Preference for Commitment

Mark Dean

Behavioral Economics G6943
Autumn 2019

- In order to discuss preference for commitment we need to be able to discuss **preferences over menus**
- Interpretation: choosing a set of alternatives from which you will make a choice at a later date.
- What would be the standard way of assessing a menu of options $A = \{a_1, a_2, a_3, ...\}$?
- Assume that you will choose the best option from the menu at the later date
- Then a menu A is preferred to menu B if the best option in A is better than the best option in B
- i.e.

$$\begin{aligned} A &\succeq B \text{ if and only if} \\ \max_{a \in A} u(a) &\geq \max_{b \in B} u(b) \end{aligned}$$

- For a 'standard' decision maker, more options to choose from is always (weakly) better
- Add alternative a to a choice set A
  - Either a is preferred to all the options already in A
    - a will be chosen from the expanded choice set
    - $\{a\} \cup A$ is better than A
  - Or there is some b in A which is preferred to a
    - a will not be chosen from the expanded choice set
    - $\{a\} \cup A$ is no better, and no worse than A
- DM will always prefer to have a bigger menu to choose from

$$\begin{aligned} B &\subset A \\ &\Rightarrow A \succeq B \end{aligned}$$

- This may not be the case if the DM suffers from problems of temptation:
- Classic example: A dieter might prefer to a restaurant with the menu

fish
salad

rather than one with the menu

fish
burger
salad

- Why?
- (At least) two possible reasons
  1. Would prefer to not eat the burger, but worries they will succumb to temptation if the burger is available
  2. Thinks they will be able to overcome the temptation to eat the burger, but it will be costly to do so

- We are going to discuss a model of menu preferences and choice that captures both these forces
- Based on the classic work of Gul and Pesendorfer [2001]
- Updated (and better explained) by Lipman and Pesendorfer [2013]

Set Up

- Let C be a compact metric space
- $\Delta(C)$ set of all measures on the Borel σ-algebra of C (i.e. all lotteries)
  - Use lotteries because it means set of choice objects is convex
- Endow $\Delta(C)$ with topology of weak convergence
- Z all non empty compact subsets of $\Delta(C)$ (Hausdorff topology)
- Let $\succeq$ be a preference relation on Z
  - Interpretation: preference over menus from which you will later get to choose
- Let $\trianglerighteq$ be a preference relation on $\Delta(C)$
  - Interpretation: preferences when asked to choose from a menu

- For $x, y \in Z$ and $\alpha \in (0,1)$ define

$$\begin{aligned} & \alpha x + (1-\alpha) y \\ = \; & \{p = \alpha q + (1-\alpha) r | q \in x, r \in y, \} \end{aligned}$$

- E.g. if $x = \{\delta_a\}$, $y = \{\delta_b, \delta_c\}$ the

$$\begin{aligned} & \alpha x + (1-\alpha) y \\ = \; & \left\{ \begin{array}{c} \alpha a + (1-\alpha) b \\ \alpha a + (1-\alpha) c \end{array} \right\} \end{aligned}$$

- Mixture of all elements in menu x with all elements in menu y

Modelling Preference over Menus

- Using this set up we will place axioms on $\succeq$ and $\trianglerighteq$
- First, we will consider conditions which are necessary and sufficient for the standard model
  - Single utility function
  - Represents $\trianglerighteq$ (choice from menus)
  - $\succeq$ (choice between menus) represented using largest utility in the set
- Next, consider how to alter these axioms in order to generate the 'Gul Pesendorfer' model
  - Allows for both 'temptation' and 'self control' to be expressed in menu preferences

Axiom 1 (Preference Relations) $\succeq$, $\trianglerighteq$ are complete preference relations

Axiom 2 (Independence) $x \succeq y$ implies
$$\alpha x + (1-\alpha)z \succeq \alpha y + (1-\alpha)z \;\forall\; x, y, z \in Z, \alpha \in (0,1)$$

- Notice that this is not the same as 'standard' independence
- Mixing operation is different
- Need to think a bit about how to interpret it

Basic Axioms

- Interpretation of independence: Standard Independence + Indifference to Timing of Uncertainty
  - Imagine we extended $\succeq$ to preferences over lotteries over menus
  - Independence would now say that, if we prefer choosing from x to choosing from y then we prefer choosing from x α% of the time (and z $(1-\alpha)$% of the time) to choosing from y α% of the time (and z $(1-\alpha)$% of the time)
  - Randomization occurs before choosing at second stage
- Claim: choosing contingent plans in this set up gives rise to the same probability distribution over outcomes as come about from 'Gul Pesendorfer' mixing

