Time Preferences

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Two Standard Ways

- In the introductory lecture we suggested two possible ways of spotting temptation
  1. Preference for Commitment
  2. Time inconsistency
- Previously we covered Preference for Commitment
- Now, time preferences!

- Imagine you are asked to make a choice for today
  1. Salad or burger for lunch
  2. 10 minute massage today or 11 minute massage tomorrow
- And a choice for next Monday
  1. Salad or burger for lunch
  2. 10 minute massage on the 18th or 11 minute massage 19th
- Choice {burger,salad} or {10,11} is a 'preference reversal'
- Interpretation: you are tempted by the burger, but would 'prefer' to choose the salad

- This is inconsistent with standard intertemporal choice theory
- Utility given by

$$\sum_{t=1}^{T} \delta^t u(c_t)$$

  - δ is the discount rate
  - c_t is consumption in period t
  - u is stable utility function
- If $u(s) > u(b)$ then salad should be chosen over burger both today and next Monday
- If $u(s) < u(b)$ then burger should be chosen over salad both today and next Monday
- If $u(10) > \delta u(11)$ then 10 minute earlier massage should be chosen over 11 minute later massage both today and next week
- If $u(10) < \delta u(11)$ then 11 minute later massage should be chosen over 10 minute earlier massage both today and next week

- Are preference reversals evidence for temptation?
- Not necessarily - could be changing tastes
  - Maybe just had a salad, so fancied a burger today but salad next week
  - Maybe know they are going to be busy tomorrow, so would prefer the 10 minute massage today but 11 minute massage in a week and one day
- Such changes should be distributed randomly
- But in many cases choices vary *consistently*
- Thirsty subjects
  - Juice now (60%) or twice amount in 5 minutes (40%)
  - Juice in 20 minutes (30%) or twice amount in 25 minutes (70%)
- Hard to explain with changing tastes

- In order to model time preferences we need to decide what *data set* we are working with
- Initially consider preference over *consumption streams*
    - Allow clean theoretical statements
- However, often we do not observe preference over consumption streams
- Instead we observe repeated consumption/savings choices
- Will next consider this data set
- Relate to preference for commitment

Preference Over Consumption Streams

- Object of choice are now consumption streams:

$$C = \{c_1, c_2, ...\}$$

- c_i is consumption at date i
- Standard model: Exponential Discounting

$$U(C) = \sum_{i=1}^{\infty} \delta^i u(c_i)$$

- Characterized by two conditions
- Trade off consistency

$$\begin{aligned} \{x, y, c_3, c_4, ...\} &\succ \{z, w, c_3, c_4, ...\} \\ &\Rightarrow \\ \{x, y, d_3, d_4, ...\} &\succ \{z, w, d_3, d_4, ...\} \end{aligned}$$

- Stationarity

$$\begin{aligned} \{c_1, c_2, ...\} &\succ \{d_1, d_2, ...\} \\ &\Rightarrow \\ \{e, c_1, c_2, ...\} &\succ \{e, d_1, d_2, ..\} \end{aligned}$$

- Trade off consistency: necessary for separable utility function

$$
\begin{array}{rcl}
\{x, y, c_3, c_4, ...\} & \succ & \{z, w, c_3, c_4, ...\} \\
& \Rightarrow & \\
\{x, y, d_3, d_4, ...\} & \succ & \{z, w, d_3, d_4, ...\}
\end{array}
$$

- Assuming exponential discounting

$$
\begin{array}{rcl}
u(x) + \delta u(y) + \sum_{i=2}^{\infty} \delta^i u(c_i) & \geq & u(w) + \delta u(z) + \sum_{i=2}^{\infty} \delta^i u(c_i) \Rightarrow \\
u(x) + \delta u(y) & \geq & u(w) + \delta u(z) \Rightarrow \\
u(x) + \delta u(y) + \sum_{i=2}^{\infty} \delta^i u(d_i) & \geq & u(w) + \delta u(z) + \sum_{i=2}^{\infty} \delta^i u(d_i)
\end{array}
$$

