Consumer Choice 1

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GR6211 - Microeconomic Analysis 1

- We are now going to think a lot more about a particular type of choice we introduced last lecture
- **Choice from Budget Sets**
  - Objects of choice are **commodity bundles**

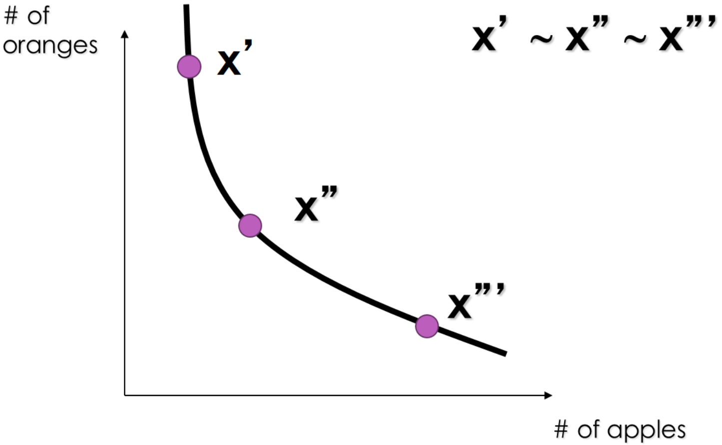
$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

  - Consumers are **price takers**
    - Treat prices and incomes as fixed
  - They can choose any bundle which satisfies their budget constraint

$$\left\{ x \in \mathbb{R}^n_+ \middle| \sum_{i=1}^{n} p_i x_i \leq w \right\}$$

- Why are such choices so interesting?
    - Many economic interactions can be characterized this way
    - Will form the basis of the study of equilibrium in the second half of the class

- When dealing with preferences over commodity bundles it will be useful to think about **Indifference Curves**
- These are curves that link bundles that are considered indifferent by the consumer
    - i.e. the set of points $\{x | x \sim y \text{ for some } y\}$
- Useful for presenting 3 dimensional information on a two dimensional graph

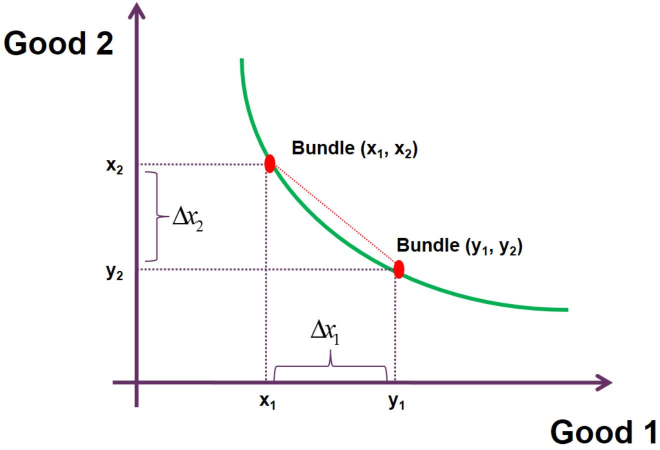


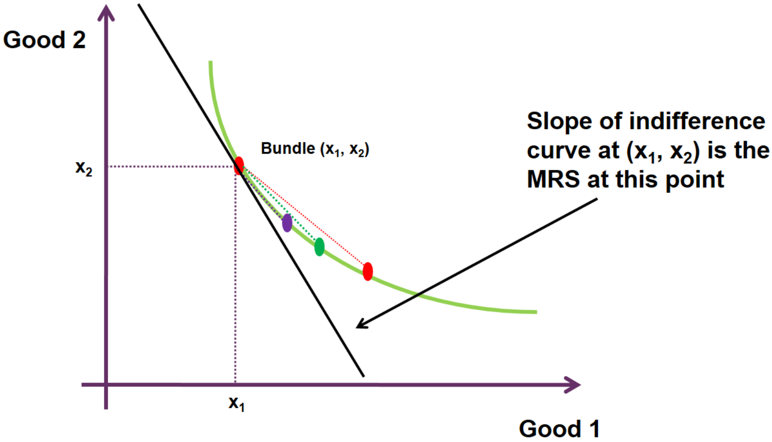
# of oranges
x’
x”
x”’
# of apples
x’ ~ x” ~ x”’

- A couple of properties of indifference curves

1. Two different indifference curves cannot cross (why?)
2. The 'slope' of the indifference curve represents the (negative of the) **marginal rate of substitution**
   - The rate at which two goods can be traded off while keeping the subject indifferent

$$\begin{aligned} MRS(x_2, x_1) &= -\lim_{\Delta(x_1)\to 0} \frac{\Delta(x_2)}{\Delta(x_1)} \\ \text{such that } (x_1, x_2) &\sim (x_1 + \Delta(x_1), x_2 + \Delta(x_2)) \end{aligned}$$


Good 2
Bundle (x_1, x_2)
x_2
Δx_2
Bundle (y_1, y_2)
y_2
Δx_1
x_1
y_1
Good 1



- Question: Is MRS always well defined?