- Example

$$\frac{1}{2}x + \frac{1}{2}z$$
$$x = \{x_1, x_2\},\ z = \{z_1, z_2\}$$

- Gul-Pesendorfer mixing: a menu of

$$\left\{ \begin{array}{c} \frac{1}{2}x_1 + \frac{1}{2}z_1 \\ \frac{1}{2}x_2 + \frac{1}{2}z_1 \\ \frac{1}{2}x_1 + \frac{1}{2}z_2 \\ \frac{1}{2}x_2 + \frac{1}{2}z_2 \end{array} \right\}$$

- 'Standard' Mixing: 50% chance of menu x, 50% chance of menu y
  - Contingent plan: choose either x_1 or x_2 from x and either y_1 or y_2 from y
  - Uncertainty decided before second stage choice
  - Set of contingent plans gives rise to same menu of lotteries over outcomes as does GP mixing

- If timing of resolution of uncertainty is not important there is an equivalence between
  - Choosing a contingent plan for a lottery over menus
  - Choosing from a menu of lotteries generated by 'Gul Pesendorfer' mixing
- Thus, 'standard' independence and indifference to timing of uncertainty give rise to GP independence

Axiom 3 (Sophistication) $x \cup \{p\} \succ x \Leftrightarrow p \rhd q \; \forall \; q \in x$

- This is the axiom that links together first and second stage choice.
- Whether or not people are sophisticated is going to be an important empirical question
  - Do they understand the choices they will make from a given menu?
  - If not, may underestimate their degree of self control
  - e.g. sign up for gym memberships they do not use
  - or make costly commitments which they subsequently do not stick to.

Axiom 4 (Continuity) Three continuity conditions:

1. (Upper Semi Continuity): The sets $\{z \in Z | z \succeq x\}$ and $\{p \in \Delta(C) | p \trianglerighteq q\}$ are closed for all x and q
2. (Lower vNM Continuity): $x \succ y \succ z$ implies $\alpha x + (1 - a)z \succ y$ for some $\alpha \in (0, 1)$
3. (Lower Singleton Continuity): The sets $\{p : \{q\} \succeq \{p\}\}$ are closed for every q

- The Standard Model of preference over menus

$$U(z) = \max_{p \in z} u(p)$$

for some linear, continuous utility $u : \Delta(C) \rightarrow \mathbb{R}$ such that

  - U represents $\succeq$
  - u represents $\trianglerighteq$

- Equivalent to axioms 1-4 and

$$x \succeq y \Rightarrow x \cup y \sim x$$

  - $x \succeq y$ implies that the best alternative in x is weakly better than the best alternative in y
  - The best alternative in $x \cup y$ is the same as the best alternative in x
  - Thus $x \cup y \sim x$
- Note that this implies

$$x \supset y \Rightarrow x \succeq y$$

- Say $y \succ x$
  - either $x/y \succeq y$ in which case

$$x = x/y \cup y \sim x/y \succeq y \succ x$$

  - or $y \succeq x/y$

$$x = x/y \cup y \sim y \succ x$$

The Gul Pesendorfer Model

- Preference over menus given by

$$U(x) = \max_{p \in x} [u(p) + v(p)] - \max_{q \in x} v(q)$$

- u : 'long run' utility
- v : 'temptation' utility
- Interpretation:
  - Choose p to maximize $u(p) + v(p)$
  - Suffer temptation cost $v(p) - v(q)$
- Unlike the standard model, the Gul Pesendorfer model can lead to strict preference for smaller choice sets

$$x \supset y \text{ but } x \prec y$$

Object	u	v
Salad	4	0
Fish	2	1
Burger	1	4

- Which menu would the DM prefer? $\{s\}$ or $\{s, b\}$?

$$\begin{aligned} U(\{s\}) &= \max_{x \in \{s\}} (u(x) + v(x)) - \max_{y \in \{s\}} v(y) \\ &= 4 + 0 - 0 \\ &= 4 \end{aligned}$$

$$\begin{aligned} U(\{s, b\}) &= \max_{x \in \{s,b\}} (u(x) + v(x)) - \max_{y \in \{s,b\}} v(y) \\ &= 1 + 4 - 4 \\ &= 1 \end{aligned}$$

Object	u	v
Salad	4	0
Fish	2	1
Burger	1	4

- Menu $\{s\}$ preferred to $\{s.b\}$
- Interpretation: b would be chosen from the latter menu
  - $u(b) + v(b) > u(s) + v(s)$
- But s has higher long run utility
  - $u(s) > u(b)$
- The DM would rather not have b in their menu, because if it is available they will choose it.

