

# Preference for Commitment

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Behavioral Economics G6943  
Fall 2016

- In order to discuss preference for commitment we need to be able to discuss **preferences over menus**
- Interpretation: choosing a set of alternatives from which you will make a choice at a later date.
- What would be the standard way of assessing a menu of options  $A = \{a_1, a_2, a_3, \dots\}$ ?
- Assume that you will choose the best option from the menu at the later date
- Then a menu  $A$  is preferred to menu  $B$  if the best option in  $A$  is better than the best option in  $B$
- i.e.

$$A \succeq B \text{ if and only if}$$
$$\max_{a \in A} u(a) \geq \max_{b \in B} u(b)$$

- For a 'standard' decision maker, more options to choose from is always (weakly) better
- Add alternative  $a$  to a choice set  $A$ 
  - Either  $a$  is preferred to all the options already in  $A$ 
    - $a$  will be chosen from the expanded choice set
    - $\{a\} \cup A$  is better than  $A$
  - Or there is some  $b$  in  $A$  which is preferred to  $a$ 
    - $a$  will not be chosen from the expanded choice set
    - $\{a\} \cup A$  is no better, and no worse than  $A$
- DM will always prefer to have a bigger menu to choose from

$$\begin{aligned} B &\subset A \\ \Rightarrow A &\succeq B \end{aligned}$$

- This may not be the case if the DM suffers from problems of temptation:
- Classic example: A dieter might prefer to a restaurant with the menu

fish
salad

rather than one with the menu

fish
burger
salad

- Why?
- (At least) two possible reasons
  - ① Would prefer to not eat the burger, but worries they will succumb to temptation if the burger is available
  - ② Thinks they will be able to overcome the temptation to eat the burger, but it will be costly to do so

- We are going to discuss a model of menu preferences and choice that captures both these forces
- Based on the classic work of Gul and Pesendorfer [2001]
- Updated (and better explained) by Lipman and Pesendorfer [2013]

- Let  $C$  be a compact metric space
- $\Delta(C)$  set of all measures on the Borel  $\sigma$ -algebra of  $C$  (i.e. all lotteries)
  - Use lotteries because it means set of choice objects is convex
- Endow  $\Delta(C)$  with topology of weak convergence
- $Z$  all non empty compact subsets of  $\Delta(C)$  (Hausdorff topology)
- Let  $\succeq$  be a preference relation on  $Z$ 
  - Interpretation: preference over menus from which you will later get to choose
- Let  $\triangleright$  be a preference relation on  $\Delta(C)$ 
  - Interpretation: preferences when asked to choose from a menu

- For  $x, y \in Z$  and  $\alpha \in (0, 1)$  define

$$\begin{aligned} & \alpha x + (1 - \alpha)y \\ = & \{p = \alpha q + (1 - \alpha)r \mid q \in x, r \in y, \} \end{aligned}$$

- E.g. if  $x = \{\delta_a\}$ ,  $y = \{\delta_b, \delta_c\}$  the

$$\begin{aligned} & \alpha x + (1 - \alpha)y \\ = & \left\{ \begin{array}{l} \alpha a + (1 - \alpha)b \\ \alpha a + (1 - \alpha)c \end{array} \right\} \end{aligned}$$

- Mixture of all elements in menu  $x$  with all elements in menu  $y$

# Modelling Preference over Menus

- Using this set up we will place axioms on  $\succeq$  and  $\triangleright$
- First, we will consider conditions which are necessary and sufficient for the standard model
  - Single utility function
  - Represents  $\triangleright$  (choice from menus)
  - $\succeq$  (choice between menus) represented using largest utility in the set
- Next, consider how to alter these axioms in order to generate the 'Gul Pesendorfer' model
  - Allows for both 'temptation' and 'self control' to be expressed in menu preferences



Axiom 1 (Preference Relations)  $\succeq, \supseteq$  are complete preference relations

