Microeconomic Analysis

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Homework 2

Due Wednesday 27th September

Question 1 Some quetions about preferences

1. Consider the revealed preference relation R and strictly revealed preferred relation S generated by a choice correspondence C observed on some subset of possible choice sets $D \subset 2^X/\varnothing$ from a finite set X. Use a result from class to show that if C satisfies GARP there exists a utility function $u : X \to \mathbb{R}$ that represents revealed preferences in the sense that

$$\begin{aligned} xRy &\rightarrow u(x) \geq u(y) \\ xSy &\rightarrow u(x) > u(y) \end{aligned}$$

2. As part of the proof, you will have used the fact that a reflexive, transitive (but not necessarily complete) binary relation $\succeq$ has a utility representation in the sense that

$$\begin{aligned} x &\succeq y \rightarrow u(x) \geq u(y) \\ x &\succ y \rightarrow u(x) > u(y) \end{aligned}$$

   This representation is worse than the standard one, in that we cannot uniquely recover preferences from the utility function. Show that (i) If we know that $\succeq$ is complete then there is a unique preference ordering associated with any utility representation. but (ii) if we do not know that $\succeq$ is complete then there will be many preference orderings associated with a given utility function

3. Does this matter for choices? i.e. If we are told the utility function u that represents the revealed preference information from C defined on $D \subset 2^X/\varnothing$, can we uniqely recover the choices that must have been made in each $A \in D$?

Question 2 Prove the following lemma which we stated in class

Lemma 1 *Let x^j and x^k be two commodity bundles such that $p^j x^k < p^j x^j$. If the DM's choices can be rationalized by a complete locally non-satiated preference relation, then it must be the case that $x^j \succ x^k$*

Question 3 Some questions on continuity

1. We used the following definition of continuity of preferences on some metric space X: for any $x, y \in X$ such that $x \succ y$, there exists an $\varepsilon > 0$ such that, for any $x' \in B(x, \varepsilon)$ and $y' \in B(y, \varepsilon)$, $x' \succ y'$. Show that this is equivalent to the assumption that the set $\{(x, y) \,|x \succeq y\} \subset X \times X$ is closed
2. Consider the lexicographic preferences we introduced in class. Let the distance between $(a, b) \in X$ and $(c, d) \in X$ be given by $max\,\{|a - c|, |b - d|\}$. Are the lexicographic preferences continuous under this metric?
3. Show that if the preferences $\succeq$ can be represented by a continuous utility function they are continuous.