Why Preference for Smaller Choice Sets?

Case 1: Commitment

- More generally, consider p, q, such that

$$\begin{aligned} u(p) &> u(q) \\ u(q)+v(q) &> u(p)+v(p) \end{aligned}$$

- Then

$$\begin{aligned} U(\{p\}) &= u(p) \\ U(\{p,q\}) &= u(q)+v(q)-v(q)=u(q) \\ U(\{q\}\} &= u(q) \end{aligned}$$

- Interpretation: give in to temptation and choose q
- 'Weak set betweenness'

$$\{p\} \succ \{p,q\} \sim \{q\}$$

Object	u	v
Salad	4	0
Fish	2	1
Burger	1	4

- Which menu would the DM prefer? $\{s\}$ or $\{s, f\}$?

$$\begin{aligned} U(\{s\}) &= \max_{x \in \{s\}} (u(x) + v(x)) - \max_{y \in \{s\}} v(y) \\ &= 4 + 0 - 0 \\ &= 4 \end{aligned}$$

$$\begin{aligned} U(\{s, f\}) &= \max_{x \in \{s,f\}} (u(x) + v(x)) - \max_{y \in \{s,f\}} v(y) \\ &= 4 + 0 - 1 \\ &= 3 \end{aligned}$$

Why Preference for Smaller Choice Sets?

Case 2: Avoid 'Willpower Costs'

Object	u	v
Salad	4	0
Fish	2	1
Burger	1	4

- Menu $\{s\}$ is preferred to menu $\{s, f\}$
- However, this time, s would be chosen from both menus, as

$$u(s) + v(s) > u(f) + v(f)$$

- The DM still prefers to have f removed from the menu because it is more tempting: $v(f) > v(s)$
- The DM is able to exert self control if both options are on the menu, but it is costly to do so

- More generally, consider p, q, such that

$$\begin{aligned} u(p) &> u(q) \\ v(q) &> v(p) \\ u(p) + v(p) &> u(q) + v(q) \end{aligned}$$

- Then

$$\begin{aligned} U(\{p\}) &= u(p) \\ U(\{p, q\}) &= u(p) + v(p) - v(q) \\ U(\{q\}) &= u(q) \end{aligned}$$

- Interpretation: fight temptation, but this is costly
- 'Strict set betweenness'

$$\{p\} \succ \{p, q\} \succ \{q\}$$

- We say that q tempts p if $\{p\} \succ \{p, q\}$
- We say that a decision maker exhibits self control at y if there exists x, z such that $x \cup z = y$ and

$$\{x\} \succ \{y\} \succ \{z\}$$

- $\{x\} \succ \{y\}$ implies there exists something in z which is tempting relative to items in x
- $\{y\} \succ \{z\}$ implies tempting item not chosen
- if it were then

$$\begin{aligned}
\max_{p \in y} u(p) + v(p) &= \max_{p \in z} u(p) + v(p) \Rightarrow \\
U(y) &= \max_{p \in y} \left(u(p) + v(p)\right) - \max_{q \in y} v(q) \\
&\leq \max_{p \in z} \left(u(p) + v(p)\right) - \max_{q \in z} v(q) \\
&= U(z)
\end{aligned}$$

Why 'Long Run' and 'Temptation' Utilities?

- So far we have described u as 'long run' utility and v as 'temptation' utility
- Why is this a behaviorally appropriate description?
- u describes choices over singleton menus:

$$U(\{p\}) = u(p) + v(p) - v(p) = u(p)$$

  and so describes preferences when the DM is not tempted

Why 'Long Run' and 'Temptation' Utilities?