Axiom 2 (Independence)  $x \succeq y$  implies

$$\alpha x + (1 - \alpha)z \succeq \alpha y + (1 - \alpha)z \quad \forall x, y, z \in Z, \\ \alpha \in (0, 1)$$

- Notice that this is not the same as 'standard' independence
- Mixing operation is different
- Need to think a bit about how to interpret it

- Interpretation of independence: Standard Independence + Indifference to Timing of Uncertainty
  - Imagine we extended  $\succeq$  to preferences over lotteries over menus
  - Independence would now say that, if we prefer choosing from  $x$  to choosing from  $y$  then we prefer choosing from  $x$   $\alpha\%$  of the time (and  $z$   $(1 - \alpha)\%$  of the time) to choosing from  $y$   $\alpha\%$  of the time (and  $z$   $(1 - \alpha)\%$  of the time)
  - Randomization occurs before choosing at second stage
- Claim: choosing contingent plans in this set up gives rise to the same probability distribution over outcomes as come about from 'Gul Pesendorfer' mixing

- Example

$$\frac{1}{2}x + \frac{1}{2}z$$

$$x = \{x_1, x_2\}, z = \{z_1, z_2\}$$

- Gul-Pesendorfer mixing: a menu of

$$\left\{ \begin{array}{l} \frac{1}{2}x_1 + \frac{1}{2}z_1 \\ \frac{1}{2}x_2 + \frac{1}{2}z_1 \\ \frac{1}{2}x_1 + \frac{1}{2}z_2 \\ \frac{1}{2}x_2 + \frac{1}{2}z_2 \end{array} \right\}$$

- 'Standard' Mixing: 50% chance of menu  $x$ , 50% chance of menu  $y$ 
  - Contingent plan: choose either  $x_1$  or  $x_2$  from  $x$  and either  $y_1$  or  $y_2$  from  $y$
  - Uncertainty decided before second stage choice
  - Set of contingent plans gives rise to same menu of lotteries over outcomes as does GP mixing

- If timing of resolution of uncertainty is not important there is an equivalence between
  - Choosing a contingent plan for a lottery over menus
  - Choosing from a menu of lotteries generated by 'Gul Pesendorfer' mixing
- Thus, 'standard' independence and indifference to timing of uncertainty give rise to GP independence

Axiom 3 (Sophistication)  $x \cup \{p\} \succ x \Leftrightarrow p \triangleright q \ \forall \ q \in x$

- This is the axiom that links together first and second stage choice.
- Whether or not people are sophisticated is going to be an important empirical question
  - Do they understand the choices they will make from a given menu?
  - If not, may underestimate their degree of self control
  - e.g. sign up for gym memberships they do not use
  - or make costly commitments which they subsequently do not stick to.

Axiom 4 (Continuity) Three continuity conditions:

- ① (Upper Semi Continuity): The sets  $\{z \in Z | z \succeq x\}$  and  $\{p \in \Delta(C) | p \succeq q\}$  are closed for all  $x$  and  $q$
- ② (Lower vNM Continuity):  $x \succ y \succ z$  implies  $\alpha x + (1 - \alpha)z \succ y$  for some  $\alpha \in (0, 1)$
- ③ (Lower Singleton Continuity): The sets  $\{p : \{q\} \succeq \{p\}\}$  are closed for every  $q$

- The Standard Model of preference over menus

$$U(z) = \max_{p \in z} u(p)$$

for some linear, continuous utility  $u : \Delta(C) \rightarrow \mathbb{R}$  such that

