Utility Maximization
A Gentle Introduction

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Introduction

- In the first 4/5 lectures or so we are going to talk about the relationship between
  - Two fundamental models of economic behavior
    - Utility maximization
    - Preference maximization
  - And the data they are designed to explain
    - Choices
- What I want to get across in this introduction is an idea of why there is anything of interest here
  - i.e. why are we going to have to study this for 5 lectures?
  - Surely utility maximization is fairly straightforward?
- This introduction is going to be very 'light'
  - So relax!

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Utility Maximization

- The model of utility maximization is probably the most pervasive in all of economics
- I am sure you have come across it
- The question I want to ask today is: how can we test it?
  - i.e. if I observe someone's behavior, how can I tell if they are in fact a utility maximizer
  - Equivalently, what predictions about behavior does the model of utility maximization make?

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Testing Utility Maximization

- In order to understand how to test the model of utility maximization (or indeed any model) we need two elements

1. The data we are going to use
2. A precise description of the model

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The Data

- We observe:
  - The choices someone makes
  - What they were choosing from
- Example: choices from different sets of snack foods

Available Snacks	Chosen Snack
Jaffa Cakes, Kit Kat	Jaffa Cakes
Kit Kat, Lays	Kit Kat
Lays, Jaffa Cakes	Jaffa Cakes
Kit Kat, Jaffa Cakes, Lays	Jaffa Cakes

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The Model

- We want to test the model of utility maximization
- Every object has a fixed utility value attached to it
- For example:
  - U(jaffa cakes)=10
  - U(kit kat) =5
  - U(lays)=2
- In any choice set, choose the object with highest utility

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The Question

- Is our data set consistent with the model of utility maximization?
- Problem: Our model contains 'unobservables'
  - We do not observe utilities
  - Kit Kats do not come with utility numbers stamped on them
  - Model says that people maximize utility, but as the experimenter I do not observe utility
- Notice that this is a problem that will get worse as our models get more complicated
  - For example, a model of subjective expected utility also assumes that people have unobservable beliefs
- How can we proceed?

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Approach 1

- Pick a particular utility function
  - e.g. utility=calories
- Test whether this utility function can explain the data
  - e.g. Do people pick the option with the most calories?
- This is now a testable prediction
- And this is indeed how early economists proceeded
  - Bentham: Felicific Calculus
  - Proposed a classification of 12 pains and 14 pleasures, by which we might test the "happiness factor" of any action
- Problem: What does failure tell us?
  - Perhaps people do not maximize utility
  - Or perhaps utility is not equal to calories
  - Maybe Bentham overlooked a pleasure!

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Approach 2

- Ask the question: Is there ANY utility function that can explain the data?
- i.e. we are agnostic about what utility is
- We require only that the person behaves as if they have some consistent utility function that they are using to make their choices
- Note that this is what is sometimes referred to as 'as if' modelling
  - We don't observe utility directly
  - Only ask that we can find some utility function that explains choices
  - Subject behaves 'as if' they are maximizing utility
  - But they might be doing something completely different

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Algenon's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Lays
4	Kit Kat, Jaffa Cakes, Lays	Jaffa Cakes

- Is there any utility function that can explain Algenon's choices
- No!
  - Choice 1 implies u(jaffa cake)>u(kit kat)
  - Choice 2 implies u(kit kat)>u(lays)
  - Choice 3 implies u(lays)>u(jaffa cakes)
- Implies u(jaffa cake)>u(jaffa cake): Contradiction

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Brittney's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Jaffa Cakes
4	Kit Kat, Jaffa Cakes, Lays	Kit Kat

- What about Brittney's Choices?
- No!
  - Choice 1 implies u(jaffa cake)>u(kit kat)
  - Choice 4 implies u(kit kat)>u(jaffa cakes)
- Contradiction

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Colvin's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Jaffa Cakes
4	Kit Kat, Jaffa Cakes, Lays	Jaffa Cakes

- How about Colvin's Choices?
- Yes!
  - u(jaffa cakes)>u(kit kat)>u(lays)
- Eg
  - u(jaffa cakes)=3
  - u(kit kat)=2
  - U(lays)=1

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A General Rule

- Question: Is there a general rule that differentiates data sets that can be explained by some utility function from those that can't?

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The Independence of Irrelevant Alternatives
Say x is chosen from a set of alternatives A
B is a subset of A that contains x
Then x must be chosen from B

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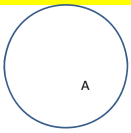

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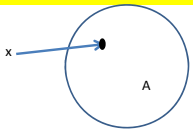

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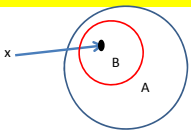

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Independence of Irrelevant Alternatives

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	
2	Kit Kat, Lays	
3	Lays, Jaffa Cakes	
4	Kit Kat, Jaffa Cakes, Lays	

- In our example, whatever is chosen in set 4 must always be chosen when it is available

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Algenon's Choices

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1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Lays
4	Kit Kat, Jaffa Cakes, Lays	Jaffa Cakes

- Algenon's choices violate these condition
  - Jaffa cakes chosen in set 4
  - Lays chosen in set 3

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Brittney's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Jaffa Cakes
4	Kit Kat, Jaffa Cakes, Lays	Kit Kat

- Also violated by Brittney's choices
  - Kit Kat chosen in set 4
  - Jaffa cakes chosen in set 1

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Colvin's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
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- Colvin's choices satisfy IIA
  - Jaffa cakes chosen in 4
  - Also chosen in 3 and 1

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A Necessary Condition

The Independence of Irrelevant Alternatives
Say x is chosen from a set of alternatives A
B is a subset of A that contains x
Then x must be chosen from B

- If we observe a utility maximizer, then they must satisfy IIA
  - If x is chosen from A, must have a higher utility than anything in A
  - B is a subset of A
  - X must have higher utility than anything in B
  - Should be chosen from B

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A Sufficient Condition?

The Independence of Irrelevant Alternatives
Say x is chosen from a set of alternatives A
B is a subset of A that contains x
Then x must be chosen from B

- Is it the case that, if IIA holds, there exists some utility function such that choices maximize utility according to that utility function?
- This would be great!
  - It means testing the condition is the same as testing the model of utility maximum
  - If the condition is satisfied then the person looks like a utility maximizer
  - If not, then they don't

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