Indifference Curves

- If preferences can be represented by a utility function, then the equation of an indifference curve is given by

$$u(x) = \bar{u}$$

- Thus, if the utility function is differentiable we have

$$\sum_{i=1}^{N} \frac{\partial u(x)}{\partial x_i} dx_i = 0$$

- And so, in the case of two goods, the slope of the indifference curve is

$$\frac{dx_2}{dx_1} = -\frac{\frac{\partial u(x)}{\partial x_1}}{\frac{\partial u(x)}{\partial x_2}} = -MRS$$

which is another way of characterizing the MRS

Preferences over Commodity Bundles

- When thinking about preferences over commodity bundles it might be natural to assume that preferences have properties other than just
  - Completeness
  - Transitivity
  - Reflexivity
- Some of these we have come across before
  - (Strict) Monotonicity

$$\begin{aligned} x_n &\geq y_n \text{ for all } n \text{ and } x_n > y_n \text{ for some } n \\ \text{implies that } x &\succ y \end{aligned}$$

  - Monotonicity

$$\begin{aligned} x_n &\geq y_n \text{ for all } n \text{ implies } x \succeq y \\ x_n &> y_n \text{ for all } n \text{ implies } x \succ y \end{aligned}$$

  - Local Non-Satiation
- Examples?

- Another property often assumed is **convexity**

Definition

The preference relation $\succeq$ is convex if the upper contour set $U_{\succeq}(x) = \{y \in X | y \succeq x\}$ is convex
i.e. for any x, z, y such that $y \succeq x$ and $z \succeq x$ and $\alpha \in (0, 1)$

$$(\alpha y + (1-\alpha)z) \succeq x$$

- What do convex indifference curves look like?
- Some alternative (equivalent) definitions of convexity
  - If $x \succeq y$ then for any α $(0, 1)$ $\alpha x + (1-\alpha)y \succeq y$
  - For all $w = \alpha x + (1-\alpha)y$ either $w \succeq x$ or $w \succeq y$

- What is convexity capturing?
  1. Mixtures are better than extremes
  2. If x is better than y, then going towards x is an improvement

Definition

A preference relation is **strictly convex** if x, z, y such that $y \succeq x$ and $z \succeq x$ and $\alpha \in (0, 1)$

$$(\alpha y + (1 - \alpha) z) \succ x$$

- Examples of preferences that are convex but not strictly so?

- What does the utility function of convex preferences look like?

Fact

A complete preference relation with a utility representation is convex if and only if it can be represented by a quasi concave utility function - i.e., for every x the set

$$\{y \in X | u(y) \geq u(x)\}$$

is convex

- Note that q-concave is weaker than concave
- It is **not** the case that continuous, convex preferences can necessarily be represented by concave utility functions
  - See homework

- A another property that preferences **can** have is **homotheticity**
  - The preference relation $\succeq$ is homothetic if $x \succeq y$ implies $\alpha x \succeq \alpha y$ for any $\alpha \geq 0$

Fact

A complete, increasing, continuous homothetic preference relation with a utility representation can be represented with a utility function which is homogenous of degree 1, i.e.

$$u(\alpha x_1, ...\alpha x_n) = \alpha u(x_1, ...x_n)$$

- What do homothetic indifference curves look like?
    - Note that $\frac{\partial u(\alpha x)}{\partial(\alpha x_i)} = \frac{\partial u(x)}{\partial x_i}$ and so

$$\frac{dx_2}{dx_1} = -\frac{\frac{\partial u(x)}{\partial x_1}}{\frac{\partial u(x)}{\partial x_2}} = -\frac{\frac{\partial u(\alpha x)}{\partial \alpha x_1}}{\frac{\partial u(\alpha x)}{\partial \alpha x_2}}$$

    - Slope of indifference curves remains the same along rays
- What is their economic intuition?
    - MRS depends only on the ratio of two goods
    - Going to nullify income effects in a handy way

- Finally, we might be interested in preferences that are **quasi linear**
  - The preference relation $\succeq$ is quasi linear in commodity 1 if $x \succeq y$ implies

$$(x + \varepsilon e_1) \succeq (y + \varepsilon e_1)$$

for all $\varepsilon > 0$ and

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Fact

A complete, continuous strictly monotonic, quasi linear preference relation with a utility representation can be represented with a utility function of the form

$$u(x) = v(x_2, ...x_k) + x_1$$

Quasi Linear Preferences

- What do quasi linear indifference curves look like?
  - MRS is unaffected by changes in q-linear good
  - Indifference curves are *parallel*
- What is their economic intuition?
  - Marginal utility of q-linear good is constant
  - q-linear good does not affect marginal utility of other goods
  - Often interpreted as 'wealth'