- $U$  represents  $\succsim$
- $u$  represents  $\triangleright$



- Equivalent to axioms 1-4 and

$$x \succeq y \Rightarrow x \cup y \sim x$$

- $x \succeq y$  implies that the best alternative in  $x$  is weakly better than the best alternative in  $y$
- The best alternative in  $x \cup y$  is the same as the best alternative in  $x$
- Thus  $x \cup y \sim x$
- Note that this implies

$$x \supset y \Rightarrow x \succeq y$$

- Say  $y \succ x$ 
  - either  $x/y \succeq y$  in which case

$$x = x/y \cup y \sim x/y \succeq y \succ x$$

- or  $y \succeq x/y$

$$x = x/y \cup y \sim y \succ x$$

- Preference over menus given by

$$U(x) = \max_{p \in x} [u(p) + v(p)] - \max_{q \in x} v(q)$$

- $u$  : 'long run' utility
- $v$  : 'temptation' utility
- Interpretation:
  - Choose  $p$  to maximize  $u(p) + v(p)$
  - Suffer temptation cost  $v(p) - v(q)$
- Unlike the standard model, the Gul Pesendorfer model can lead to strict preference for smaller choice sets

$$x \supset y \text{ but } x \prec y$$

# Why Preference for Smaller Choice Sets?

Case 1: Commitment

Object	$u$	$v$
Salad	4	0
Fish	2	1
Burger	1	4

- Which menu would the DM prefer?  $\{s\}$  or  $\{s, b\}$ ?

$$\begin{aligned}U(\{s\}) &= \max_{x \in \{s\}} (u(x) + v(x)) - \max_{y \in \{s\}} v(y) \\&= 4 + 0 - 0 \\&= 4\end{aligned}$$

$$\begin{aligned}U(\{s, b\}) &= \max_{x \in \{s, b\}} (u(x) + v(x)) - \max_{y \in \{s, b\}} v(y) \\&= 1 + 4 - 4 \\&= 1\end{aligned}$$

# Why Preference for Smaller Choice Sets?

Case 1: Commitment

Object	$u$	$v$
Salad	4	0
Fish	2	1
Burger	1	4

- Menu  $\{s\}$  preferred to  $\{s, b\}$
- Interpretation:  $b$  would be chosen from the latter menu
  - $u(b) + v(b) > u(s) + v(s)$
- But  $s$  has higher long run utility
  - $u(s) > u(b)$
- The DM would rather not have  $b$  in their menu, because if it is available they will choose it.

# Why Preference for Smaller Choice Sets?

Case 1: Commitment

- More generally, consider  $p, q$ , such that

$$\begin{aligned}u(p) &> u(q) \\ u(q) + v(q) &> u(p) + v(p)\end{aligned}$$

- Then

$$\begin{aligned}U(\{p\}) &= u(p) \\ U(\{p, q\}) &= u(q) + v(q) - v(q) = u(q) \\ U(\{q\}) &= u(q)\end{aligned}$$

- Interpretation: give in to temptation and choose  $q$
- 'Weak set betweenness'

$$\{p\} \succ \{p, q\} \sim \{q\}$$

# Why Preference for Smaller Choice Sets?

Case 2: Avoid 'Willpower Costs'

Object	$u$	$v$
Salad	4	0
Fish	2	1
Burger	1	4

- Which menu would the DM prefer?  $\{s\}$  or  $\{s, f\}$ ?

$$\begin{aligned}U(\{s\}) &= \max_{x \in \{s\}} (u(x) + v(x)) - \max_{y \in \{s\}} v(y) \\&= 4 + 0 - 0 \\&= 4\end{aligned}$$

$$\begin{aligned}U(\{s, f\}) &= \max_{x \in \{s, f\}} (u(x) + v(x)) - \max_{y \in \{s, f\}} v(y) \\&= 4 + 0 - 1 \\&= 3\end{aligned}$$

# Why Preference for Smaller Choice Sets?

Case 2: Avoid 'Willpower Costs'

Object	$u$	$v$
Salad	4	0
Fish	2	1
Burger	1	4

- Menu  $\{s\}$  is preferred to menu  $\{s, f\}$
- However, this time,  $s$  would be chosen from both menus, as

$$u(s) + v(s) > u(f) + v(f)$$

- The DM still prefers to have  $f$  removed from the menu because it is more tempting:  $v(f) > v(s)$
- The DM is able to exert self control if both options are on the menu, but it is costly to do so

# Why Preference for Smaller Choice Sets?

Case 2: Avoid 'Willpower Costs'

