Time Preferences

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Two Standard Ways

- In the introductory lecture we suggested two possible ways of spotting temptation
  1. Preference for Commitment
  2. Time inconsistency
- Previously we covered Preference for Commitment
- Now, time preferences!

Time Inconsistency

- Imagine you are asked to make a choice for today
  1. Salad or burger for lunch
  2. 10 minute massage today or 11 minute massage tomorrow
- And a choice for next Wednesday
  1. Salad or burger for lunch
  2. 10 minute massage on the 16th or 11 minute massage 17th
- Choice {burger,salad} or {10,11} is a 'preference reversal'
- Interpretation: you are tempted by the burger, but would 'prefer' to choose the salad
- In terms of previous model
  - burger maximizes $u + v$
  - salad maximizes u

- This is inconsistent with standard intertemporal choice theory
- Utility given by

$$\sum_{t=1}^{T} \delta^t u(c_t)$$

  - δ is the discount rate
  - c_t is consumption in period t
  - u is stable utility function
- If $u(s) > u(b)$ then salad should be chosen over burger both today and next Wednesday
- If $u(s) < u(b)$ then burger should be chosen over salad both today and next Wednesday
- If $u(10) > \delta u(11)$ then 10 minute earlier massage should be chosen over 11 minute later massage both today and next week
- If $u(10) < \delta u(11)$ then 11 minute later massage should be chosen over 10 minute earlier massage both today and next week

- Are preference reversals evidence for temptation?
- Not necessarily - could be changing tastes
  - Maybe just had a salad, so fancied a burger today but salad next week
  - Maybe know they are going to be busy tomorrow, so would prefer the 10 minute massage today but 11 minute massage in a week and one day
- Such changes should be distributed randomly
- But in many cases choices vary *consistently*
- Thirsty subjects
  - Juice now (60%) or twice amount in 5 minutes (40%)
  - Juice in 20 minutes (30%) or twice amount in 25 minutes (70%)
- Hard to explain with changing tastes

- In order to model time preferences we need to decide what *data set* we are working with
- Initially consider preference over *consumption streams*
  - Allow clean theoretical statements
- However, often we do not often observe preference over consumption streams
- Instead we observe repeated consumption/savings choices
- Will next consider this data set
- Relate to preference for commitment

Preference Over Consumption Streams

- Object of choice are now consumption streams:

$$C = \{c_1, c_2, ...\}$$

- c_i is consumption at date i
- Standard model: Exponential Discounting

$$U(C) = \sum_{i=1}^{\infty} \delta^i u(c_i)$$

- Characterized by two conditions
- Trade off consistency

$$\begin{array}{ccc} \{x, y, c_3, c_4, ...\} & \succ & \{z, w, c_3, c_4, ...\} \\ & \Rightarrow & \\ \{x, y, d_3, d_4, ...\} & \succ & \{z, w, d_3, d_4, ...\} \end{array}$$

- Stationarity

$$\begin{array}{ccc} \{c_1, c_2, ...\} & \succ & \{d_1, d_2, ...\} \\ & \Rightarrow & \\ \{e, c_1, c_2, ...\} & \succ & \{e, d_1, d_2, ..\} \end{array}$$

- Trade off consistency: necessary for separable utility function

$$\begin{aligned} \{x, y, c_3, c_4, ...\} &\succ \{z, w, c_3, c_4, ...\} \\ &\Rightarrow \\ \{x, y, d_3, d_4, ...\} &\succ \{z, w, d_3, d_4, ...\} \end{aligned}$$

- Assuming exponential discounting

$$\begin{aligned} u(x) + \delta u(y) + \sum_{i=2}^{\infty} \delta^i u(c_i) &\geq u(w) + \delta u(z) + \sum_{i=2}^{\infty} \delta^i u(c_i) \Rightarrow \\ u(x) + \delta u(y) &\geq u(w) + \delta u(z) \Rightarrow \\ u(x) + \delta u(y) + \sum_{i=2}^{\infty} \delta^i u(d_i) &\geq u(w) + \delta u(z) + \sum_{i=2}^{\infty} \delta^i u(d_i) \end{aligned}$$