- v leads to temptation: q tempts p only if $v(q) > v(p)$
  - Case 1: $u(p) + v(p) \geq u(q) + v(q)$

$$
\begin{aligned}
U(\{p\}) &> u(\{p, q\}) \\
&\Rightarrow u(p) > u(p) + v(p) - \max_{r \in \{p,q\}} v(r) \\
&\Rightarrow \max_{r \in \{p,q\}} v(r) > v(p) \\
&\Rightarrow v(q) = \max_{r \in \{p,q\}} v(r) > v(p)
\end{aligned}
$$

Why 'Long Run' and 'Temptation' Utilities?

- v leads to temptation: q tempts p only if $v(q) > v(p)$
  - Case 2: $u(q) + v(q) > u(p) + v(p)$

$$\begin{aligned} U(\{p\}) &> u(\{p,q\}) \\ &\Rightarrow u(p) > u(q) + v(q) - \max_{r \in \{p,q\}} v(r) \\ &\Rightarrow u(p) + \max_{r \in \{p,q\}} v(r) > u(q) + v(q) \\ &\Rightarrow \max_{r \in \{p,q\}} v(r) = v(q) > v(p) \end{aligned}$$

  - Last line follows from assumption $u(q) + v(q) > u(p) + v(p)$

Limiting Case: No Willpower

- Imagine that differences in v are large relative to differences in u
- In the limit, model reduces to

$$U(x) = \max_{p \in x} u(p) \text{ s.t. } v(p) \geq v(q) \ \forall \ q \in x$$

- This is the 'Strolz' model
- Implies no strict set betweenness, and not self control
- $\beta - \delta$ model is of this class

Axiomatic Characterization of GP Model

- Set Betweenness: for any x, y s.t $x \succeq y$

$$x \succeq x \cup y \succeq y$$

- Notice the difference to the 'standard' model

$$x \succeq y \Rightarrow x \cup y \sim x$$

- Smaller sets can be strictly preferred

Axiomatic Characterization of GP Model

- Set Betweenness: for any x, y s.t $x \succeq y$

$$x \succeq x \cup y \succeq y$$

- Necessity:
  - $x \succeq y$ implies that

$$u(p^x) + v(p^x) - v(q^x) \geq u(p^y) + v(p^y) - v(q^y)$$

where

$$p^i = \arg\max_{p \in i} u(p) + v(p)$$

and

$$q^i = \arg\max_{q \in i} v(q)$$

  - NTS $x \succeq x \cup y$

- Two cases:
- Case 1: $u(p^x) + v(p^x) \geq u(p^y) + v(p^y)$

$$\begin{aligned} u(p^x) + v(p^x) &\geq u(p^y) + v(p^y) \Rightarrow \\ u(p^x) + v(p^x) &= u(p^{x\cup y}) + v(p^{x\cup y}) \Rightarrow \\ u(p^x) + v(p^x) - v(q^x) &\geq u(p^{x\cup y}) + v(p^{x\cup y}) - v(q^{x\cup y}) \end{aligned}$$

- Case 2: $u(p^x) + v(p^x) < u(p^y) + v(p^y)$
  - implies $v(q^x) \leq v(q^y)$ as x is preferred to y

$$\begin{aligned} u(p^y) + v(p^y) &= u(p^{x\cup y}) + v(p^{x\cup y}) \\ v(q^{x\cup y}) &= v(q^y) \Rightarrow \\ u(p^{x\cup y}) + v(p^{x\cup y}) - v(q^{x\cup y}) &= u(p^y) + v(p^y) - v(q^y) \\ &\leq u(p^x) + v(p^x) - v(q^x) \end{aligned}$$

Axiomatic Characterization of GP Model

Theorem

$\succeq$ satisfies Axioms 1, 2, 4 and set betweenness if and only if it has a Strolz representation or a G-P representation

Theorem

The proper relation $\succeq$ and $\trianglerighteq$ satisfy Axioms 1-4 and set betweenness if and only if

- *$\succeq$ has a Stroltz representation and $p \trianglerighteq q$ if and only if $v(p) > v(q)$ or $v(p) = v(q)$ and $u(p) \geq u(q)$*
- *or $\succeq$ has a G-P representation and $u(p) + v(p)$ represents $\trianglerighteq$*

Sketch of Proof that Axioms Imply Representation

- **Lemma 1:** Axioms 1, 2, 4 imply a linear $U : Z \longrightarrow \mathbb{R}$ that represents $\succeq$ and is continuous on singleton sets
  - This is standard, and makes use of the mixture space axioms