- More generally, consider  $p, q$ , such that

$$u(p) > u(q)$$

$$v(q) > v(p)$$

$$u(p) + v(p) > u(q) + v(q)$$

- Then

$$U(\{p\}) = u(p)$$

$$U(\{p, q\}) = u(p) + v(p) - v(q)$$

$$U(\{q\}) = u(q)$$

- Interpretation: fight temptation, but this is costly
- 'Strict set betweenness'

$$\{p\} \succ \{p, q\} \succ \{q\}$$



# Temptation and Self Control

- We say that  $q$  tempts  $p$  if  $\{p\} \succ \{p, q\}$
- We say that a decision maker exhibits self control at  $y$  if there exists  $x, z$  such that  $x \cup z = y$  and

$$\{x\} \succ \{y\} \succ \{z\}$$

- $\{x\} \succ \{y\}$  implies there exists something in  $z$  which is tempting relative to items in  $x$
- $\{y\} \succ \{z\}$  implies tempting item not chosen
- if it were then

$$\begin{aligned} \max_{p \in y} u(p) + v(p) &= \max_{p \in z} u(p) + v(p) \Rightarrow \\ U(y) &= \max_{p \in y} (u(p) + v(p)) - \max_{q \in y} v(q) \\ &\leq \max_{p \in z} (u(p) + v(p)) - \max_{q \in z} v(q) \\ &= U(z) \end{aligned}$$

# Why 'Long Run' and 'Temptation' Utilities?

- So far we have described  $u$  as 'long run' utility and  $v$  as 'temptation' utility
- Why is this a behaviorally appropriate description?
- $u$  describes choices over singleton menus:

$$U(\{p\}) = u(p) + v(p) - v(p) = u(p)$$

and so describes preferences when the DM is not tempted

# Why 'Long Run' and 'Temptation' Utilities?

- $v$  leads to temptation:  $q$  tempts  $p$  only if  $v(q) > v(p)$ 
  - Case 1:  $u(p) + v(p) \geq u(q) + v(q)$

$$\begin{aligned}U(\{p\}) &> u(\{p, q\}) \\ \Rightarrow u(p) &> u(p) + v(p) - \max_{r \in \{p, q\}} v(r) \\ \Rightarrow \max_{r \in \{p, q\}} v(r) &> v(p) \\ \Rightarrow v(q) = \max_{r \in \{p, q\}} v(r) &> v(p)\end{aligned}$$

# Why 'Long Run' and 'Temptation' Utilities?

- $v$  leads to temptation:  $q$  tempts  $p$  only if  $v(q) > v(p)$ 
  - Case 2:  $u(q) + v(q) > u(p) + v(p)$

$$\begin{aligned}U(\{p\}) &> u(\{p, q\}) \\ \Rightarrow u(p) &> u(q) + v(q) - \max_{r \in \{p, q\}} v(r) \\ \Rightarrow u(p) + \max_{r \in \{p, q\}} v(r) &> u(q) + v(q) \\ \Rightarrow \max_{r \in \{p, q\}} v(r) &= v(q) > v(p)\end{aligned}$$

- Last line follows from assumption  $u(q) + v(q) > u(p) + v(p)$

- Imagine that differences in  $v$  are large relative to differences in  $u$
- In the limit, model reduces to

$$U(x) = \max_{p \in x} u(p) \text{ s.t. } v(p) \geq v(q) \quad \forall q \in x$$

- This is the 'Stolz' model
- Implies no strict set betweenness, and not self control
- $\beta - \delta$  model is of this class

# Axiomatic Characterization of GP Model

- Set Betweenness: for any  $x, y$  s.t  $x \succeq y$

$$x \succeq x \cup y \succeq y$$

- Notice the difference to the 'standard' model

$$x \succeq y \Rightarrow x \cup y \sim x$$

- Smaller sets can be strictly preferred

# Axiomatic Characterization of GP Model

- Set Betweenness: for any  $x, y$  s.t  $x \succeq y$

$$x \succeq x \cup y \succeq y$$

- Necessity:

