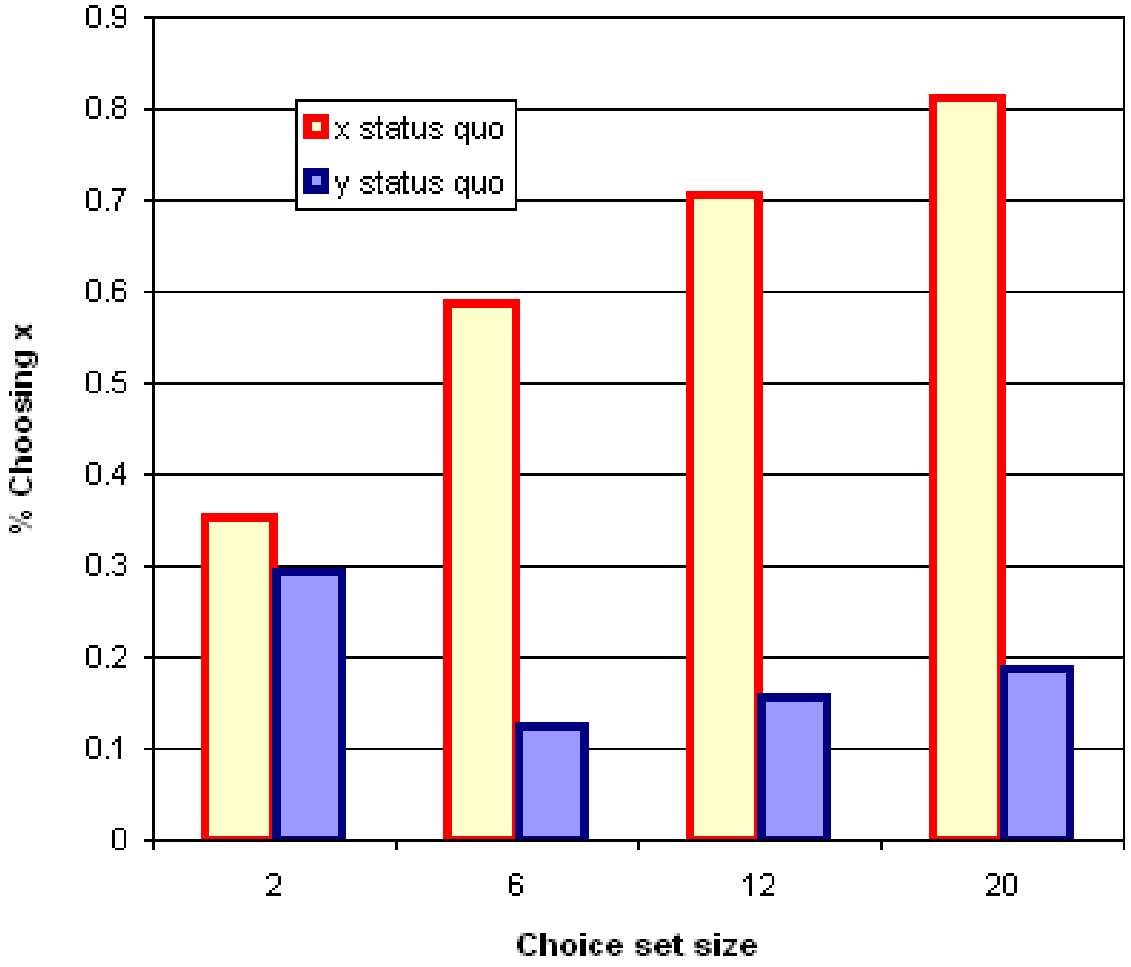
Behavioral Implications of Preference-Based Models

- For a *fixed* status quo, DM maximizes a *fixed* set of preferences
- Looks like a 'standard' decision maker
- **Status Quo Conditional Consistency (SQCC):**
- *For any* (A, s), (B, s)
  - *Independence of Irrelevant Alternatives: If* $x \in A \subset B$ *and* $x \in C(B, s)$ *then* $x \in C(A, s)$

The Problem with Preference-Based Models

- This cannot capture **too much choice** effects
  - e.g. Iyengar and Lepper
  - People switch to choosing the status quo in larger choice sets
- Violates Independence of Irrelevant Alternatives for a fixed status quo
  - Status quo chosen in bigger choice set
  - Still available in smaller choice set
  - Yet not chosen in smaller choice set

Example 2


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
x status quo
y status quo
% Choosing x
2
6
12
20
Choice set size

Decision Avoidance

- One possible solution: models of decision avoidance
  - Try to avoid hard choices
- 'Easy' choice:
  - Make an active decision to select an alternative
  - May move away from the status quo
- 'Difficult' choice
  - May avoid thinking about the decision
  - End up with the status quo
- May cause switching to the SQ in larger choice sets
  - If this leads to more difficult choices