Sketch of Proof that Axioms Imply Representation

- **Lemma 2:** Show that

$$\begin{aligned} U(x) &= \max_{p\in x}\min_{q\in x} U(\{p,q\}) \\ &= \min_{q\in x}\max_{p\in x} U(\{p,q\}) \end{aligned}$$

  - Utility depends only on 'chosen element', and 'most tempting element

- **Proof: Let** $\bar{u} = \max_{p\in x}\min_{q\in x} U(\{p,q\}) = U(\{p^*,q^*\})$
- Note that $U(\{p^*,q\}) \geq U(\{p^*,q^*\}) = \bar{u}\ \forall\ q \in A$
- Set betweenness implies $\bar{u} \leq U(\cup_{q\in x}\{p^*,q\}) = U(x)$
- Also, for every $p \in A$, $\exists\ q_p \in A$ such that $U(\{p,q_p\}) \leq \bar{u}$
- By set betweenness $\bar{u} \geq U(\cup_{p\in A}\{p,q_p\}) = U(x)$

Sketch of Proof that Axioms Imply Representation

- **Lemma 3:** Show that

$$\begin{aligned} U(\{x\}) &> U(\{x,y\}) > U(\{y\}) \\ U(\{a\}) &> U(\{a,b\}) > U(\{b\}) \end{aligned}$$

implies

$$\begin{aligned} & U(\alpha\{x,y\} + (1-\alpha)\{a,b\}) \\ = \; & U(\{\alpha x + (1-\alpha)a), \alpha y + (1-\alpha)b)\}) \end{aligned}$$

  - This comes straight from super independence and the fact that $\alpha x + (1-\alpha)a$ is the best and $\alpha y + (1-\alpha)b$ the most tempting element

Sketch of Proof that Axioms Imply Representation

- Define

$$
\begin{aligned}
u(p) &= U(\{p\}) \\
v(s; p, q, \delta) &= \frac{U(\{p, q\}) - U(\{p, (1-\delta)q + \delta s\})}{\delta}
\end{aligned}
$$

  - u is the long run utility
  - v is a measure of how tempting s is relative to p and q (under the assumption p is chosen)

Sketch of Proof that Axioms Imply Representation

- **Lemma 4:** Show that, if

$$U(\{p\}) > U(\{p, (1-\delta)r + \delta s\}) > U(\{(1-\delta)r + \delta s\})$$

for all $s \in \Delta(C)$, then

1. $U(\{p\}) > U(\{p, s\}) > U(s) \Rightarrow v(s; p, q, \delta) = U(\{p, q\}) - U(\{p, s\})$
2. $v(p; p, q, \delta) = U(\{p, q\}) - U(\{p\})$

- Follows from Lemma 3

Sketch of Proof that Axioms Imply Representation

- **Lemma 5:** Show that, if

$$U(\{p\}) \geq U(\{p, q\}) \geq U(\{q\})$$

and for some r and δ

$$U(\{p\}) > U(\{p, (1-\delta)r + \delta s\}) > U(\{(1-\delta)r + \delta s\})$$

for all $s \in \Delta(C)$, then

$$\begin{aligned} & U(\{p, q\}) \\ = \quad & \max_{w \in \{p,q\}} [u(w) + v(w; p, r, \delta)] - \max_{z \in \{p,q\}} [v(z; p, r, \delta)] \end{aligned}$$

Sketch of Proof that Axioms Imply Representation

- **Proof** (assuming)

$$U(\{p\}) > U(\{p,q\}) > U(\{q\})$$

  - By previous lemma

$$\begin{aligned} v(q;p,r,\delta) &= U(\{p,r\}) - U(\{p,q\}) \\ &\geq U(\{p,r\}) - U(\{p\}) \\ &= v(p;p,r,\delta) \end{aligned}$$

  and so

$$\max_{z\in\{p,q\}} [v(z;p,r,\delta)] = v(q;p,r,\delta)$$

  - Also

$$\begin{aligned} u(p) + v(p;p,r,\delta) &= U(\{p\}) + U(\{p,r\}) - U(\{p\}) = U(\{p,r\} \\ u(q) + v(q;p,r,\delta) &= U(\{q\}) + U(\{p,r\}) - U(\{p.q\}) \end{aligned}$$

  and so

$$\max_{w\in\{p,q\}} [u(w) + v(w;p,r,\delta)] = u(p) + v(p;p,r,\delta)$$

Sketch of Proof that Axioms Imply Representation

- This then implies

$$
\begin{aligned}
& \max_{w \in \{p,q\}} \left[u(w) + v(w; p, r, \delta) \right] - \max_{z \in \{p,q\}} \left[v(z; p, r, \delta) \right] \\
= \quad & u(p) + v(p; p, r, \delta) - v(q; p, r, \delta) & (1) \\
= \quad & U(\{p\}) + U(\{p, r\}) - U(\{p\}) - U(\{p, r\}) + U(\{p, q\}) \\
= \quad & U(\{p, q\}) & (2)
\end{aligned}
$$