- Stationarity: necessary for exponential discounting

$$\begin{array}{rcl} \{c_1, c_2, ...\} & \succ & \{d_1, d_2, ...\} \\ & \Rightarrow & \\ \{e, c_1, c_2, ...\} & \succ & \{e, d_1, d_2, ..\} \end{array}$$

- Assuming exponential discounting

$$\begin{array}{rcl} \sum_{i=0}^{\infty} \delta^i u(c_i) & \geq & \sum_{i=0}^{\infty} \delta^i u(d_i) \Rightarrow \\ u(e) + \delta \left(\sum_{i=0}^{\infty} \delta^i u(c_i) \right) & \geq & u(e) + \delta \left(\sum_{i=0}^{\infty} \delta^i u(d_i) \right) \end{array}$$

Sufficiency

- Trade Off Consistency and Stationarity clearly necessary for an exponential discounting representation
- Turns out that they are also sufficient (along with some technical axioms)
  - Stationarity propagates Trade Off Consistency to future periods
- See Koopmans [1960] (or for an easier read Bleichrodt, Rohde and Wakker [2008])
- Which of these axioms is violated by time inconsistency?

- Time inconsistency violates Stationarity

$$\begin{aligned} \{10, 0, 0, ...\} &\succ \{0, 11, 0, ...\} \\ &\text{but} \\ \{0, 10, 0, 0, ...\} &\prec \{0, 0, 11, 0, ...\} \end{aligned}$$

- In general this is dealt with by replacing exponential discounting with some other form
  - Hyperbolic

$$U(C) = \sum_{i=1}^{\infty} \frac{1}{1+ki} u(c_i)$$

  - quasi hyperbolic

$$U(C) = u(c_1) + \sum_{i=2}^{\infty} \beta\delta^i u(c_i)$$

- Hyperbolic discounting is a pain to use, so people generally work with quasi hyperbolic discounting [Laibson 1997]

Quasi Hyperbolic Discounting

- Implication of quasi hyperbolic discounting: Only the first period is special
- Otherwise the DM looks standard
- Weaken stationarity to 'quasi-stationarity' [Olea and Strzalecki 2014]

$$\begin{array}{ccc} \{f, c_1, c_2, ...\} & \succ & \{f, d_1, d_2, ...\} \\ & \Rightarrow & \\ \{f, e, c_1, c_2, ...\} & \succ & \{f, e, d_1, d_2, ..\} \end{array}$$

- Stationarity holds after first period

Clearly necessary for quasi-hyperbolic discounting

$$\begin{aligned} \{f, c_1, c_2, ...\} &\succ \{f, d_1, d_2, ...\} \\ &\Rightarrow \\ \{f, e, c_1, c_2, ...\} &\succ \{f, e, d_1, d_2, ..\} \end{aligned}$$

$$\begin{aligned} u(f) + \beta \sum_{i=1}^{\infty} \delta^i u(c_i) &\geq u(f) + \beta \sum_{i=1}^{\infty} \delta^i u(d_i) \Rightarrow \\ & u(f) + \beta\delta \left(u(e) + \sum_{i=1}^{\infty} \delta^i u(c_i) \right) \\ &\geq u(f) + \beta\delta \left(u(e) + \sum_{i=1}^{\infty} \delta^i u(d_i) \right) \end{aligned}$$

- Olea and Strzalecki show that quasistationarity plus a slight modification to trade off consistency (plus technical axioms) is equivalent to

$$u(c_0) + \beta \sum_{i=1}^{\infty} \delta^i v(c_i)$$

- Note that u may be different from v

Quasi Hyperbolic Discounting

- To get to Quasihyperbolic discounting, need to add something else.
- If

$$\begin{aligned} \{b, e_2, e_{2,\ldots}\} &\succeq \{a, e_1, e_{1,\ldots}\} \\ \{c, e_1, e_{1,\ldots}\} &\succeq \{d, e_2, e_{2,\ldots}\} \\ \{e_3, a, a_{,\ldots}\} &\sim \{e_4, b, b_{,\ldots}\} \end{aligned}$$

  then

$$\{e_3, c, c_{,\ldots}\} \succeq \{e_4, d, d_{,\ldots}\}$$

- First two conditions say that, according to u, c is 'more better' than d than a is to b
- Second two conditions says that this has to be preserved by v
- This ensures that u and v are the same