- Stationarity: necessary of exponential discounting

$$
\begin{array}{rcl}
\{c_1, c_2, ...\} & \succ & \{d_1, d_2, ...\} \\
& \Rightarrow & \\
\{e, c_1, c_2, ...\} & \succ & \{e, d_1, d_2, ..\}
\end{array}
$$

- Assuming exponential discounting

$$
\begin{array}{rcl}
\sum_{i=0}^{\infty} \delta^i u(c_i) & \geq & \sum_{i=0}^{\infty} \delta^i u(d_i) \Rightarrow \\
u(e) + \delta \left(\sum_{i=0}^{\infty} \delta^i u(c_i) \right) & \geq & u(e) + \delta \left(\sum_{i=0}^{\infty} \delta^i u(d_i) \right)
\end{array}
$$

Sufficiency

- Trade Off Consistency and Stationarity clearly necessary for an exponential discounting representation
- Turns out that they are also sufficient (along with some technical axioms)
  - Stationarity propagates Trade Off Consistency to future periods
- See Koopmans [1960] (or for an easier read Bleichrodt, Rohde and Wakker [2008])
- Which of these axioms is violated by time consistency?

- Time inconsistency violates Stationarity

$$\begin{array}{ccc} \{10, 0, 0, ...\} & \succ & \{0, 11, 0, ...\} \\ & & \text{but} \\ \{0, 10, 0, 0, ...\} & \prec & \{0, 0, 11, 0, ...\} \end{array}$$

- In general this is dealt with by replacing exponential discounting with some other form
  - Hyperbolic
$$U(C) = \sum_{i=1}^{\infty} \frac{1}{1+ki} u(c_i)$$
  - quasi hyperbolic
$$U(C) = u(c_1) + \sum_{i=2}^{\infty} \beta\delta^i u(c_i)$$
- Hyperbolic discounting is a pain to use, so people generally work with quasi hyperbolic discounting [Laibson 1997]

- Implication of quasi hyperbolic discounting: Only the first period is special
- Otherwise the DM looks standard
- Weaken stationarity to 'quasi-stationarity' [Olea and Strzalecki 2014]

$$\begin{array}{ccc} \{f, c_1, c_2, ...\} & \succ & \{f, d_1, d_2, ...\} \\ & \Rightarrow & \\ \{f, e, c_1, c_2, ...\} & \succ & \{f, e, d_1, d_2, ..\} \end{array}$$

- Stationarity holds after first period

Clearly necessary for quasi-hyperbolic discounting

$$\begin{array}{rcl} \{f, c_1, c_2, ...\} & \succ & \{f, d_1, d_2, ...\} \\ & \Rightarrow & \\ \{f, e, c_1, c_2, ...\} & \succ & \{f, e, d_1, d_2, ..\} \end{array}$$

$$\begin{aligned} u(f) + \beta \sum_{i=1}^{\infty} \delta^i u(c_i) & \geq & u(f) + \beta \sum_{i=1}^{\infty} \delta^i u(d_i) \Rightarrow \\ & & u(f) + \beta\delta \left(u(e) + \sum_{i=1}^{\infty} \delta^i u(c_i) \right) \\ & \geq & u(f) + \beta\delta \left(u(e) + \sum_{i=1}^{\infty} \delta^i u(d_i) \right) \end{aligned}$$