Sketch of Proof that Axioms Imply Representation

- Finally, pick p, q such that

$$U(\{p\}) > U(\{p, q\}) > U(\{q\})$$

(if such exists) and pick δ such that

$$U(\{p\}) > U(\{p, (1-\delta)q + \delta s\}) > U(\{(1-\delta)q + \delta s\})$$

for all s (which we can do by continuity)

- Define $v(s)$ as $v(s; p, q, \delta)$, and show that $v(s; p, q, \delta)$ doesn't depend on the specifics of the last three parameters.
- Lemma 5 therefore gives

$$U(\{p, q\}) = \max_{w \in \{p,q\}} [u(w) + v(w)] - \max_{z \in \{p,q\}} [v(z)]$$

- Lemma 2 then extends this result to an arbitrary set A

Discussion: Linearity

- Imagine

$$\{p\} \succ \{p,q\} \succ \{q\} \succ \{q,r\} \succ \{r\}$$

- DM can resist q for p and resist r for q.
  - Can they resist r for p?
- Under the GP model, the above implies

$$\begin{aligned} u(p) &> u(q) > u(r) \\ v(r) &> v(q) > v(p) \\ u(p)+v(p) &> u(q)+v(q) > u(r)+v(r) \end{aligned}$$

- Which in turn implies

$$\{p\} \succ \{p,r\} \succ \{r\}$$

- 'Self Control is Linear'
  - See Noor and Takeoka [2010]

Discussion: What is Willpower?

- It seems that the following statement is meaningful:
  - Person A has the same long run preferences as person B
  - Person A has the same temptation as person B
  - Person A has more willpower than person B
- Yet this is not possible in the GP model
- Alternative: Masatlioglu, Nakajima and Ozdenoren [2019]

$$\begin{aligned} U(z) &= \max_{p\in z} u(p) \\ \text{subject to } \max_{q\in z} v(q) - v(p) &\leq w \end{aligned}$$

Discussion: Strict Set Betweenness and Random Strolz

- Does $\{p\} \succ \{p, q\} \succ \{q\}$ imply self control?
- Imagine that you are a Strolz guy with $u(p) > u(q)$, but are not sure that you will be tempted
- Half the time

  $$v(p) = v(q)$$

  half the time

  $$v(p) < v(q)$$

- Implies

$$\begin{aligned} U(\{p\}) &= u(p) \\ U(\{p, q\}) &= \frac{u(p) + u(q)}{2} \\ U(\{q\}) &= u(q) \end{aligned}$$

- Strict set betweenness without self control

- Say with probability ε won't be tempted so

$$\hat{U}(z) = (1-\varepsilon)U(z) + \varepsilon \max_{p \in z} u(p)$$

- Can lead to violations of set betweenness.
- Let $g = gym$, $j = jog$, $t = tv$

$$\begin{aligned} u(g) &> u(j) > u(t) \\ v(g) &< v(j) < v(t) \\ u(j) + v(j) &> u(t) + v(t) > u(g) + v(g) \end{aligned}$$

- For ε small

$$\{t, j\} \succ \{t, g\}$$

as

$$\begin{aligned} U(\{t, j\}) &= u(j) + v(j) - v(t) \\ U(\{t, g\}) &= u(t) \end{aligned}$$

- but

$$\{t, j, g\} \succ \{t, j\}$$

as with probability ε no temptation and will go to the gym

Preference for Flexibility

- Consider choice between menus of drinks cocoa or lemonade
- Must choose between menus now, but your choice from those menus will occur on March 1st
- Which would you prefer?

$$\{c\}, \{l\} \text{ or } \{c, l\}?$$

- Choice of $\{c, l\}$ over both $\{c\}$ and $\{l\}$ is a violation of set betweenness