- $x \succeq y$  implies that

$$u(p^x) + v(p^x) - v(q^x) \geq u(p^y) + v(p^y) - v(q^y)$$

where

$$p^i = \arg \max_{p \in i} u(p) + v(p)$$

and

$$q^i = \arg \max_{q \in i} v(q)$$

- NTS  $x \succeq x \cup y$

# Axiomatic Characterization of GP Model

- Two cases:
- Case 1:  $u(p^x) + v(p^x) \geq u(p^y) + v(p^y)$

$$\begin{aligned}u(p^x) + v(p^x) &\geq u(p^y) + v(p^y) \Rightarrow \\u(p^x) + v(p^x) &= u(p^{x \cup y}) + v(p^{x \cup y}) \Rightarrow \\u(p^x) + v(p^x) - v(q^x) &\geq u(p^{x \cup y}) + v(p^{x \cup y}) - v(q^{x \cup y})\end{aligned}$$

- Case 2:  $u(p^x) + v(p^x) < u(p^y) + v(p^y)$ 
  - implies  $v(q^x) \leq v(q^y)$  as  $x$  is preferred to  $y$

$$\begin{aligned}u(p^y) + v(p^y) &= u(p^{x \cup y}) + v(p^{x \cup y}) \\v(q^{x \cup y}) &= v(q^y) \Rightarrow \\u(p^{x \cup y}) + v(p^{x \cup y}) - v(q^{x \cup y}) &= u(p^y) + v(p^y) - v(q^y) \\&\leq u(p^x) + v(p^x) - v(q^x)\end{aligned}$$



# Axiomatic Characterization of GP Model

## Theorem

$\succeq$  satisfies Axioms 1, 2, 4 and set betweenness if and only if it has a Stolz representation or a G-P representation

## Theorem

The proper relation  $\succeq$  and  $\triangleright$  satisfy Axioms 1-4 and set betweenness if and only if

- $\succeq$  has a Stolz representation and  $p \triangleright q$  if and only if  $v(p) > v(q)$  or  $v(p) = v(q)$  and  $u(p) \geq u(q)$
- or  $\succeq$  has a G-P representation and  $u(p) + v(p)$  represents  $\triangleright$

# Sketch of Proof that Axioms Imply Representation

- **Lemma 1:** Axioms 1, 2, 4 imply a linear  $U : Z \rightarrow \mathbb{R}$  that represents  $\succeq$  and is continuous on singleton sets
  - This is standard, and makes use of the mixture space axioms

# Sketch of Proof that Axioms Imply Representation

- **Lemma 2:** Show that

$$\begin{aligned}U(x) &= \max_{p \in x} \min_{q \in x} U(\{p, q\}) \\&= \min_{q \in x} \max_{p \in x} U(\{p, q\})\end{aligned}$$

- Utility depends only on 'chosen element', and 'most tempting element'
- **Proof: Let**  $\bar{u} = \max_{p \in x} \min_{q \in x} U(\{p, q\}) = U(\{p^*, q^*\})$
- Note that  $U(\{p^*, q\}) \geq U(\{p^*, q^*\}) = \bar{u} \ \forall \ q \in A$
- Set betweenness implies  $\bar{u} \leq U(\cup_{q \in x} \{p^*, q\}) = U(x)$
- Also, for every  $p \in A$ ,  $\exists \ q_p \in A$  such that  $U(\{p, q_p\}) \leq \bar{u}$
- By set betweenness  $\bar{u} \geq U(\cup_{p \in A} \{p, q_p\}) = U(x)$

# Sketch of Proof that Axioms Imply Representation

- **Lemma 3:** Show that

$$\begin{aligned}U(\{x\}) &> U(\{x, y\}) > U(\{y\}) \\U(\{a\}) &> U(\{a, b\}) > U(\{b\})\end{aligned}$$

implies

$$\begin{aligned}&U(\alpha \{x, y\} + (1 - \alpha) \{a, b\}) \\&= U(\{\alpha x + (1 - \alpha)a, \alpha y + (1 - \alpha)b\})\end{aligned}$$

