Models of Reference Dependent Preferences

Mark Dean

Behavioral Economics G6943
Autumn 2019
Likely that there are many different causes of reference dependence

- As we discussed in the introduction

Broadly speaking two classes of models

1. Preference-based reference dependence
   - Reference points affect preferences which affect choices

2. ‘Rational’ reference dependence
   - Reference dependence as a rational response to costs
     - Effort costs
     - Attention Costs

Focus on the former, say a little about the latter
• In 1979 Kahneman and Tversky introduced the idea of ‘Loss Aversion’

• Basic idea: Losses loom larger than gains
  • Utility calculated on changes, not levels
  • The magnitude of the utility loss associated with losing $x$ is greater than the utility gain associated with gaining $x$

• Initially applied to risky choice

• Later also applied to riskless choice [Tversky and Kahneman 1991]

• Can explain
  • Endowment effect
  • Increased risk aversion for lotteries involving gains and losses
  • Status quo bias
A Simple Loss Aversion Model

- World consists of different dimensions
  - e.g cash and mugs
- Will be asked to choose between alternatives that provide different amount of each dimension
  \[
  \begin{pmatrix}
  x_c \\
  x_m \\
  \end{pmatrix}
  \]
- Has a reference point for each dimension
  \[
  \begin{pmatrix}
  r_c \\
  r_m \\
  \end{pmatrix}
  \]
- **Key Point:** Utility depends on changes, not on levels
A Simple Loss Aversion Model

- Utility of an alternative comes from comparison of output to reference point along each dimension

\[
\begin{pmatrix}
  x_c \\
  x_m 
\end{pmatrix}, \quad \begin{pmatrix}
  r_c \\
  r_m 
\end{pmatrix}
\]

- Utility for gains relative to \( r \) given by a utility function \( u \)

\[
\begin{align*}
  u_c(x_c) - u_c(r_c) & \text{ if } x_c > r_c \\
  u_m(x_m) - u(r_m) & \text{ if } x_m > r_m
\end{align*}
\]

- Utility of losses relative to \( r \) given buy \( u \) of the equivalent gain multiplied by \(-\lambda\) with \( \lambda > 1 \)

\[
\begin{align*}
  \lambda \left( u_c(x_c) - u_c(r_c) \right) & \text{ if } x_c < r_c \\
  \lambda \left( u_m(x_m) - u(r_m) \right) & \text{ if } x_m < r_m
\end{align*}
\]

- For simplicity assume that utilities are linear: \( u_c(x_c) = x_c \), \( u_m(x_m) = u_m x_m \)
A Simple Loss Aversion Model

- $x$ is a gain of $1$ and loss of $1$ mug relative to $r$
- Utility of $x$

$$1 - \lambda u_m$$
Loss Aversion and the Endowment Effect

• How can loss aversion explain the Endowment Effect (i.e. WTP/WTA gap)?

• Willingness to pay:
  • Let \((r_c, r_m)\) be the reference point with no mug
  • How much would they be willing to pay for the mug?
  • i.e. what is the \(z\) such that

\[
0 = U\begin{pmatrix} r_c \\ r_m \end{pmatrix} = U\begin{pmatrix} r_c - z \\ r_m + 1 \end{pmatrix}
\]

• Utility of buying a mug given by

\[
U\begin{pmatrix} r_c - z \\ r_m + 1 \end{pmatrix} = u_m - \lambda z
\]

• Break even buying price given by \(z = \frac{u_m}{\lambda}\)
Buying is a loss of $z$ and gain of 1 mug relative to $r$.

