Behavioral Economics

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Homework 4

Due Tuesday 3rd March

PLEASE DO QUESTION 1 AND QUESTION 2 ON SEPARATE SHEETS OF PAPER

Question 1 We are going to to think about how Gertrude chooses how to get to work. She has a choice between three routes, A, B and C. Gertrude prefers shorter routes to longer routes, and routes with less traffic to more. C is shorter than A and A is shorter than B. B has less traffic than route C, but she does not know how much traffic there is on route A.

1. In any given choice set, Gertrude first rules out any routes such that there is an available route which definitely has less traffic. Then of the remaining routes she chooses the shortest
   (a) Under this procedure, figure out what Gertrude will choose from each choice set
   (b) Do the resulting choices satisfy condition α?
2. Here is a more general way of describing the procedure above: The decision maker has two rationales P_1 and P_2 (for example traffic and length). P_2 is antisymmetric (if xP_2y and yP_2x then $y = x$), transitive (xP_2y, yP_2z implies xP_2z) and complete (xP_2y or yP_2x for all x, y). P_1 is antisymmetric, but not necessarily complete and transitive. When choosing from a set S the decision maker first identifies all the elements that are not dominated by some other element according to P_1 - i.e. the set S_1 such that

$$S_1 = \{x \in S | \text{ there is no } y \in S \text{ such that } y \neq x \text{ and } yP_1x\}$$

Then chooses the best thing from this set according to P_2, so if $C(S)$ is the choice function

$$C(S) = \{x \in S_1 | \text{there is no } y \in S_1 \text{ such that } y \neq x \text{ and } yP_2x\}$$

We call this the rational shortlist method. Show that the procedure described in 2 is a rational shortlist method.

3. Imagine that we observed Gertrude make the following choices:

$$\begin{aligned} C(\{A,B\}) &= B \\ C(\{A,C\}) &= A \\ C(\{B,C\}) &= B \\ C(\{A,B,C\}) &= A \end{aligned}$$

Show that this behavior is not consistent with the rational shortlist method (i.e. there is no P_1, P_2 that would rationalize these choices according to the rational shortlist method)

4. Here are two properties of choices

- **WEAK WARP:** If x is chosen from $\{x, y\}$ and is also chosen from $\{x, y, z_1, ..., z_n\}$, then y is not chosen from any set consisting of x, y and some subset of $\{z_1, .., z_n\}$. i.e.

$$\begin{aligned} \{x, y\} &\subset S \subset T \\ C(\{x, y\}) &= C(T) = x \\ &\Rightarrow y \neq C(S) \end{aligned}$$

- **EXPANSION:** if x is chosen from each of two sets is also chosen from their union. i.e.

$$\begin{aligned} x &= C(S) = C(T) \\ &\Rightarrow x = C(S \cup T) \end{aligned}$$

Show that

(a) If choices satisfy property α they will also satisfy expansion and weak WARP (remember that we are assuming a choice function here, not a choice correspondence)

(b) If choices are made according to the rational shortlist method then they will satisfy expansion and weak WARP.

Question 1 In the lectures, I set up the optimization problem for a decision maker who has Shannon Entropy costs of attention in the case there were two states and two acts to choose from. I claimed that, if both acts were chosen, then posterior beliefs would satisfy

$$\frac{\gamma_1^t}{\gamma_1^s} = \exp\left(\frac{U_1^a - U_1^b}{\kappa}\right)$$
$$\frac{\gamma_2^t}{\gamma_2^s} = \exp\left(\frac{U_2^a - U_2^b}{\kappa}\right)$$

1. Prove this result
2. Use this result (and Bayes rule) to find an expression from $\pi_1(t)$ and $\pi_2(t)$ - i.e. the probability of receiving signal t in each state (note you should write a formula that describes these values as a function of the primitives of the model - the utilities of the different acts in different states and the costs κ and the prior μ_1)
3. Show that, in the simple case in which $U_1^a = U_2^b = c$, $U_2^a = U_1^b = 0$, and $\mu_1 = 0.5$ the probability of choosing the correct act in each state is given by $\frac{\exp\left(\frac{c}{\kappa}\right)}{1+\exp\left(\frac{c}{\kappa}\right)}$
4. In a recent experiment, I recorded the fraction of correct responses in each state for four different levels of reward. The results of the experiment are given in the following table

Reward ($)	% Correct
2	74.8
10	81.9
20	83.3
30	86.7

assume $U(\$x) = x$. Is this data consistent with your findings from section 3 (i.e. can the same κ explain behavior at the 4 different reward levels?

5. If not, are my subjects increasing their attention as rewards increase more quickly or more slowly than the Shannon model predicts?