- This comes straight from super independence and the fact that  $\alpha x + (1 - \alpha)a$  is the best and  $\alpha y + (1 - \alpha)b$  the most tempting element

# Sketch of Proof that Axioms Imply Representation

- Define

$$\begin{aligned}u(p) &= U(\{p\}) \\v(s; p, q, \delta) &= \frac{U(\{p, q\}) - U(\{p, (1 - \delta)q + \delta s\})}{\delta}\end{aligned}$$

- $u$  is the long run utility
- $v$  is a measure of how tempting  $s$  is relative to  $p$  and  $q$  (under the assumption  $p$  is chosen)

# Sketch of Proof that Axioms Imply Representation

- **Lemma 4:** Show that, if

$$U(\{p\}) > U(\{p, (1 - \delta)r + \delta s\}) > U(\{(1 - \delta)r + \delta s\})$$

for all  $s \in \Delta(C)$ , then

- ①  $U(\{p\}) > U(\{p, s\}) > U(s) \Rightarrow v(s; p, q, \delta) = U(\{p, q\}) - U(\{p, s\})$
- ②  $v(p; p, q, \delta) = U(\{p, q\}) - U(\{p\})$

- Follows from Lemma 3

# Sketch of Proof that Axioms Imply Representation

- **Lemma 5:** Show that, if

$$U(\{p\}) \geq U(\{p, q\}) \geq U(\{q\})$$

and for some  $r$  and  $\delta$

$$U(\{p\}) > U(\{p, (1 - \delta)r + \delta s\}) > U(\{(1 - \delta)r + \delta s\})$$

for all  $s \in \Delta(C)$ , then

$$\begin{aligned} & U(\{p, q\}) \\ = & \max_{w \in \{p, q\}} [u(w) + v(w; p, r, \delta)] - \max_{z \in \{p, q\}} [v(z; p, r, \delta)] \end{aligned}$$

# Sketch of Proof that Axioms Imply Representation

- **Proof** (assuming)

$$U(\{p\}) > U(\{p, q\}) > U(\{q\})$$

- By previous lemma

$$\begin{aligned}v(q; p, r, \delta) &= U(\{p, r\}) - U(\{p, q\}) \\&\geq U(\{p, r\}) - U(\{p\}) \\&= v(p; p, r, \delta)\end{aligned}$$

and so

$$\max_{z \in \{p, q\}} [v(z; p, r, \delta)] = v(q; p, r, \delta)$$

- Also

$$\begin{aligned}u(p) + v(p; p, r, \delta) &= U(\{p\}) + U(\{p, r\}) - U(\{p\}) = U(\{p, r\}) \\u(q) + v(q; p, r, \delta) &= U(\{q\}) + U(\{p, r\}) - U(\{p, q\})\end{aligned}$$

and so

$$\max_{w \in \{p, q\}} [u(w) + v(w; p, r, \delta)] = u(p) + v(p; p, r, \delta)$$



# Sketch of Proof that Axioms Imply Representation

- This then implies

$$\begin{aligned} & \max_{w \in \{p, q\}} [u(w) + v(w; p, r, \delta)] - \max_{z \in \{p, q\}} [v(z; p, r, \delta)] \\ &= u(p) + v(p; p, r, \delta) - v(q; p, r, \delta) \quad (1) \\ &= U(\{p\}) + U(\{p, r\}) - U(\{p\}) - U(\{p, r\}) + U(\{p, q\}) \\ &= U(\{p, q\}) \quad (2) \end{aligned}$$

# Sketch of Proof that Axioms Imply Representation

- Finally, pick  $p, q$  such that

$$U(\{p\}) > U(\{p, q\}) > U(\{q\})$$

(if such exists) and pick  $\delta$  such that

$$U(\{p\}) > U(\{p, (1 - \delta)q + \delta s\}) > U(\{(1 - \delta)q + \delta s\})$$

for all  $s$  (which we can do by continuity)