Models of Decision Avoidance

- What makes choice difficult?
- Conflict model
  - Difficulty in comparing two alternatives
- Information overload model
  - Ability to compare objects reduces with the size of the choice set

The Conflict Model

- DM endowed with a possibly incomplete preference ordering
- In any given choice set
  - If one alternative is preferred to all others, the DM chooses it
  - If not, may avoid decision by choosing the status quo
- If no suitable status quo, uses other decision making mechanism
  - 'Think harder' about the problem
  - Complete their preference ordering

The Conflict Decision Avoidance Model

- **Formal Representation:**

1. Choice is defined for any $\{Z, s\}$ by
   1. $C(Z, s) = \{x \in Z | x \succeq y \ \forall \ y \in Z\}$ if such set is non-empty
   2. otherwise $C(Z, s) = s$ if $s \in Z / T(Z)$
   3. otherwise $C(Z, s) = \{x \in Z | x \trianglerighteq y \ \forall \ y \in Z\}$

A Multi-Utility Representation

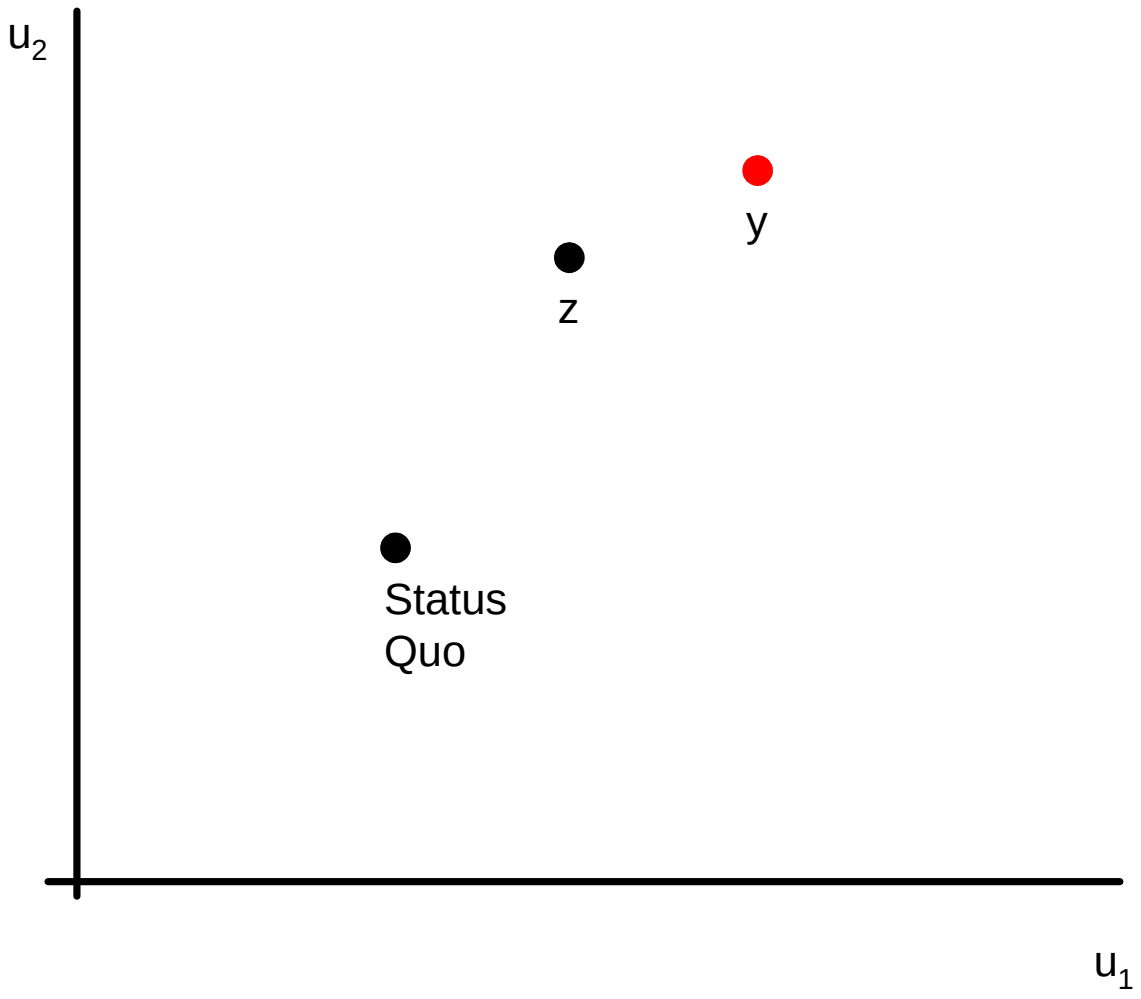
- Incomplete preference ordering $\succeq$ can be represented by a vector-valued utility function:

$$u(z) = \begin{pmatrix} u_1(z) \\ \vdots \\ u_n(z) \end{pmatrix}$$

- Such that

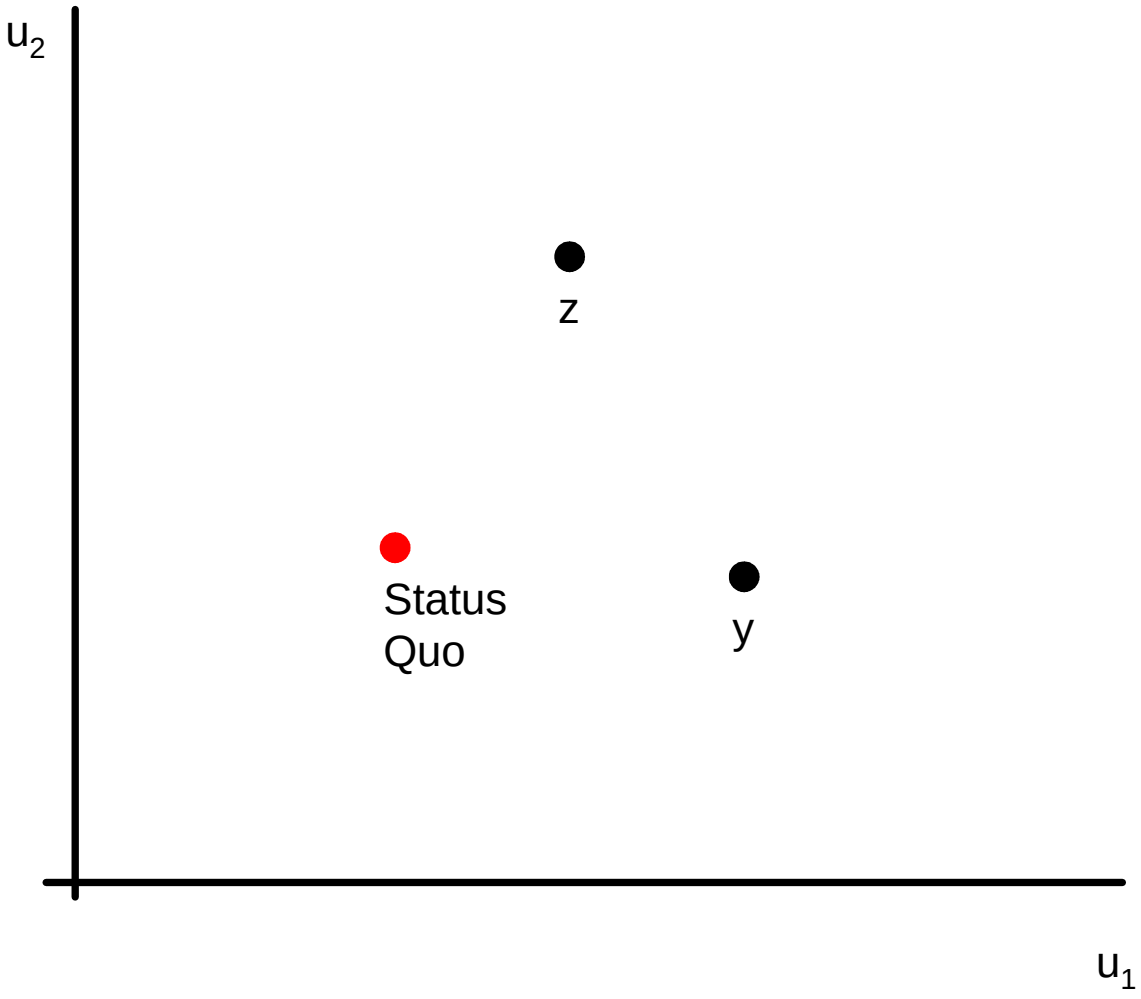
$$\begin{aligned} z &\succeq w \\ \text{if and only if } u_i(z) &\geq u_i(w) \ \forall \ i \in 1..n \end{aligned}$$

A Multi-Utility Representation



- Choose y as y is best object along all dimensions

A Multi-Utility Representation



- Choose status quo to avoid having to decide between z and y

Information Overload

- Alternative hypothesis: Information Overload
  - Large choice sets are inherently more difficult than small choice sets
    - Iyengar and Lepper [2000]
  - DM can compare all available options on a bilateral basis,
  - May still find large choice set difficult

Nested Preferences

- Modify Conflict model to allow for information overload
- Preferences may become less complete in large choice sets
- Replace fixed preference relation of Conflict model with nested preference relation
- **Nested Preferences:**
  - For every Z a preference relation $\succeq_Z$
  - Such that, for every $W \subset Z$

$$x \succeq_Z y \Rightarrow x \succeq_W y$$

  - but not

$$x \succeq_Z y \Longleftarrow x \succeq_W y$$

The Information Overload Model

- Modifies the Conflict Decision Avoidance Model....