Utility of buying

$$u_m = \lambda z$$
Loss Aversion and the Endowment Effect

- Willingness to accept:
  - Let \((r_c, r_m)\) be the reference point with mug
  - How much would they be willing to sell your mug for?
  - i.e. what is the \(y\) such that

\[
0 = U \left( \frac{r_c}{r_m}, \frac{r_c}{r_m} \right) = U \left( \frac{r_c + y}{r_m - 1}, \frac{r_c}{r_m} \right)
\]

- Utility of selling a mug given by

\[
U \left( \frac{r_c + y}{r_m - 1}, \frac{r_c}{r_m} \right) = -\lambda u_m + y
\]

- Break even selling price given by \(y = \lambda u_m(1)\)
Selling is a gain of $y$ and loss of 1 mug relative to $r$.

Utility of selling

$$-\lambda u_m + y$$
Loss Aversion and the Endowment Effect

- Willingness to pay
  \[ z = \frac{u_m}{\lambda} \]

- Willingness to accept
  \[ y = \lambda u_m \]

- WTP/WTA ratio
  \[ \frac{z}{y} = \frac{1}{\lambda^2} \]

- Less that 1 for \( \lambda > 1 \)
Tversky and Kahneman [1991] provide an axiomatization of this model

**Axiom 1: Cancellation** if, for some reference point

\[
\left( \begin{array}{c} x_1 \\ z_2 \end{array} \right) \succeq \left( \begin{array}{c} z_1 \\ y_2 \end{array} \right) \quad \text{and} \quad \left( \begin{array}{c} z_1 \\ x_2 \end{array} \right) \succeq \left( \begin{array}{c} y_1 \\ z_2 \end{array} \right)
\]

then

\[
\left( \begin{array}{c} x_1 \\ x_2 \end{array} \right) \succ \left( \begin{array}{c} y_1 \\ y_2 \end{array} \right)
\]

(guarantees additivity)
Define the 'quadrant' that $x$ is in relative to $r$
Axiom 2: Sign Dependence  Let options $x$ and $y$ and reference points $s$ and $r$ be such that

1. $x$ and $y$ are in the same quadrant with respect to $r$ and with respect to $s$
2. $s$ and $r$ are in the same quadrant with respect to $x$ and with respect to $y$

Then $x \succeq y$ when $r$ is the status quo $\iff x \succeq y$ when $s$ is the status quo

- Guarantees that only the ‘sign’ matters
Axiom 3: Preference Interlocking  Say that, for some reference point \( r \), we saw that

\[
\begin{pmatrix}
  x_1 \\
  x_2 
\end{pmatrix}
\sim
\begin{pmatrix}
  w_1 \\
  w_2 
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
  z_1 \\
  x_2 
\end{pmatrix}
\sim
\begin{pmatrix}
  y_1 \\
  w_2 
\end{pmatrix}
\]

And, for another reference point \( s \) (that puts everything in the same quadrant, but maybe a different quadrant to \( r \))

\[
\begin{pmatrix}
  x_1 \\
  \bar{x}_2 
\end{pmatrix}
\sim
\begin{pmatrix}
  w_1 \\
  \bar{w}_2 
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
  z_1 \\
  \bar{x}_2 
\end{pmatrix}
\sim
\begin{pmatrix}
  y_1 \\
  \bar{w}_2 
\end{pmatrix}
\]

- Ensures that the same trade offs that work in the gain domain also work in the loss domain
Loss Aversion in Risky Choice

- Loss aversion can also lead to increased risk aversion for lotteries that involve gains and losses
- Now there is only 1 dimension (money)
- Lotteries evaluated as gains/losses relative to some reference point
- See also Kosegi and Rabin [2007]
- Again, assume linear utility for money
  - Utility of winning $x$ is $x$
  - Utility of losing $x$ is $-\lambda x$
Loss Aversion in Risky Choice
What is the certainty equivalence of
- 50% chance of gaining $10
- 50% chance of gaining $0

\[ x \] such that

\[ u_c(x) = 0.5 \times u_c(10) + 0.5 \times u_c(10) \]
\[ x = 0.5 \times 10 + 0.5 \times 0 = \$5 \]

What is the certainty equivalence of
- 50% chance of gaining $5
- 50% chance of losing $5

\[ y \] such that

\[ -\lambda u_c(-y) = 0.5 \times u_c(5) + 0.5 \times (-\lambda)) u_c(5) \]
\[ -\lambda y = 0.5 \times 5 - \lambda 0.5 \times 5 \]
\[ y = \frac{(1 - \lambda)}{\lambda} < 0 \]
Loss Aversion in Risky Choice

The graph illustrates the relationship between utility functions $U(-x)$ and $U(x)$, showing how loss aversion affects decision-making under risk.
A Unified Theory of Loss Aversion?