Quasi Hyperbolic Discounting

- Present bias: if $a \succ c$ then

$$\begin{aligned} \{g, a, b, e, ...\} &\sim \{g, c, d, f, ...\} \Rightarrow \\ \{a, b, e, ...\} &\succeq \{c, d, f, ...\} \end{aligned}$$

- Ensures $\beta \leq 1$

Consumption and Savings

- In general, we do not observe choice over consumption streams
- Instead, observe choices over consumption levels *today*, which determine savings levels tomorrow
- Consumption streams 'fix' level of future consumption
  - Implicitly introduce commitment
- In consumption/savings problems, no commitment
  - Consumption level at time t decided at time t
- What does quasi-hyperbolic discounting look like in this case?

Consumption and Savings - Example

- Three period cake eating problem, with initial endowment $3y$
- Formulate two versions of the problem
  - a single agent chooses c_0, c_1 and c_2 in order to maximize

$$U(C) = u(c_0) + \beta \sum_{i=1}^{2} \delta^i u(c_i) \text{ st } \sum_{i=0}^{2} c_i \leq 3y$$

  - a game between 3 agents $k = 0, 1, 2$ where agent k chooses c_k to max

$$U(C) = u(c_k) + \beta \sum_{i=k+1}^{2} \delta^i u(c_i) \text{ st } c_k \leq s_{k-1}$$

  - where s_{k-1} is remaining cake, and taking other agents strategies as given

Consumption and Savings with Exponential Discounting

- Under exponential discounting (i.e. $\beta = 1$), these two approaches give same outcome
- Assuming CRRA utility

$$
\begin{aligned}
c_0 &= \frac{3y}{1+(\delta)^{\frac{1}{\sigma}}+\left(\delta^2\right)^{\frac{1}{\sigma}}} \\
c_1 &= (\delta)^{\frac{1}{\sigma}}\, c_0 \\
c_2 &= (\delta)^{\frac{1}{\sigma}}\, c_1
\end{aligned}
$$

- Agents are time consistent: period i agent will stick to the plan of period $i-1$ agent
- Only exponential discounting function has this feature [Strotz 1955]

Consumption and Savings with Quasi Hyperbolic Discounting

- Now assume that the agent has a quasi-hyperbolic utility function: agent k chooses c_k to max

$$U(C) = u(c_k) + \sum_{i=k+1}^{2} \beta\delta^i u(c_i) \text{ st } c_k \leq s_{k-1}$$

- Now the solutions are different:
- Consider three cases
  1. Commitment: time 0 agent gets to choose c_0, c_1, c_2
  2. Sophistication: each player solves the game by backward induction and chooses optimally, correctly anticipating future behavior
  3. Naive: each player acts as if future plans will be followed

Consumption and Savings with Quasi Hyperbolic Discounting

- Case 1: Commitment

$$
\begin{aligned}
c_0 &= \left(1 + (\beta\delta)^{\frac{1}{\sigma}} + \left(\beta\delta^2\right)^{\frac{1}{\sigma}}\right)^{-1} 3y \\
c_2 &= \delta^{\frac{1}{\sigma}} c_1
\end{aligned}
$$

- Case 2: Sophistication

$$
\begin{aligned}
\bar{c}_0 &= \left[1 + \left(\frac{\beta\delta}{\left(1 + (\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}} + \frac{\delta\left(\beta\delta\right)^{\frac{1}{\sigma}}}{\left(1 + (\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}}\right)^{\frac{1}{\sigma}}\right]^{-1} 3y \\
\bar{c}_2 &= (\beta\delta)^{\frac{1}{\sigma}} c_1
\end{aligned}
$$

- Without commitment, period 2 consumption lower relative to period 1 consumption