Preference for Flexibility

- X : set of alternatives
- S : set of states
- $\mu \in \Delta(S)$: probability distribution over states
- $u : X \times S \to \mathbb{R}$: utility function
  - $u(x, s)$ utility of alternative x in state s
- Preference uncertainty driven by uncertainty about s

- Let A be a menu of alternatives
- Choice from A will take place **after** the state is known
- Value of A **before** the state is known given by

$$U(A) = \sum_{s \in S} \mu(s) \max_{x \in A} u(x, s)$$

- U represents **choice between menus**

Preference for Flexibility

- The 'preference uncertainty' model implies a (potentially strict) preference for larger choice sets

$$A \succeq B \Rightarrow A \cup B \succeq A$$

- Compare to 'standard' model

$$A \succeq B \Rightarrow A \cup B \sim A$$

- And Set Betweenness

$$A \succeq B \Rightarrow A \cup B \preceq A$$

- Preference uncertainty can provide a powerful force that works against a preference for commitment

Amador Angelitos and Wernig [2005]

- Amador Angelitos and Wernig consider the optimal form of commitment in the face of time inconsistency and a need for flexibility
- Find conditions under which a 'minimum savings rule' is optimal
  - Must save a minimum amount s
  - Free to choose any level of consumption that is consistent with this
- More generally, optimal commitment always exhibits 'bunching at the top'

Set Up

- Two periods with c consumed in the first period and k consumed in the second
- Total resource constraint is y, $B(y)$ is the budget set
- Utility of time 1 self is given by

$$\theta U(c) + \beta W(k)$$

- Utility of time 0 self is given by

$$E\left[\theta U(c) + W(k)\right]$$

- θ is an (uncontractible) taste shock, unknown at time 0, distributed according to F

- Assume distribution of types is represented by continuous θ on $\Theta = [\theta_*, \bar{\theta}]$
- For convenience, assume we are choosing $u(\theta) = U(c(\theta))$ and $w(\theta) = W(k(\theta))$ directly
- Value of plan for type θ is

$$V(\theta) = \max_{\theta' \in \Theta} \left[\frac{\theta}{\beta} u(\theta') + w(\theta') \right]$$

- Assuming truth telling, and by envelope theorem

$$V'(\theta) = \frac{u(\theta)}{\beta}$$

- Integrating $V'(\theta^*)$ tells us that

$$\begin{aligned} V(\theta) &= \frac{\theta}{\beta}u(\theta) + w(\theta) \\ &= \int_{\theta_*}^{\theta} \frac{1}{\beta}u(\theta')d\theta' + \frac{\theta_*}{\beta}u(\theta_*) + w(\theta_*) \end{aligned}$$

- As is standard in principle agent problems, this condition plus monotonicity are necessary and sufficient for incentive compatibility

The Principle's Problem

- Choose $\{u, w\}$ to maximize

$$\int (\theta u(\theta) + w(\theta)) f(\theta) d(\theta)$$

subject to

$$\begin{aligned} & \frac{\theta}{\beta} u(\theta) + w(\theta) \\ = \quad & \int_{\theta_*}^{\theta} \frac{1}{\beta} u(\theta') d\theta' + \frac{\theta_*}{\beta} u(\theta_*) + w(\theta_*) \end{aligned}$$

$$C(u(\theta)) + K(w(\theta)) \leq y$$

$$u(\theta') \geq u(\theta) \text{ for } \theta' \geq \theta$$

The Principle's Problem

- Can use the IC constraint to get rid of w
- Objective function becomes

$$\frac{\theta_*}{\beta}u(\theta_*) + w_* + \frac{1}{\beta}\int_{\theta_*}^{\hat{\theta}}(1 - G(\theta))u(\theta)d\theta \tag{3}$$

subject to

$$W(y - Cu(\theta)) + \frac{\theta}{\beta}u(\theta) - \int_{\theta_*}^{\theta}\frac{1}{\beta}u(\theta')d\theta' - \frac{\theta_*}{\beta}u(\theta_*) - w(\theta_*) \geq 0$$

and monotonicity, where

$$G(\theta) = F(\theta) + \theta(1 - \beta)f(\theta)$$

- It is always optimal to have some bunching at the top

Theorem

An optimal allocation (w, u^*) *satisfies* $u^*(\theta) = u^*(\theta_p)$ *for* $\theta \geq \theta_p$*, where* θ_p *is the lowest value in* Θ *such that*

$$\int_{\theta}^{\bar{\theta}} (1 - G(\theta'))d(\theta') \leq 0$$

for $\theta \geq \theta_p$

- It is always optimal to have some bunching at the top

Theorem

Proof.