- Olea and Strzalecki show that quasistationarity plus a slight modification to trade off consistency (plus technical axioms) is equivalent to

$$u(c_0) + \beta \sum_{i=1}^{\infty} \delta^i v(c_i)$$

- Note that u may be different from v

Quasi Hyperbolic Discounting

- To get to Quasihyperbolic discounting, need to add something else.
- If

$$\begin{aligned} \{b, e_2, e_{2,\ldots}\} &\succeq \{a, e_1, e_{1,\ldots}\} \\ \{c, e_1, e_{1,\ldots}\} &\succeq \{d, e_2, e_{2,\ldots}\} \\ \{e_3, a, a_{,\ldots}\} &\sim \{e_4, b, b_{,\ldots}\} \end{aligned}$$

  then

$$\{e_3, c, c_{,\ldots}\} \succeq \{e_4, d, d_{,\ldots}\}$$

- First two conditions say that, accodding to u, c is 'more better' than d than a is to b
- Second two conditions says that this has to be preserved by v
- This ensures that u and v are the same

- Present bias: if $a \succ c$ then

$$\begin{aligned} \{g, a, b, e, ...\} &\sim \{g, c, d, f, ...\} \Rightarrow \\ \{a, b, e, ...\} &\succeq \{c, d, f, ...\} \end{aligned}$$

- Ensures $\beta \leq 1$

Consumption and Savings

- In general, we do not observe choice over consumption streams
- Instead, observe choices over consumption levels *today*, which determine savings levels tomorrow
- Consumption streams 'fix' level of future consumption
  - Implicitly introduce commitment
- In consumption/savings problems, no commitment
  - Consumption level at time t decided at time t
- What does quasi-hyperbolic discounting look like in this case?

Consumption and Savings - Example

- Three period cake eating problem, with initial endowment $3y$
- Formulate two versions of the problem
  - a single agent chooses c_0, c_1 and c_2 in order to maximize

$$U(C) = u(c_0) + \beta \sum_{i=1}^{2} \delta^i u(c_i) \text{ st } \sum_{i=0}^{2} c_i \leq 3y$$

  - a game between 3 agents $k = 0, 1, 2$ where agent k chooses c_k to max

$$U(C) = u(c_k) + \beta \sum_{i=k+1}^{2} \delta^i u(c_i) \text{ st } c_k \leq s_{k-1}$$

  - where s_{k-1} is remaining cake, and taking other agents strategies as given

Consumption and Savings with Exponential Discounting

- Under exponential discounting (i.e. $\beta = 1$), these two approaches give same outcome
- Assuming CRRA utility

$$
\begin{aligned}
c_0 &= \frac{3y}{1 + (\delta)^{\frac{1}{\sigma}} + \left(\delta^2\right)^{\frac{1}{\sigma}}} \\
c_1 &= (\delta)^{\frac{1}{\sigma}} c_0 \\
c_2 &= (\delta)^{\frac{1}{\sigma}} c_1
\end{aligned}
$$

- Agents are time consistent: period i agent will stick to the plan of period $i-1$ agent
- Only exponential discounting function has this feature [Strotz 1955]

Consumption and Savings with Quasi Hyperbolic Discounting

- Now assume that the agent has a quasi-hyperbolic utility function: agent k chooses c_k to max

$$U(C) = u(c_k) + \sum_{i=k+1}^{2} \beta\delta^i u(c_i) \text{ st } c_k \leq s_{k-1}$$

- Now the solutions are different:
- Consider three cases
  1. Commitment: time 0 agent gets to choose c_0, c_1, c_2
  2. Sophistication: each player solves the game by backward induction and chooses optimally, correctly anticipating future behavior
  3. Naive: each player acts as if future plans will be followed

Consumption and Savings with Quasi Hyperbolic Discounting

- Case 1: Commitment

$$
\begin{aligned}
c_0 &= \left(1+(\beta\delta)^{\frac{1}{\sigma}}+\left(\beta\delta^2\right)^{\frac{1}{\sigma}}\right)^{-1} 3y \\
c_2 &= \delta^{\frac{1}{\sigma}} c_1
\end{aligned}
$$

- Case 2: Sophistication

$$
\begin{aligned}
\bar{c}_0 &= \left[1+\left(\frac{\beta\delta}{\left(1+(\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}}+\frac{\delta\left(\beta\delta\right)^{\frac{1}{\sigma}}}{\left(1+(\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}}\right)^{\frac{1}{\sigma}}\right]^{-1} 3y \\
\bar{c}_2 &= (\beta\delta)^{\frac{1}{\sigma}} c_1
\end{aligned}
$$