Consumption and Savings with Quasi Hyperbolic Discounting

- Case 1: Commitment

$$c_0 = \left(1 + (\beta\delta)^{\frac{1}{\sigma}} + \left(\beta\delta^2\right)^{\frac{1}{\sigma}}\right)^{-1} 3y$$

$$c_2 = \delta^{\frac{1}{\sigma}} c_1$$

- Case 2: Sophistication

$$\bar{c}_0 = \left[1 + \left(\frac{\beta\delta}{\left(1 + (\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}} + \frac{\delta\,(\beta\delta)^{\frac{1}{\sigma}}}{\left(1 + (\beta\delta)^{\frac{1}{\sigma}}\right)^{1-\sigma}}\right)^{\frac{1}{\sigma}}\right]^{-1} 3y$$

$$\bar{c}_2 = (\beta\delta)^{\frac{1}{\sigma}} c_1$$

- Period 0 consumption can be lower or higher depending on σ
  - Two offsetting effects:
    - Less efficient use of savings
    - Agent in period 2 gets screwed

Discounting and Preference for Commitment

- Note that an exponential discounter will not have a preference for commitment
  - Agent at time 1 will follow plan made at time 0
- A sophisticated non-exponential discounter will have a preference for commitment
  - Agent at time 1 will not follow preferred plan of agent at time 0
- Thus, under sophistication

$$\begin{aligned} & \text{Non-exponential discounting} \\ \Leftrightarrow\; & \text{Preference reversals} \\ \Leftrightarrow\; & \text{Demand for commitment} \end{aligned}$$

Consumption and Savings with Quasi Hyperbolic Discounting

- Case 3: Naivete

$$\begin{aligned} c_0 &= \left(1 + (\beta\delta)^{\frac{1}{\sigma}} + \left(\beta\delta^2\right)^{\frac{1}{\sigma}}\right)^{-1} 3y \\ c_2 &= (\beta\delta)^{\frac{1}{\sigma}} c_1 \end{aligned}$$

- Period 0 consumption will be the same as commitment case (unsurprisingly)
- Period 1 consumption will be unambiguously higher
- Period 2 consumption will be unambiguously lower
- A naive q-hyperbolic discounter will not have a preference for commitment
    - Will *expect* agent at time 1 to follow plan made at time 0

Discounting and Preference for Commitment

- This provides a link between preference reversals and demand for commitment
  - A sophisticated q-hyperbolic agent would like to make use of illiquid assets, cut up credit cards, etc
- Next lecture we will examine whether there is an empirical link between the two
- A separate question: how valuable is commitment in consumption savings problems?
  - Not very (Laibson [2015])

Strong Hyperbolic Euler Equation

- For sophisticated consumers with no commitment optimal behavior can be characterized by the SHEE

$$\frac{\partial u(c_t)}{\partial c_t} = RE_t \left[\left(\beta\delta c'_{t+1} + (1 - c'_{t+1})\delta \right) \frac{\partial u(c_{t+1})}{\partial c_{t+1}} \right]$$

- Where c'_{t+1} is the marginal propensity to consume in period $t + 1$
- Modification of 'standard' Euler equation:
  - Standard case: effective discount rate $d_t = \delta$
  - SHEE: effective discount rate $d_t = \beta\delta c'_{t+1} + (1 - c'_{ct+1})\delta$
  - If MPC is low, two models look similar
- Requires consumers not to be 'too' hyperbolic (see Harris and Laibson 2001)

Observing Time Inconsistency in a Consumption/Savings Problem

- What are the observable implications of quasi-hyperbolic discounting?
- If we observe a sequences of
  - consumptions choices
  - one period interest rates
  - prices
  - Incomes

  under what circumstances are they consistent with q-hyperbolic discounting?
- Are these conditions different from those for the standard exponential discounting model?