- We have claimed that loss aversion can explain
  - Increased Risk aversion for ‘mixed’ lotteries
  - Endowment Effect

- Though note somewhat different assumptions re reference points

- Is the same phenomena responsible for both behaviors?

- If so we would expect to find them correlated in the population

- Dean and Ortoleva [2014] estimate
  - $\lambda$
  - WTP/WTA gap

  In the same group of subjects

- Find a correlation of 0.63 (significant $p=0.001$)
  - See also Gachter et al [2007]

- However do not find such an effect in a recent larger study
• Prospect Theory: Kahneman and Tversky [1979]
• ‘Workhorse Model’ of choice under risk
• Combines
  • Loss Aversion
  • Cumulative Probability Weighting
  • Diminishing Sensitivity
Loss Aversion in Risky Choice

- Diminishing sensitivity:
  - Differences harder to distinguish as you move away from reference point (similar to perceptual psychology)
  - Leads to risk aversion for gains, risk loving for losses
- Looks like many other perceptual phenomena
Loss Aversion in Risky Choice

- Let $p$ be a lottery with (relative) prizes
  \[ x_1 > x_2 \ldots x_k > 0 > x_{k+1} > \ldots > x_n \]

- $p_i$ probability of winning prize $x_i$

- Utility of lottery $p$ given by

\[
\pi(p_1)u(x_1) \\
+ (\pi(p_2) - \pi(p_1)) u(x_1) \\
+ \ldots \\
+ (\pi(p_1 + \ldots + p_k) - \pi(p_1 + \ldots + p_{k-1})) u(x_k) \\
- (\pi(p_1 + \ldots + p_{k+1}) - \pi(p_1 + \ldots + p_k)) \lambda u(-x_{k+1}) \\
- \ldots \\
- (\pi(p_1 + \ldots + p_n) - \pi(p_1 + \ldots + p_{n-1})) \lambda u(-x_n)
\]
Kahneman and Tversky start with a model of behavior, and then derive axioms.

Arguably, model is compelling, axioms not so much.

An alternative approach is taken by Masatlioglu and Ok [2005].

Start with some axioms, and see what model obtains.
• $X$: finite set of alternatives
• ◇: Placeholder for no status quo
• $\mathcal{D}$: set of decision problems $\{A, x\}$ where $A \subset X$ and $x \in A \cup \diamond$
  • Note the enrichment of the data set
• $C : \mathcal{D} \Rightarrow X$: choice correspondence
**Axiom 1: Status Quo Conditional Consistency**  For any $x \in X \cup \diamond$, $C(A, x)$ obeys WARP

**Axiom 2: Dominance**  If $y = C(A, x)$ for some $A \subset B$ and $y \in C(B, \diamond)$ then $y \in C(B, x)$

**Axiom 3: Status Quo Irrelevance**  If $y \in C(A, x)$ and for every $\{x\} \neq T \subset A, x \notin C(T, x)$ then $y \in C(A, \diamond)$

**Axiom 4: Status Quo Bias**  If $x \neq y \in C(A, x)$, then $y = C(A, y)$
These axioms are necessary and sufficient for two representations:

Model 1: There exists

- Preference relation $\succeq$ on $X$
- A completion $\supseteq$

such that

\[
C(A, \diamond) = \{ x \in A | x \supseteq y \forall y \in A \}
\]
\[
C(A, x) = x \text{ if } \nexists y \in A \text{ s.t. } y \succ x
\]
\[
= \{ y \in A | y \supseteq z \forall z \succ x \} \text{ otherwise}
\]

