

Microeconomic Analysis

Mark Dean

Homework 1

Due Wednesday 20th September

Question 1 Say that, rather than starting with the preference relation 'at least as good as' \succeq as a primitive, and using this to define 'strictly preferred to' \succ , we go the other way round.

1. If you start with the binary relation \succ , interpreted as 'strictly preferred to', how would you generate the relation 'at least as good as'?
2. Assume that \succ satisfies the following two properties
 - Asymmetry: if $x \succ y$ then not $y \succ x$
 - Negative Transitivity: For all x, y and z , if $x \succ y$ then either $x \succ z$ or $z \succ y$ (or both)

Show that under these conditions, the relationship you constructed in part (1) will be complete, transitive and reflexive.

Question 2 Remember that we say that choices satisfy the Weak Axiom of Revealed Preference (WARP) if the following is true: for any x and y in X , if $\{x, y\} \in A \cap B$ and $x \in C(A)$, and $y \in C(B)$ then $x \in C(B)$. Show that a choice correspondence satisfies WARP if and only if it satisfies the properties α and β described in the lecture notes.

Question 3 Utility maximization is not the only choice procedure that are consistent with α and β . There are also other choice procedures that will satisfy these conditions and so are indistinguishable from rational choice. Consider the following decision making procedures. Prove whether or not they will result in choices that satisfy α and β

1. A decision maker (DM) is choosing between books from a set B . They have a utility function $u : B \rightarrow \mathbb{R}$, and a 'threshold utility' level u^* . In any choice set, they search through the books alphabetically by title, and choose the first book that has utility level u that is equal to or above u^* . If they have not found any such book by the time they reach the end of the choice set, they will choose the book with the highest utility (to make things simpler you can assume that there is no indifference - i.e. no two books have the same utilities)
2. A DM assigns a utility number to each alternative and chooses the alternative with the lowest utility
3. The DM ranks the alternatives according to a utility function, and in any choice set chooses the median element

Question 4 Complete the various proofs of the relationship between utility maximization and choice that were left undone in the lecture notes

1. Show that the other three cases in the proof of the existence of a utility representation for choice all go through
2. Show that a decision maker who makes choices by maximizing a utility function must satisfy properties α and β
3. Prove the theorem: Let $u : X \rightarrow \mathbb{R}$ be a utility representation for a Choice Correspondence C . Then $v : X \rightarrow \mathbb{R}$ will also represent C if and only if there is a strictly increasing function T such that

$$v(x) = T(u(x)) \quad \forall x \in X$$