1. Choice is defined for any $\{Z, s\}$ by
   1. $C(Z, s) = \{x \in Z | x \succeq_Z y \ \forall \ y \in Z\}$ if such set is non-empty
   2. otherwise $C(Z, s) = s$ if $s \in Z / T(Z)$
   3. otherwise $C(Z, s) = \{x \in Z | x \trianglerighteq y \ \forall \ y \in Z\}$

Behavioral Implications of Decision Avoidance Models

- Information overload model and conflict model:
  - **A1:** Limited status quo dependence
  - **A2:** Weak status quo conditional consistency
- Conflict model only
  - **A3:** Expansion

Limited Status Quo Dependence

- Choice can only depend on status quo in a limited way
- Making an object x the status quo can lead people to switch their choices to x...
- ...but cannot lead them to choose another alternative y
- **A1: LSQD:** *In any choice set, choice must be either*
  - *The status quo*
  - *What is chosen when there is no status quo*
- Note - not implied by preference-based models

Weak Status Quo Conditional Consistency

- Decision avoidance models allow for violations of SQCC, but only of a specific type
  - People may switch to choosing the status quo in larger choice sets
- **A2: Weak SQCC:** *For a fixed status quo*
  - *if x is chosen in a larger choice set*
  - *must also be chosen in a subset*
  - *unless x is the status quo*