Observing Time Inconsistency in a Consumption/Savings Problem

- Surprisingly, this question is not well answered
- Barro [1999] shows that if utility is log then the two are observationally equivalent
- What if utility is not log?
- In the CRRA class of utilities, there are three parameters to estimate, β, δ and σ
- Intuitively, need three moments
- Above data provides two:
  - Response to changes in income
  - Response to changes in interest rates
- Need to get third moment from somewhere
- Two recent revealed preference approaches
  - Blow, Browning and Crawford [2014] (multiple goods)
  - Saito, Echenique and Imai [2015] (multiple lives)

Time Preferences as Risk Preferences

- One (quite fundamental) question is: why do we discount in the first place?
- One possible answer is that things in the future *might not happen*
- Would you prefer cake today or cake in a week?
- Before a week's time
  - You might die
  - The baker might die
  - Everyone might die!
- So might prefer cake now

Time Preferences as Risk Preferences

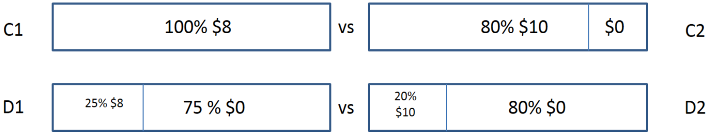
- Consider a model in which there is a constant probability $(1-\delta)$ that the world will end in each period
- What is the value of an outcome c received in t periods?
- If you are an expected utility maximizer it is

$$\delta^t u(c)$$

- Exponential discounting!

- However, in the domain of risky choices there is plentiful evidence that people violate EU

The Common Ratio Effect



- What would you choose?
- Many people choose C1 and D2

A Common Ratio Effect for Time Preferences

- Informally, we can see a link between the common ratio effect and present bias.
- Perhaps C1 is preferred because it is the only certain option?
- Outcomes received today are the only certain option in intertemporal choice
- In fact a model that gave a boost of $\frac{1}{\beta}$ for $\beta < 1$ to options that are certain would
  - Explain the common ratio effect
  - Give the $\beta - \delta$ model
- c valued as

$$\frac{1}{\beta}u(c) \text{ if received in period } 0$$

$$\delta^t u(c) \text{ if received in period } t > 0$$

A Common Ratio Effect for Time Preferences

- For various reasons such a model is not particularly popular
- But there are number of papers that have shown that models of **probability weighting** can explain behavior in both domains
  - In fact the type of probability weighting that gives present bias is exactly the same that gives common ratio effects
- See
  - Halevy, Yoram. "Strotz meets Allais: Diminishing impatience and the certainty effect." American Economic Review 98.3 (2008): 1145-62.
  - Saito, Kota. "Strotz meets allais: Diminishing impatience and the certainty effect: Comment." American Economic Review 101.5 (2011): 2271-75.
  - Chakraborty, Anujit, Yoram Halevy, and Kota Saito. "The Relation between Behavior under Risk and over Time." Unpublished manuscript (2019).
- An obvious question: are these two behaviors linked empirically?

Discounting as Perceptual Noise

- Recently, researchers have focussed on another possible mechanism for discounting
  - It might be harder to perceive the value of events that occur in the future
- Intuitively, this will mean that good events in the future will be downweighted relative to good events now
- Can give rise to ‘present bias’ choices

Discounting as Perceptual Noise

- Consider the following simple example from Gabaix and Laibson [2019]
- Imagine, that, when presented with a prospect of value u_t the DM receives a noisy signal

$$s_t = u_t + \varepsilon_t$$

  Where $\varepsilon_t \sim N(0, t\sigma_s^2)$
- Assume prior beliefs are distributed $N(0, \sigma_\mu^2)$

Discounting as Perceptual Noise

- Upon receiving signal s_t, beliefs will be given by

$$N\left(\frac{1}{1+\frac{t\sigma_s}{\sigma_\mu}}s_t, \left(1-\frac{1}{1+\frac{t\sigma_s}{\sigma_\mu}}\right)\sigma_\mu^2\right)$$

- Integrating over signals, the average mean belief is given by

$$\frac{1}{1+\frac{t\sigma_s}{\sigma_\mu}}u_t$$

- This has the hyperbolic form so assuming that $u_t > 0$
    - Future rewards will be downweighted
    - Choices will be present biased

even if there is no ‘actual’ discounting!