Interpretation:

- $\succeq$ represents ‘easy’ comparisons
- If there is nothing ‘obviously’ better than the status quo, choose the status quo
- Otherwise think more carefully about all the alternatives which are obviously better than the status quo
- An equivalent representation
- Model 2: there exists
  - \( u : X \to \mathbb{R}^N \)
  - A strictly increasing function \( f : u(X) \to \mathbb{R} \)
    such that

\[
C(A, \diamond) = \arg\max_{x \in A} f(u(x)) \\
C(A, x) = x \text{ if } U_u(A, x) \text{ is empty} \\
= \arg\max_{x \in U_u(A,x)} f(u(x)) \text{ otherwise}
\]

Where \( U_u(A, x) = \{ y \in A \mid u(y) > u(x) \} \)
• Models of reference dependence discussed so far are preference-based.
• A status quo generates a set of preferences:

\[ \succeq_s \text{ for all } s \in X \cup \diamond \]

• Decision Maker chooses to maximize these preference:

\[ C(A, s) = \{ z \in A | z \succeq_s y \text{ for all } y \in A \} \]
For a *fixed* status quo, DM maximizes a *fixed* set of preferences

Looks like a ‘standard’ decision maker

**Status Quo Conditional Consistency (SQCC):**

For any \((A, s), (B, s)\)

- *Independence of Irrelevant Alternatives:* If \(x \in A \subseteq B\) and \(x \in C(B, s)\) then \(x \in C(A, s)\)
The Problem with Preference-Based Models

- This cannot capture **too much choice** effects
  - e.g. Iyengar and Lepper
  - People switch to choosing the status quo in larger choice sets

- **Violates Independence of Irrelevant Alternatives** for a fixed status quo
  - Status quo chosen in bigger choice set
  - Still available in smaller choice set
  - Yet not chosen in smaller choice set
Example 2

![Bar chart showing % Choosing x vs. Choice set size](chart.png)

- Red bars: x status quo
- Blue bars: y status quo

% Choosing x

Choice set size
One possible solution: models of decision avoidance

Try to avoid hard choices

‘Easy’ choice:
- Make an active decision to select an alternative
- May move away from the status quo

‘Difficult’ choice
- May avoid thinking about the decision
- End up with the status quo

May cause switching to the SQ in larger choice sets
- If this leads to more difficult choices
Models of Decision Avoidance

- What makes choice difficult?
- Conflict model
  - Difficulty in comparing two alternatives
- Information overload model
  - Ability to compare objects reduces with the size of the choice set
DM endowed with a possibly incomplete preference ordering

In any given choice set
  - If one alternative is preferred to all others, the DM chooses it
  - If not, may avoid decision by choosing the status quo

If no suitable status quo, uses other decision making mechanism
  - ‘Think harder’ about the problem
  - Complete their preference ordering
The Conflict Decision Avoidance Model

- **Formal Representation:**

1. Choice is defined for any \( \{Z, s\} \) by
   - \( C(Z, s) = \{x \in Z | x \succeq y \ \forall \ y \in Z\} \) if such set is non-empty
   - otherwise \( C(Z, s) = s \) if \( s \in Z / T(Z) \)
   - otherwise \( C(Z, s) = \{x \in Z | x \succeq y \ \forall \ y \in Z\} \)
Incomplete preference ordering $\succeq$ can be represented by a vector-valued utility function:

$$u(z) = \begin{pmatrix} u_1(z) \\ \vdots \\ u_n(z) \end{pmatrix}$$

Such that $z \succeq w$ if and only if $u_i(z) \geq u_i(w) \forall \ i \in 1..n$
Choose $y$ as $y$ is best object along all dimensions
• Choose status quo to avoid having to decide between $z$ and $y$
• Alternative hypothesis: Information Overload
  • Large choice sets are inherently more difficult than small choice sets
    • Iyengar and Lepper [2000]
  • DM can compare all available options on a bilateral basis,
  • May still find large choice set difficult
• Modify Conflict model to allow for information overload
• Preferences may become less complete in large choice sets
• Replace fixed preference relation of Conflict model with nested preference relation