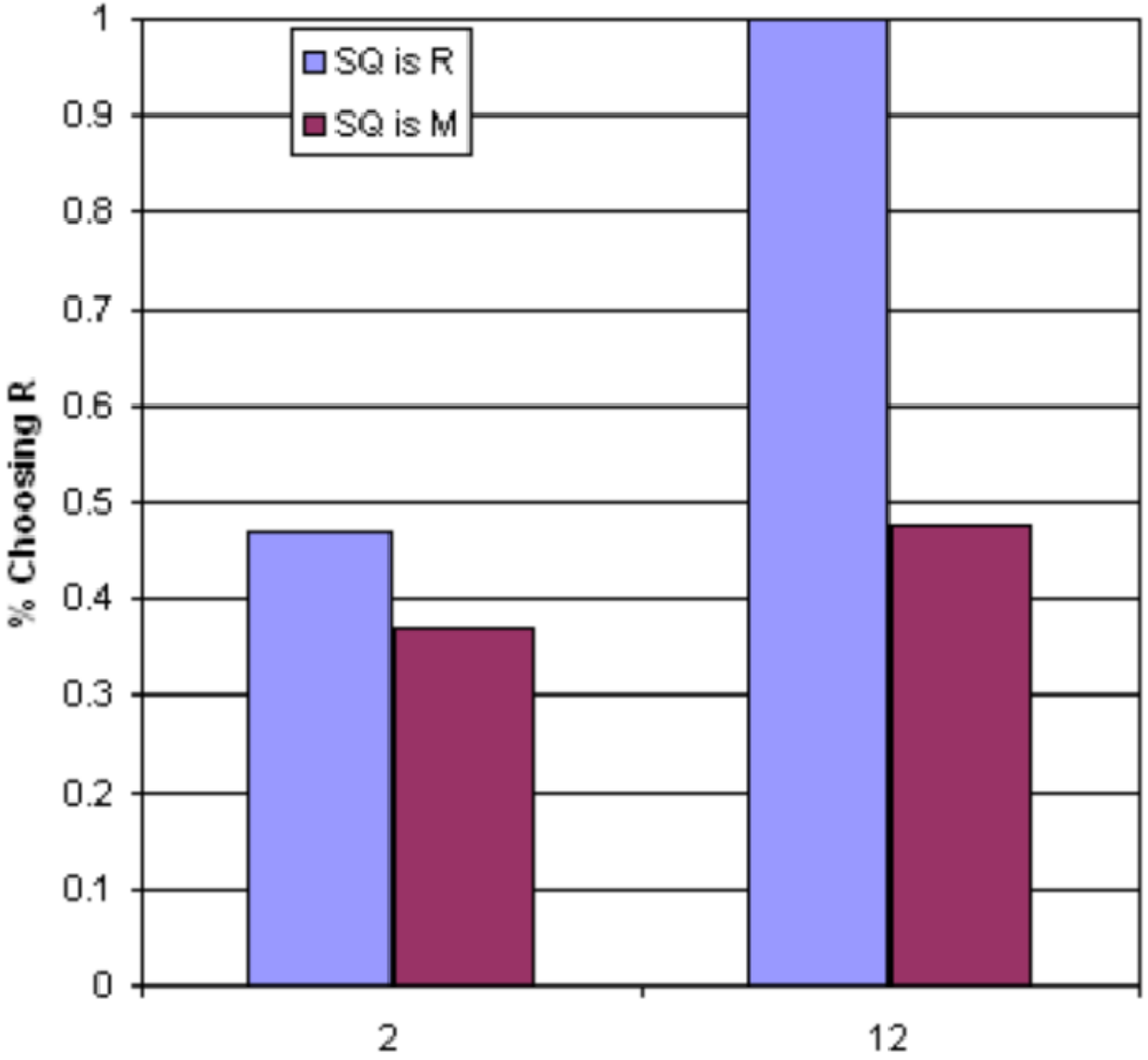
Expansion

- **A3: Expansion:** *Adding dominated options cannot lead people to switch to the status quo*
- Say x is chosen in a choice set Z when it is not the status quo
- Add option y to the choice set that is *dominated* by some $w \in Z$
  - w is chosen over y even when y is the status quo
- x must still be chosen from the larger choice set

Expansion

- Conflict model implies expansion
  - Adding dominated options does not make choice any more 'difficult'
- Information overload model does not imply expansion
  - DM may 'know' their preferred option in smaller choice set
  - Adding dominated options to the choice set degrades preferences
  - Can no longer identify preferred option in the larger choice set

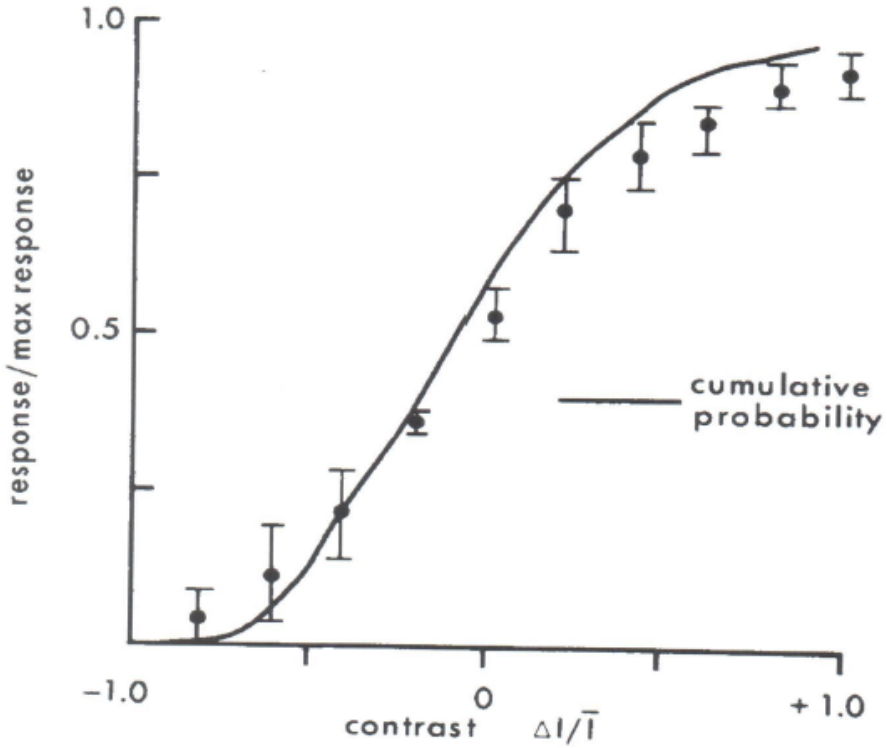
An Experimental Test of Expansion



Reference Points and Optimal Coding

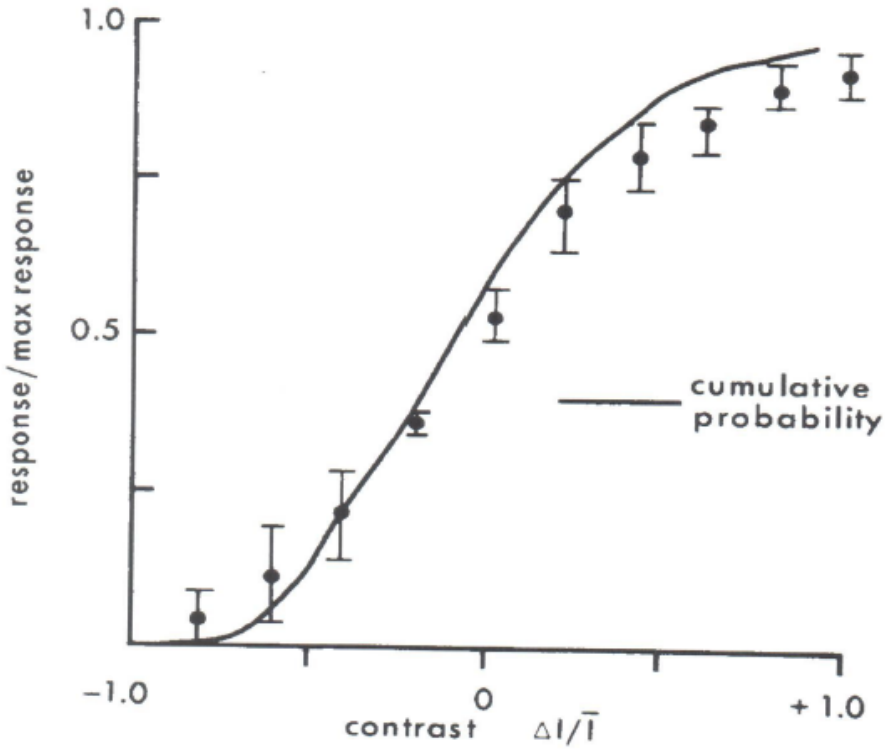
- One possible interpretation of reference point effects is that they focus attention on particular parts of the problem
- Could this be a rational use of neural resources?
  - Focus attention where it is most useful
- If so, may be a role for reference points affecting valuation and therefore choice
  - Reference points tell us what is most likely to happen
  - and so where it is most likely to be useful to make fine judgements
- This hypothesis is explored in Woodford [2012]