- Without commitment, period 2 consumption lower relative to period 1 consumption

Consumption and Savings with Quasi Hyperbolic Discounting

- Case 1: Commitment

$$
\begin{aligned}
c_0 &= \left(1+(\beta\delta)^{\frac{1}{\sigma}}+\left(\beta\delta^2\right)^{\frac{1}{\sigma}}\right)^{-1} 3y \\
c_2 &= \delta^{\frac{1}{\sigma}} c_1
\end{aligned}
$$

- Case 2: Sophistication

$$
\begin{aligned}
\bar{c}_0 &= \left[1+\left(\frac{\beta\delta}{\left(1+(\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}}+\frac{\delta\,(\beta\delta)^{\frac{1}{\sigma}}}{\left(1+(\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}}\right)^{\frac{1}{\sigma}}\right]^{-1} 3y \\
\bar{c}_2 &= (\beta\delta)^{\frac{1}{\sigma}}\, c_1
\end{aligned}
$$

- Period 0 consumption can be lower or higher depending on σ
  - Two offsetting effects:
    - Less efficient use of savings
    - Agent in period 2 gets screwed

Discounting and Preference for Commitment

- Note that an exponential discounter will not have a preference for commitment
  - Agent at time 1 will follow plan made at time 0
- A sophisticated non-exponential discounter will have a preference for commitment
  - Agent at time 1 will not follow preferred plan of agent at time 0
- Thus, under sophistication

$$\begin{aligned} & \text{Non-exponential discounting} \\ \Leftrightarrow \quad & \text{Preference reversals} \\ \Leftrightarrow \quad & \text{Demand for commitment} \end{aligned}$$

Consumption and Savings with Quasi Hyperbolic Discounting

- Case 3: Naivete

$$
\begin{aligned}
c_0 &= \left(1+(\beta\delta)^{\frac{1}{\sigma}}+\left(\beta\delta^2\right)^{\frac{1}{\sigma}}\right)^{-1} 3y \\
c_2 &= (\beta\delta)^{\frac{1}{\sigma}} c_1
\end{aligned}
$$

- Period 0 consumption will be the same as commitment case (unsurprisingly)
- Period 1 consumption will be unambiguously higher
- Period 2 consumption will be unambiguously lower
- A naive q-hyperbolic discounter will not have a preference for commitment
    - Will *expect* agent at time 1 to follow plan made at time 0

Discounting and Preference for Commitment

- This provides a link between preference reversals and demand for commitment
  - A sophisticated q-hyperbolic agent would like to make use of illiquid assets, cut up credit cards, etc
- Next lecture we will examine whether there is an empirical link between the two
- A separate question: how valuable is commitment in consumption savings problems?
  - Not very (Laibson [2015])

Strong Hyperbolic Euler Equation

- For sophisticated consumers with no commitment optimal behavior can be characterized by the SHEE

$$\frac{\partial u(c_t)}{\partial c_t} = RE_t \left[\left(\beta \delta c'_{t+1} + (1 - c'_{t+1}) \delta \right) \frac{\partial u(c_{t+1})}{\partial c_{t+1}} \right]$$

- Where c'_{t+1} is the marginal propensity to consume in period $t + 1$
- Modification of 'standard' Euler equation:
    - Standard case: effective discount rate $d_t = \delta$
    - SHEE: effective discount rate $d_t = \beta \delta c'_{t+1} + (1 - c'_{ct+1}) \delta$
    - If MPC is low, two models look similar
- Requires consumers not to be 'too' hyperbolic (see Harris and Laibson 2001)

Observing Time Inconsistency in a Consumption/Savings Problem

- What are the observable implications of quasi-hyperbolic discounting?
- If we observe a sequences of
  - consumptions choices
  - one period interest rates
  - prices
  - Incomes

  under what circumstances are they consistent with q-hyperbolic discounting?
- Are these conditions different from those for the standard exponential discounting model?

