Behavioral Economics

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Homework 7

Due Tuesday April 28th

Question 1 Here is another variant of the Ellsberg paradox: Consider an urn with 90 balls, 30 of which are red and 60 of which are either black or yellow (but you do not know the precise number of each. Consider the following choices

1. Between act f_1 which pays \$10 if the ball drawn is red (and zero otherwise) and act g_1 which pays \$10 if you draw a black ball (and zero otherwise)
2. Between act f_2 which pays \$10 if the ball drawn is red or yellow (and zero otherwise) and act g_2 which pays \$10 if you draw a black or yellow ball (and zero otherwise)

Most people strictly prefer act f_1 to g_1 and strictly prefer g_2 to f_2. Show that an SEU maximizer cannot behave in this way (i.e. figure out the state space for this problem, and show that if an SEU maximizer prefers f_1 to g_1 then they must prefer f_2 to g_2. Show that a MaxMin Expected utility person can exhibit this behavior (for simplicity, assume linear utility and find a set of beliefs that will generate this behavior)

Question 2 Consider the following (very simplified) version of prospect theory for lotteries over money. For any reference point x and lottery p, let p_-^x be the prizes worse than x (i.e.$y \in \text{supp}(p)$ such that $y < x$) and p_+^x be the prizes better that x (i.e.$y \in \text{supp}(p)$ such that $y > x$). The utility for lottery p with reference point x is given by

$$U(p,x) = \sum_{y \in p_-^x} p(y)\lambda(y-x) + \sum_{y \in p_+^x} p(y)(y-x)$$

where $p(y)$ is the probability that lottery p assigns to prize y and λ is some number greater than 1.

1. What is the utility of a lottery with a 50% chance of winning \$5, a 25% chance of winning \$3 and a 25% chance of winning \$0 if the reference point is \$5? What about if the reference point is \$3? Or if it is \$0?
2. Can this utility function explain the 'reflection effect'? I.e. consider some lottery p with two prizes $y_1 > y_2$. Is it possible for a decision maker with this utility function to be risk averse if the reference point is y_2, but risk loving when the reference point is y_1?
3. Will a Decision Maker with this utility function be more risk averse for lotteries which have both gains and losses? i.e. is it the case that, for a lottery with prizes y_1 and y_2, the decision maker will be more risk averse if the reference point x is such that $y_1 > x > y_2$ than they would be if $x \geq y_1$ or $x \leq y_2$?
4. This model can also be extended to allow for 'stochastic reference points'. For example, for a lottery p, the reference point can be another lottery q be a lottery that gives prizes z_1 and z_2. Then the utility of p if q is the reference point is given by

$$\begin{aligned} U(p,q) &= q(z_1)\left(\sum_{y \in p_-^{z_1}} p(y)\lambda(y-z_1) + \sum_{y \in p_+^{z_1}} p(y)(y-z_1)\right) \\ &\quad + q(z_2)\left(\sum_{y \in p_-^{z_2}} p(y)\lambda(y-z_2) + \sum_{y \in p_+^{z_2}} p(y)(y-z_2)\right) \end{aligned}$$

   Calculate the utility of a lottery that has a 50% chance of winning \$10 and a 50% chance of winning \$0 if the reference point is that lottery (i.e if both p and q are the lottery that has a 50% chance of winning \$10 and a 50% chance of winning \$0).
5. Show that this can lead to an 'endowment effect for risk' - which means that people may be less risk averse if they own a lottery To show this, show that, for a lottery p with a 50% chance of winning \$10 and a 50% chance of winning \$0, and a sure thing \$5, then a decision maker will prefer the sure \$5 to p if the reference point is \$5, but will be indifferent between the two if their reference point is the lottery
6. One problem in the reference dependence literature is that we do not know where reference points come from. One suggestion to address this problem is the concept of

'personal equilibrium'. We say that a lottery p is a personal equilibrium in a choice set A if

$$U(p,p) \geq U(q,p) \text{ for all } q \in A$$

This is one way of describing a 'reasonable' set of reference points: for p to be a personal equilibrium in A it must be the case that, if p is the reference point in A then the decision maker will choose p of all the available lotteries.

Consider a choice set A which consists of p with a 50% chance of winning \$10 and a 50% chance of winning \$0 and an amount x for sure. Calculate the range of x for sure such that x for sure is a personal equilibrium in A. Also calculate the range of x for which p is a personal equilibrium in A.

Question 3 - For your own pleasure only, you do not have to hand this in Read the paper "Expected utility theory and prospect theory: one wedding and a decent funeral"[1] By Glenn Harrison and E Elisabet Rutsrom. Use this (and other sources if you like) to write a two page critique of the descriptive power of prospect theory.

[1] http://link.springer.com/article/10.1007%2Fs10683-008-9203-7