A Detour Regarding Blowflys



- Shows neural response to contrast differences in light sources (black dots)
- Also CDF of contrast differences in blowfly environment (line)

A Detour Regarding Blowflys



- Sharpest distinction occurs between contrasts which are likely to occur
- i.e slope of line matches the 'slope' of the dots

Rational Coding

- Blowflies seem to use neural resources to best differentiate between states that are most likely to occur
- Does this represent 'optimal' use of resources?
- Surprisingly not if costs are based on Shannon mutual information
- Why not?

The Effect of Priors

- Remember Shannon Mutual Information costs can be written as

$$-\left[H(\Gamma) - E(H(\Gamma|\Omega))\right] =$$
$$\sum_{\gamma\in\Gamma(\pi)} P(\gamma)\ln P(\gamma) - \sum_{\omega}\mu(\omega)\left(\sum_{\gamma\in\Gamma(\pi)}\pi(\gamma|\omega)\ln\pi(\gamma|\omega)\right)$$

where

$$P(\gamma) = \sum_{\omega\in\Omega}\pi(\gamma|\omega)\mu(\omega)$$

- Changing the precision of a signal in a given state (i.e. $\pi(\gamma|\omega)$) changes info costs by

$$(\ln(P(\gamma)) + 1)\,\frac{\partial P(\gamma)}{\partial\pi(\gamma|\omega)} - \mu(\omega)\,(\ln(\pi(\gamma|s) + 1)$$

The Effect of Priors

- But $\frac{\partial P(\gamma)}{\partial \pi(\gamma|\omega)} = \mu(\omega)$, so

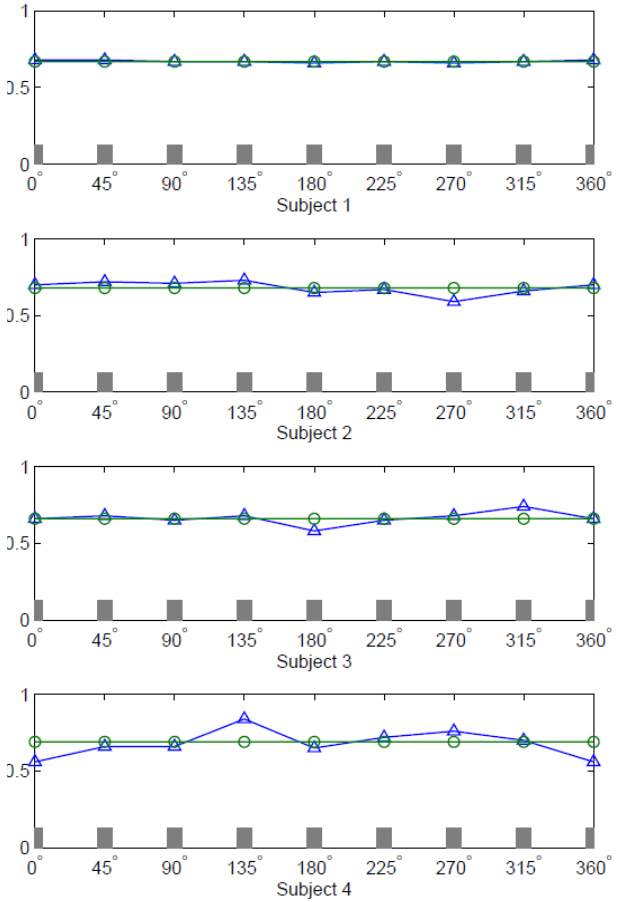
$$\mu(\omega)\left(\ln(P(\gamma)) - \ln(\pi(\gamma|s))\right)$$