- Define  $v(s)$  as  $v(s; p, q, \delta)$ , and show that  $v(s; p, q, \delta)$  doesn't depend on the specifics of the last three parameters.
- Lemma 5 therefore gives

$$U(\{p, q\}) = \max_{w \in \{p, q\}} [u(w) + v(w)] - \max_{z \in \{p, q\}} [v(z)]$$

- Lemma 2 then extends this result to an arbitrary set  $A$

- Imagine

$$\{p\} \succ \{p, q\} \succ \{q\} \succ \{q, r\} \succ \{r\}$$

- DM can resist  $q$  for  $p$  and resist  $r$  for  $q$ .
  - Can they resist  $r$  for  $p$ ?
- Under the GP model, the above implies

$$\begin{aligned}u(p) &> u(q) > u(r) \\v(r) &> v(q) > v(p) \\u(p) + v(p) &> u(q) + v(q) > u(r) + v(r)\end{aligned}$$

- Which in turn implies

$$\{p\} \succ \{p, r\} \succ \{r\}$$

- 'Self Control is Linear'

## Discussion: What is Willpower?

- It seems that the following statement is meaningful:
  - Person A has the same long run preferences as person B
  - Person A has the same temptation as person B
  - Person A has more willpower than person B
- Yet this is not possible in the GP model
- Alternative: Masatlioglu, Nakajima and Ozdenoren [2013]

$$\begin{aligned} U(z) &= \max_{p \in z} u(p) \\ \text{subject to } \max_{q \in z} v(q) - v(p) &\leq w \end{aligned}$$

# Discussion: Strict Set Betweenness and Random Strolz

- Does  $\{p\} \succ \{p, q\} \succ \{q\}$  imply self control?
- Imagine that you are a Strolz guy with  $u(p) > u(q)$ , but are not sure that you will be tempted
- Half the time

$$v(p) = v(q)$$

half the time

$$v(p) < v(q)$$

- Implies

$$\begin{aligned}U(\{p\}) &= u(p) \\U(\{p, q\}) &= \frac{u(p) + u(q)}{2} \\U(\{q\}) &= u(q)\end{aligned}$$

- Strict set betweenness without self control

- Say with probability  $\varepsilon$  won't be tempted so

$$\hat{U}(z) = (1 - \varepsilon)U(z) + \varepsilon \max_{p \in z} u(p)$$

- Can lead to violations of set betweenness.
- Let  $g = \text{gym}$ ,  $j = \text{jog}$ ,  $t = \text{tv}$

$$u(g) > u(j) > u(t)$$

$$v(g) < v(j) < v(t)$$

$$u(j) + v(j) > u(t) + v(t) > u(g) + v(g)$$

- For  $\varepsilon$  small

$$\{t, j\} \succ \{t, g\}$$

as

$$\begin{aligned}U(\{t, j\}) &= u(j) + v(j) - v(t) \\U(\{t, g\}) &= u(t)\end{aligned}$$

- but

$$\{t, j, g\} \succ \{t, j\}$$

as with probability  $\varepsilon$  no temptation and will go to the gym

## Discussion: Preference for Flexibility

- Consider choice between menus of drinks cocoa or lemonade
- Must choose between menus now, but your choice from those menus will occur on March 1st
- Which would you prefer?

$\{c\}$ ,  $\{l\}$  or  $\{c, l\}$ ?