- Obviously there are many special assumptions about this set up but the logic is quite strong
- For example, Gabaix and Laibson show that as long as $\sigma^2_{s_t}$ is a weakly concave function of time, the 'discount rate' $\frac{1}{1+\frac{\sigma_{s_t}}{\sigma_\mu}}$ will generate increasing patience
- Also a recent paper shows that present bias comes naturally out of an optimal choice of attention
  - "Optimal similarity judgments in intertemporal choice" by Adriani and Sonderegger [2019]
- Key mechanism: when time periods are further in the future it is less worthwhile distinguishing between them

- Q-hyperbolic model still difficult to solve for many periods
- Game between two long run players
- Multiple equilibria [Laibson 1997, Harris and Laibson 2004]
- Fudenberg and Levine come up with a simpler model

Fudenberg and Levine [2006]

- Long run self plays a game against a sequence of short lived self
- Short run self gets to choose what action to take $a \in A$
- Long run self chooses 'self control' $r \in R$ which modifies utility function of short run self
- State y evolves according to some (stochastic) process depending on history of y,a and r
- $\Gamma(y)$ available options in state y

- Each short run player chooses an action a to maximize

$$u(y, r, a)$$

- Long run player chooses a mapping from histories h to R to maximize

$$\sum_{i=1}^{\infty} \delta^{t-1} \int u(y(h), r(h), a(h)) d\pi(h)$$

where

  - $r(h)$ is the strategy of the long run player
  - $a(.)$ is strategy of each short run player
  - $y(.)$ is the state following history h
  - π is the probability distribution over h given strategies

Fudenberg and Levine [2006]

- Define $C(y, a)$ as the self control cost of choosing a in state y

$$C(y, a) = u(y, 0, a) - \sup_{r \text{ s.t. } u(y,r,a) \geq u(y,r,b) \ \forall \ b \in \Gamma(y)} u(y, r, a)$$

- Then we can rewrite long run's self problem as a decision problem
- choose mapping from h to A in order to maximize

$$\sum_{i=1}^{\infty} \int u(y(h), 0, a(h)) - c(y(h), a(h)) d\pi(h)$$

Fudenberg and Levine [2006]

- Further assume that self control costs are
  - Linear
  - Depend only on the chosen object and most tempting object in choice set

$$c(y, a) = \lambda\left(\max_{b \in \Gamma(y)} u(b, 0, y) - u(a, 0, y)\right)$$

- This is a Gul-Pesendorfer type model
  - Reducing choice set reduces self control costs

A Consumption/Saving Example

- State y represents wealth
- a is fraction of wealth saved
- Return on wealth is R
- Instantaneous utility is log

$$u(y, 0, a) = \log((1-a)\, y)$$

- Temptation utility in each period is $\log(y)$
- Objective function becomes

$$\begin{aligned}
& \sum_{i=1}^{\infty} \delta^{t-1} \left[\log((1-a)\, y_i) - \lambda(\log(y_i) - \log((1-a_i)\, y_i)\right] \\
= \; & \sum_{i=1}^{\infty} \delta^{t-1} \left[(1+\lambda) \log((1-a_i)\, y_i) - \lambda(\log(y_i)\right] \\
& \text{subject to} \\
a_i \; \in \; & [0, 1] \\
y_{i+1} \; = \; & R a_i y_i
\end{aligned}$$

- Solution. It turns out that optimal policy is constant savings rate, so $y_i = (Ra)^{i-1} y_1$

$$\sum_{i=1}^{\infty} \delta^{t-1} \left[\begin{array}{c} (1+\lambda)\log((1-a) + (i-1)\log Ra + \log y_1) \\ -\lambda((i-1)\log Ra + \log y_1) \end{array} \right]$$

$$= \quad (1+\lambda)\frac{\log(1-a)}{(1-\delta)} + \frac{\log y_1}{(1-\delta)} + \frac{\delta\log(Ra)}{(1-\delta)^2}$$

- FOC wrt a

$$\frac{(1+\lambda)}{(1-\delta)(1-a)} = \frac{\delta}{(1-\delta)^2 a}$$

$$a = \frac{\delta}{1 + (1 - \delta)\lambda}$$

- As self control costs increase, savings go down
- As δ increases, effect of self control increases