- It is cheaper to get information about states that are less likely to occur
  - Intuition: you only pay the expected cost of information
  - Expected cost information about states that are unlikely to occur is low
- This offsets the lower value of gathering information about such states
  - In the Shannon model the two offset exactly
  - Prior probability of state should not matter for optimal coding

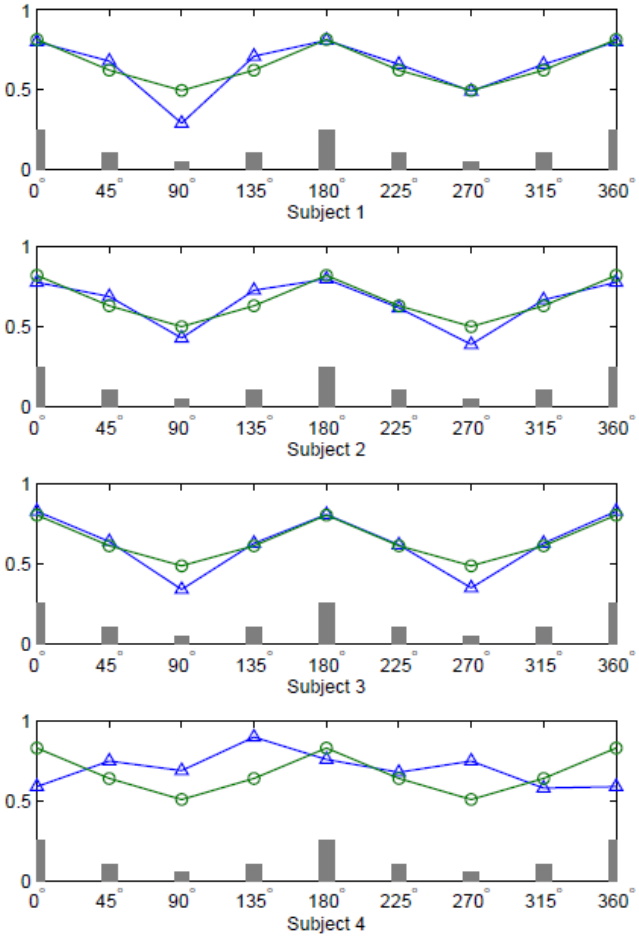
The Effect of Priors

- Does this hold up in practice?
- Experiment: Shaw and Shaw [1977]
  - Subjects had to report which of three letters had flashed onto a screen
  - Letter could appear at one of 8 locations (points on a circle)
- Two treatments
  - All positions equally likely
  - 0 and 180 degrees more likely
- Shannon prediction: behavior the same in both cases

Shaw and Shaw [1977]: Treatment 1



Shaw and Shaw [1977]: Treatment 2



Shannon Capacity

- This observation lead Woodford [2012] to consider an alternative cost function
  - Shannon Capacity
- Let

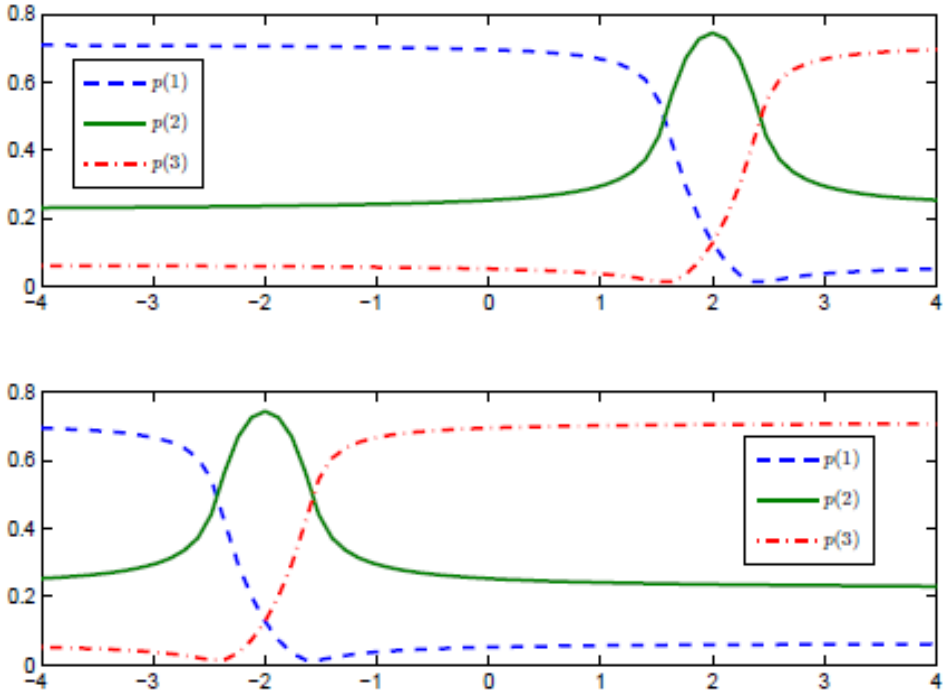
$$I_\mu(\Gamma, \Omega)$$

  be the Mutual Information between signal and state under prior beliefs μ
- Shannon Capacity is given by

$$\max_{\mu \in \Delta(\Omega)} I_\mu(\Gamma, \Omega)$$

- i.e. the maximal mutual information across all possible prior beliefs
- True priors no longer affect costs
- Signals on less likely states no cheaper than signals on more likely states