- Choice of  $\{c, l\}$  over both  $\{c\}$  and  $\{l\}$  is a violation of set betweenness



## Discussion: Preference for Flexibility

- $X$  : set of alternatives
- $S$  : set of states
- $\mu \in \Delta(S)$ : probability distribution over states
- $u : X \times S \rightarrow \mathbb{R}$  : utility function
  - $u(x, s)$  utility of alternative  $x$  in state  $s$
- Preference uncertainty driven by uncertainty about  $s$

## Discussion: Preference for Flexibility

- Let  $A$  be a menu of alternatives
- Choice from  $A$  will take place **after** the state is known
- Value of  $A$  **before** the state is known given by

$$U(A) = \sum_{s \in S} \mu(s) \max_{x \in A} u(x, s)$$

- $U$  represents **choice between menus**

## Discussion: Preference for Flexibility

- The 'preference uncertainty' model implies a (potentially strict) preference for larger choice sets

$$A \succeq B \Rightarrow A \cup B \succeq A$$

- Compare to 'standard' model

$$A \succeq B \Rightarrow A \cup B \sim A$$

- And Set Betweenness

$$A \succeq B \Rightarrow A \cup B \preceq A$$

- Preference uncertainty can provide a powerful force that works against a preference for commitment
  - See Amador, Werning and Angeletos [2006]

- So far, we have assumed that a DM is sophisticated
  - They understand their second stage choice
  - Implemented by the axiom  $x \cup \{p\} \succ x \Leftrightarrow p \triangleright q \ \forall \ q \in x$
- What about a DM who is not sophisticated?

- Example 1: A DM who ignores temptation

Object	$u$	$v$
Salad	4	0
Fish	2	1
Burger	1	4

- Assume these preferences represent choices that the DM will make from the menu
- But they believe that their choices will be governed by  $u$
- Such a DM will prefer  $\{s, b\}$  to  $\{s\}$ , but when faced with the choice from  $\{s, b\}$  will choose  $b$
- Such a DM will violate sophistication
  - Never exhibit a preference for commitment

- Example 2: A DM who underestimates temptation

Object	$u$	$v$	$v'$
Salad	5	0	0
Fish	2	1	1
Burger	1	9	5

- Assume that a DM has temptation driven by  $v$ , but believes that they have temptation driven by  $v'$
- They are offered the chance to buy a 'commitment contract' where they have to pay \$2 if they eat the burger
- Assume that  $u(2) = 2$ ,  $v(2) = 2$  the  $u$  of money is additive with  $u$  of consumption and the  $v$  of money is additive with the  $v$  of consumption
- Let  $b + c$  be the burger with the commitment contract

- Example 2: A DM who underestimates temptation

Object	$u$	$v$	$v'$
Salad	5	0	0
Fish	2	1	1
Burger	1	9	5
B+C	-1	7	3

- The DM will have preferences

$$\{b + c, s\} \succ \{b, s\}$$

as

$$\begin{aligned} U(\{b + c, s\}) &= u(s) + v'(s) - v'(b + c) = 2 \\ &> 1 = u(b) = U(\{b, s\}) \end{aligned}$$

- But the DM will actually choose  $b + c$  over  $s$  at the second stage as

$$u(b + c) + v(b + c) = 6 > 5 = u(s) + v(s)$$

- Example 2: A DM who underestimates temptation

Object	$u$	$v$	$v'$
Salad	5	0	0
Fish	2	1	1
Burger	1	9	5
B+C	-1	7	3

- End up with lower 'long run' utility
- Also a violation of sophistication as

$$\{b + c, s\} \succ \{b + c\}$$

but  $b + c$  will be chosen from the former menu



- Menu preferences allow us to formalize a model of preference for commitment
- We argued that this is a sign that people have problems with temptation

- Temptation: Preference for Commitment

$$A \succeq B \Rightarrow A \cup B \preceq A$$

- Preference uncertainty: Preference for Flexibility

$$A \succeq B \Rightarrow A \cup B \succeq A$$

- Compare to 'standard' model

$$A \succeq B \Rightarrow A \cup B \sim A$$

- Gul and Pesendorfer provide a model which allows for both temptation and self control

$$U(x) = \max_{p \in x} [u(p) + v(p)] - \max_{q \in x} v(q)$$

- Characterized by set betweenness:  $x \succeq y \Rightarrow x \succeq x \cup y \succeq y$