Risk Aversion in the Large and Small

- Rabin [2000] argued that lab risk aversion cannot be due to curvature of utility function
  - Would lead to absurd levels of risk aversion in the large
- Can be explained by probability weighting
- F and L offer another explanation
  - For small wins, prize will be consumed immediately - compare to current spending
  - For large wins prize will be saved - compare to current wealth

Risk Aversion in the Large and Small

- Each period split in two
- Bank
  - No consumption, but savings
  - No temptation (nothing to consume)
  - Choose amount x to take out of bank
- Casino
  - Choose how much of x to consume
  - Return remainder to the Bank

- If everything is deterministic then can implement first best outcome
  - Set $a^* = \delta$
- Now assume that with some small probability will be asked to choose between gambles at casino
- Assume probability is 'small' so still set $a^* = \delta$ in the bank
- Consider receiving prize z
- Wealth in period 2 given by

$$y_2 = R(y_1 + z_1 - c_1)$$

Risk Aversion in the Large and Small

- Utility of y_2 in period 2 is given by

$$
\begin{aligned}
& \sum_{i=1}^{\infty} \delta^{t-1} \left[(1+\lambda) \log((1-a^*) + (i-1) \log Ra^* + \log y_2) \right] \\
= \quad & \frac{\log(1-a^*)}{(1-\delta)} + \frac{\log y_2}{(1-\delta)} + \frac{\delta \log(Ra^*)}{(1-\delta)^2} \\
= \quad & \frac{1}{(1-\delta)} \left[\log(1-\delta) + \log y_2 + \frac{\delta}{1+\delta} \log(R\delta) \right]
\end{aligned}
$$

- Total utility from consuming c_1

$$
\begin{aligned}
& (1+\lambda) \log c_1 - \lambda \log(x_1 + z_1) \\
& + \frac{1}{(1-\delta)} \left[\log(1-\delta) + \log R(y_1 + z_1 - c_1) + \frac{\delta}{1+\delta} \log(R\delta) \right]
\end{aligned}
$$

- Gives First Order Conditions

$$\begin{aligned} c^* &= \frac{(1-\delta)(1+\lambda)(y_1+z_1)}{\delta+(1+\lambda)(1-\delta)} \\ &= \left(1-\frac{\delta}{\delta+(1+\lambda)(1-\delta)}\right)(y_1+z_1) \end{aligned}$$

- Consumption is constrained by $x_1 + z_1 = (1-\delta)y_1 + z_1$. Define z^* as

$$\left(1-\frac{\delta}{\delta+(1+\lambda)(1-\delta)}\right)(y_1+z^*) = (1-\delta)y_1 + z^*$$

- For $z_1 > z^*$, consume c^*, otherwise consume $(1-\delta)y_1 + z_1$

Risk Aversion in the Large and Small

- Utility of prize less than z^*

$$\log(x_1 + z_1) + \frac{1}{(1-\delta)}\left[\log(1-\delta) + \log(y_1 - x_1) + \frac{\delta}{1+\delta}\log(R\delta)\right]$$

- Utility of prize greater than z^*

$$(1+\lambda)\log\frac{(1-\delta)(1+\lambda)}{1+\lambda(1-\delta)}(y_1+z_1) - \lambda\log(x_1+z_1) + \frac{1}{(1-\delta)}\left[\log(1-\delta) + \log R\frac{\delta(y_1+z_1)}{1+\lambda(1-\delta)} + \frac{\delta}{1+\delta}\log(R\delta)\right]$$

- For 'small' wins, constant relative risk aversion relative to pocket cash
- For 'large' wins (approximately) constant relative risk aversion relative to wealth

Summary

- Systematic preference reversals present a challenge to the standard model of time separable, exponential discounting
  - A violation of stationarity
- There is a strong theoretical link between preference reversals, non-exponential discounting and preference for commitment
- Quasi-hyperbolic discounting model a popular alternative used to explain the data
  - Treats today as special
- Can be used to model a wide variety of phenomena
  - Demand for liquid assets
  - Procrastination
- Pinning down the precise implications of the q-hyperbolic model is
  - Easy in choice over consumption streams
  - Harder in choice in consumption savings problems