Shannon Capacity



- Optimal behavior when objective is linear in squared error
- Upper panel prior is $N(2, 1)$, lower panel prior is $N(-2, 1)$

Coding Values

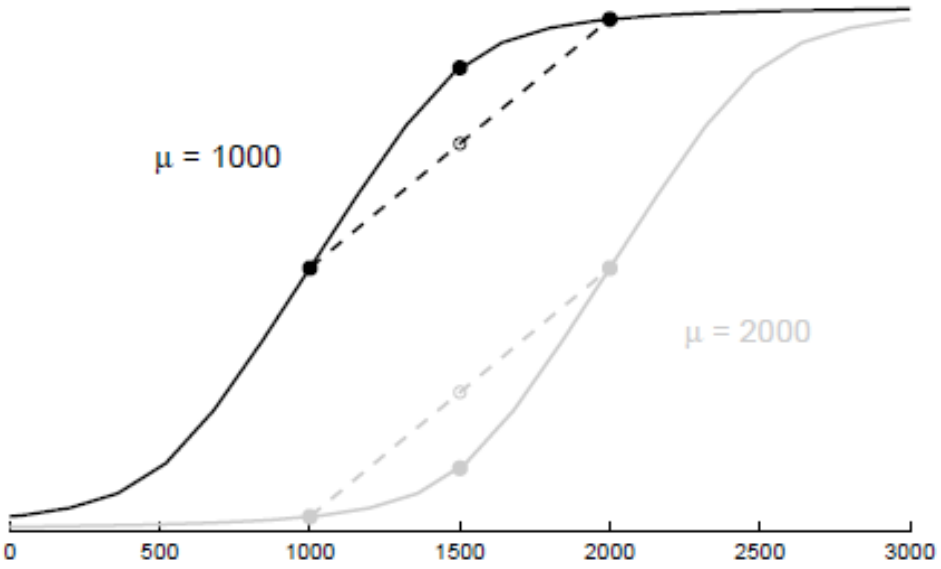
- One can apply this model to economic choice
- Assume that DM have to encode the value of a given alternative
- Assume alternative is characterized along different dimensions
- Has a limited capacity to encode value along each dimension
- Chooses optimal encoding given costs, prior beliefs and the task at hand

Reference Dependence

- This model can explain diminishing sensitivity
  - But not, in an obvious way, loss aversion
- Remember, diminishing sensitivity predicts
  - Risk aversion for gains
  - Risk seeking for losses
- E.g.
  - Choice 1: start with 1000, choose between a gain of 500 for sure or a 50% chance of a gain of 1000
  - Choice 2: start with 2000, choose between a loss of 500 for sure or a 50% chance of a loss of 1000

Reference Dependence

- Assume that the change in the reference point changes the prior distribution over final outcomes
  - Choice 2 has a mean which is 1000 higher than choice 1
  - Assume that prior is normal
- In Choice 1 1000 most likely, then 1500, then 2000
  - 1000 most precisely encoded, then 1500 then 2000
  - More 'sensitive' to the change between 1000 and 1500 than between 1500 and 2000
  - Leads to risk aversion
- In Choice 2 2000 most likely, then 1500, then 1000
  - 2000 most precisely encoded, then 1500 then 1000
  - More 'sensitive' to the change between 2000 and 1500 than between 1500 and 1000
  - Leads to risk loving



- Plot of Mean Squared Normalized Value under the two different coding schemes

Framing and Perception

- This is part of a developing literatature looking at behavioral biases from a perceptual standpoint
  - Khaw, Mel Win, Ziang Li, and Michael Woodford. **Risk aversion as a perceptual bias.** No. w23294. National Bureau of Economic Research, 2017.
  - Gabaix, Xavier, and David Laibson. **Myopia and discounting**. No. w23254. National bureau of economic research, 2017.
  - Adriani, Fabrizio, and Silvia Sonderegger. **Optimal similarity judgements in intertemporal choice**, 2015 .

Reference Dependent Preferences

- Strong evidence that people evaluate options relative to some reference point
- Change in reference point can change preferences
  - Endowment Effect
  - Risk aversion
- One robust finding is loss aversion
  - Losses loom larger than gains
  - Can explain the endowment effect and increased risk aversion for mixed choice
- One open question is where reference points come from
- Prospect theory is a workhorse model of choice under risk
  - Loss Aversion
  - Probability Weighting
  - Diminishing Sensitivity