Observing Time Inconsistency in a Consumption/Savings Problem

- Surprisingly, this question is not well answered
- Barro [1999] shows that if utility is log then the two are observationally equivalent
- What if utility is not log?
- In the CRRA class of utilities, there are three parameters to estimate, β, δ and σ
- Intuitively, need three moments
- Above data provides two:
  - Response to changes in income
  - Response to changes in interest rates
- Need to get third moment from somewhere
- Two recent revealed preference approaches
  - Blow, Browning and Crawford [2014] (multiple goods)
  - Saito, Echenique and Imai [2015] (multiple lives)

Application: Procrastination

- O'Donoghue and Rabin [1999]
- T time periods
- Have to decide in which period to perform a task
- $\{c_1, ...c_T\}$: Cost of performing the task in each period
- $\{v_1, ...v_T\}$: Value of performing the task in each period
- Two cases:
  - Immediate costs, delayed rewards
  - Immediate rewards, delayed cost

- For simplicity, assume that $\delta = 1$
- Period t utility from the task being done in period τ is:
  - Immediate costs case

$$\begin{aligned} \beta v_\tau - \beta c_\tau \text{ if } \tau &> t \\ \beta v_\tau - c_\tau \text{ if } \tau &= t \end{aligned}$$

  - Immediate rewards case

$$\begin{aligned} \beta v_\tau - \beta c_\tau \text{ if } \tau &> t \\ v_\tau - \beta c_\tau \text{ if } \tau &= t \end{aligned}$$

Application: Procrastination

- Example 1: Writing a referee report in the next 4 weeks
- Costs are immediate, rewards delayed
  - Rewards: $v = \{0, 0, 0, 0\}$
  - Costs: $c = \{3, 5, 8, 15\}$
- Report has to be done in week 4 if not done before
- Time consistent agent ($\beta = 1$) will do the report in week 1
- Sophisticated agent with $\beta = \frac{1}{2}$?
  - Will do the report in week 2
  - Would delay in week 3
  - Prefers to do it in week 2 than week 4
  - Prefers to wait till week 2 from week 1
- Naive agent with $\beta = \frac{1}{2}$?
  - will end up doing the report in week 4
  - Always thinks they will do the report next week

- Example 2: Choosing when to see a movie
- Costs are delayed, rewards immediate
  - Rewards: $v = \{3, 5, 8, 13\}$
  - Costs: $c = \{0, 0, 0, 0\}$
- Movie has to be seen in week 4 if not done before
- Time consistent agent ($\beta = 1$) will see the movie in week 4
- Sophisticated agent with $\beta = \frac{1}{2}$?
  - Will see the movie in week 1
  - Would see it in week 3 if given the choice
  - Prefers to see it in week 2 than week 3
  - Prefers to see it in week 1 than week 2
- Naive agent with $\beta = \frac{1}{2}$?
  - Will end up seeing the movie in week 3
  - Prefers to see it in week 3 than week 4
  - In week 2, thinks will wait till week 4, so delays
  - In week 1 thinks will wait till week 4 so delays

- Proposition: Naive decision makers will always take action later than sophisticates
  - Immediate costs: Sophisticates recognize future procrastination and act to avoid it
  - Immediate rewards: Sophisticates recognize future 'greed', and act to preempt it

Summary

- Systematic preference reversals present a challenge to the standard model of time separable, exponential discounting
  - A violation of stationarity
- There is a strong theoretical link between preference reversals, non-exponential discounting and preference for commitment
- Quasi-hyperbolic discounting model a popular alternative used to explain the data
  - Treats today as special
- Can be used to model a wide variety of phenomena
  - Demand for liquid assets
  - Procrastination
- Binning down the precise implications of the q-hyperbolic model is
  - Easy in choice over consumption streams
  - Harder in choice in consumption savings problems