**Nested Preferences:**

• For every $Z$ a preference relation $\succeq_Z$
• Such that, for every $W \subset Z$

\[ x \succeq_Z y \Rightarrow x \succeq_W y \]

• but not

\[ x \succeq_Z y \not\Rightarrow x \succeq_W y \]
The Information Overload Model

- Modifies the Conflict Decision Avoidance Model.

1. Choice is defined for any \( \{Z, s\} \) by
   
   1. \( C(Z, s) = \{x \in Z | x \geq_Z y \ \forall \ y \in Z\} \) if such set is non-empty
   2. otherwise \( C(Z, s) = s \) if \( s \in Z / T(Z) \)
   3. otherwise \( C(Z, s) = \{x \in Z | x \geq y \ \forall \ y \in Z\} \)
• Information overload model and conflict model:
  • **A1:** Limited status quo dependence
  • **A2:** Weak status quo conditional consistency

• Conflict model only
  • **A3:** Expansion
Choice can only depend on status quo in a limited way.

Making an object $x$ the status quo can lead people to switch their choices to $x$...

...but cannot lead them to choose another alternative $y$.

**A1: LSQD:** *In any choice set, choice must be either*

- *The status quo*
- *What is chosen when there is no status quo*

Note - not implied by preference-based models.
Decision avoidance models allow for violations of SQCC, but only of a specific type
- People may switch to choosing the status quo in larger choice sets

**A2: Weak SQCC:** *For a fixed status quo*
- *if* $x$ *is chosen in a larger choice set*
- *must also be chosen in a subset*
- *unless* $x$ *is the status quo*
A3: Expansion: Adding dominated options cannot lead people to switch to the status quo

Say $x$ is chosen in a choice set $Z$ when it is not the status quo

Add option $y$ to the choice set that is dominated by some $w \in Z$

  - $w$ is chosen over $y$ even when $y$ is the status quo

$x$ must still be chosen from the larger choice set
• Conflict model implies expansion
  • Adding dominated options does not make choice any more ‘difficult’

• Information overload model does not imply expansion
  • DM may ‘know’ their preferred option in smaller choice set
  • Adding dominated options to the choice set degrades preferences
  • Can no longer identify preferred option in the larger choice set
An Experimental Test of Expansion
One possible interpretation of reference point effects is that they focus attention on particular parts of the problem.

Could this be a rational use of neural resources?

- Focus attention where it is most useful.

If so, may be a role for reference points affecting valuation and therefore choice.

- Reference points tell us what is most likely to happen.
- And so where it is most likely to be useful to make fine judgements.

This hypothesis is explored in Woodford [2012].
A Detour Regarding Blowflys

- Shows neural response to contrast differences in light sources (black dots)
- Also CDF of contrast differences in blowfly environment (line)
A Detour Regarding Blowflys

- Sharpest distinction occurs between contrasts which are likely to occur
  - i.e. slope of line matches the 'slope' of the dots
Blowflies seem to use neural resources to best differentiate between states that are most likely to occur.

- Does this represent ‘optimal’ use of resources?
- Surprisingly not if costs are based on Shannon mutual information.
- Why not?
The Effect of Priors

• Remember Shannon Mutual Information costs can be written as

\[- [H(\Gamma) - E(H(\Gamma|\Omega))] = \]

\[
\sum_{\gamma \in \Gamma(\pi)} P(\gamma) \ln P(\gamma) - \sum_{\omega} \mu(\omega) \left( \sum_{\gamma \in \Gamma(\pi)} \pi(\gamma|\omega) \ln \pi(\gamma|\omega) \right)
\]

where

\[P(\gamma) = \sum_{\omega \in \Omega} \pi(\gamma|\omega) \mu(\omega)\]