The contribution of $\theta \geq \theta_p$ *to the objective function is*

$$\frac{1}{\beta}\int_{\theta_p}^{\bar{\theta}}(1 - G(\theta))u(\theta)d\theta$$

rewriting $u(\theta) = u(\theta_p) + \int_{\theta_p}^{\theta} u'(\theta)d(\theta)$ *gives*

$$\frac{1}{\beta}u(\theta_p)\int_{\theta_p}^{\bar{\theta}}(1 - G(\theta))u(\theta)d\theta + \int_{\theta_p}^{\bar{\theta}}\int_{\theta}^{\bar{\theta}}(1 - G(\theta''))u'(\theta')d\theta''d\theta'$$

□

- It is always optimal for all types above a certain threshold consume the same amount
- This does not imply that a minimum savings rule is necessarily optimal
- For that we need one further condition

$$G(\theta) = F(\theta) + \theta(1-\beta)f(\theta)$$

  is increasing for all $\theta \leq \theta_p$
- If (and only if) this condition is satisfied, a simple minimal savings rule is optimal

- So far, we have assumed that a DM is sophisticated
  - They understand their second stage choice
  - Implemented by the axiom $x \cup \{p\} \succ x \Leftrightarrow p \rhd q \; \forall \; q \in x$
- What about a DM who is not sophisticated?

Sophistication

- Example 1: A DM who ignores temptation

Object	u	v
Salad	4	0
Fish	2	1
Burger	1	4

- Assume these preferences represent choices that the DM will make from the menu
- But they believe that their choices will be governed by u
- Such a DM will prefer $\{s, b\}$ to $\{s\}$, but when faced with the choice from $\{s, b\}$ will choose b
- Such a DM will violate sophistication
    - Never exhibit a preference for commitment

- Example 2: A DM who underestimates temptation

Object	u	v	v'
Salad	5	0	0
Fish	2	1	1
Burger	1	9	5

- Assume that a DM has temptation driven by v, but believes that they have temptation driven by v'
- They are offered the chance to buy a 'commitment contract' where they have to pay $2 if they eat the burger
- Assume that $u(2) = 2$, $v(2) = 2$ the u of money is additive with u of consumption and the v of money is additive with the v of consumption
- Let $b + c$ be the burger with the commitment contract

Discussion: Sophistication

- Example 2: A DM who underestimates temptation

Object	u	v	v'
Salad	5	0	0
Fish	2	1	1
Burger	1	9	5
B+C	-1	7	3

- The DM will have preferences

$$\{b+c, s\} \succ \{b, s\}$$

as

$$\begin{aligned} U(\{b+c, s\}) &= u(s) + v'(s) - v'(b+c) = 2 \\ &> 1 = u(b) = U(\{b, s\}) \end{aligned}$$

- But the DM will actually choose $b+c$ over s at the second stage as

$$u(b+c) + v(b+c) = 6 > 5 = u(s) + v(s)$$

- Example 2: A DM who underestimates temptation

Object	u	v	v'
Salad	5	0	0
Fish	2	1	1
Burger	1	9	5
B+C	-1	7	3

- End up with lower 'long run' utility
- Also a violation of sophistication as

$$\{b+c, s\} \succ \{b+c\}$$

but $b+c$ will be chosen from the former menu

Summary

- Menu preferences allow us to formalize a model of preference for commitment
- We argued that this is a sign that people have problems with temptation
  - Temptation: Preference for Commitment

    $$A \succeq B \Rightarrow A \cup B \preceq A$$

  - Preference uncertainty: Preference for Flexibility

    $$A \succeq B \Rightarrow A \cup B \succeq A$$

  - Compare to 'standard' model

    $$A \succeq B \Rightarrow A \cup B \sim A$$

- Gul and Pesendorfer provide a model which allows for both temptation and self control

$$U(x) = \max_{p \in x} [u(p) + v(p)] - \max_{q \in x} v(q)$$

- Characterized by set betweenness: $x \succeq y \Rightarrow x \succeq x \cup y \succeq y$