• Changing the precision of a signal in a given state (i.e. \(\pi(\gamma|\omega))\) changes info costs by

\[(\ln(P(\gamma)) + 1) \frac{\partial P(\gamma)}{\partial \pi(\gamma|\omega)} - \mu(\omega) (\ln(\pi(\gamma|s) + 1)\]
But \( \frac{\partial P(\gamma)}{\partial \pi(\gamma,|\omega)} = \mu(\omega) \), so

\[
\mu(\omega) \left( \ln(P(\gamma)) - \ln(\pi(\gamma,s)) \right)
\]

- It is cheaper to get information about states that are less likely to occur
  - Intuition: you only pay the expected cost of information
  - Expected cost information about states that are unlikely to occur is low
- This offsets the lower value of gathering information about such states
  - In the Shannon model the two offset exactly
  - Prior probability of state should not matter for optimal coding
The Effect of Priors

• Does this hold up in practice?
• Experiment: Shaw and Shaw [1977]
  • Subjects had to report which of three letters had flashed onto a screen
  • Letter could appear at one of 8 locations (points on a circle)
• Two treatments
  • All positions equally likely
  • 0 and 180 degrees more likely
• Shannon prediction: behavior the same in both cases
Shaw and Shaw [1977]: Treatment 1
Shaw and Shaw [1977]: Treatment 2
This observation lead Woodford [2012] to consider an alternative cost function

- Shannon Capacity

Let

\[ I_\mu(\Gamma, \Omega) \]

be the Mutual Information between signal and state under prior beliefs \( \mu \)

Shannon Capacity is given by

\[ \max_{\mu \in \Delta(\Omega)} I_\mu(\Gamma, \Omega) \]

i.e. the maximal mutual information across all possible prior beliefs

- True priors no longer affect costs
- Signals on less likely states no cheaper than signals on more likely states
• Optimal behavior when objective is linear in squared error
• Upper panel prior is $N(2, 1)$, lower panel prior is $N(-2, 1)$
• One can apply this model to economic choice
• Assume that DM have to encode the value of a given alternative
• Assume alternative is characterized along different dimensions
• Has a limited capacity to encode value along each dimension
• Chooses optimal encoding given costs, prior beliefs and the task at hand
• This model can explain diminishing sensitivity
  • But not, in an obvious way, loss aversion
• Remember, diminishing sensitivity predicts
  • Risk aversion for gains
  • Risk seeking for losses
• E.g.
  • Choice 1: start with 1000, choose between a gain of 500 for sure or a 50% chance of a gain of 1000
  • Choice 2: start with 2000, choose between a loss of 500 for sure or a 50% chance of a loss of 1000
• Assume that the change in the reference point changes the prior distribution over final outcomes
  • Choice 2 has a mean which is 1000 higher than choice 1
  • Assume that prior is normal

• In Choice 1 1000 most likely, then 1500, then 2000
  • 1000 most precisely encoded, then 1500 then 2000
  • More ‘sensitive’ to the change between 1000 and 1500 than between 1500 and 2000
  • Leads to risk aversion

• In Choice 2 2000 most likely, then 1500, then 1000
  • 2000 most precisely encoded, then 1500 then 1000
  • More ‘sensitive’ to the change between 2000 and 1500 than between 1500 and 1000
  • Leads to risk loving
• Plot of Mean Squared Normalized Value under the two different coding schemes
This is part of a developing literature looking at behavioral biases from a perceptual standpoint

Reference Dependent Preferences

- Strong evidence that people evaluate options relative to some reference point
- Change in reference point can change preferences
  - Endowment Effect
  - Risk aversion
- One robust finding is loss aversion
  - Losses loom larger than gains
  - Can explain the endowment effect and increased risk aversion for mixed choice
- One open question is where reference points come from
- Prospect theory is a workhorse model of choice under risk
  - Loss Aversion
  - Probability Weighting
  - Diminishing Sensitivity