Microeconomic Analysis

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Homework 1

Due Thursday 13th September

Question 1 Some things to clear up from class

1. Prove that a choice correspondence satisfies properties α and β if and only if it satisfies WARP
2. Show that if a choice function has a utility representation then it must satisfy α and β

Question 2 Say that, rather than starting with the preference relation 'at least as good as' $\succeq$ as a primitive, and using this to define 'strictly preferred to' $\succ$, we go the other way round.

1. If you start with the binary relation $\succ$, interpreted as 'strictly preferred to', how would you generate the relation 'at least as good as'?
2. Assume that $\succ$ satisfies the following two properties
   - Asymmetry: if $x \succ y$ then not $y \succ x$
   - Negative Transitivity: For all x, y and z, if $x \succ y$ then either $x \succ z$ or $z \succ y$ (or both)

   Show that under these conditions, the relationship you constructed in part (1) will be complete, transitive and reflexive.

Question 3 Utility maximization is not the only choice procedure that are consistent with α and β. There are also other choice procedures that will satisfy these conditions and so are indistinguishable from rational choice. Consider the following decision making procedures. Prove whether or not they will result in choices that satisfy α and β

1. A decision maker (DM) is choosing between books from a set B. They have a utility function $u : B \to \mathbb{R}$, and a 'threshold utility' level u^*. In any choice set, they search through the books alphabetically by title, and choose the first book that has utility level u that is equal to or above u^*. If they have not found any such book by the time they reach the end of the choice set, they will choose the book with the highest utility (to make things simpler you can assume that there is no indifference - i.e. no two books have the same utilities)
2. A DM assigns a utility number to each alternative and chooses the alternative with the lowest utility
3. The decision maker asks his two children to rank the alternatives and then chooses the alternative that is the best on average.
4. The DM ranks the alternatives according to a utility function, and in any choice set chooses the median element

Question 4 Consider the following choice procedure: A decision maker has an ordering $\succeq$ over the set X that is asymmetric transitive irreflexive and has the property for any $x \neq y$ either $x \succeq y$ or $y \succeq x$ They also assigns to each $x \in X$ a natural number $class(x)$ to be interpreted as the "class" of x. Given a choice problem A, he chooses the best element in A from those belonging to the most common class in A (i.e., the class that appears in A most often). If there is more than one most common class, he picks the best element from the members of A that belong to a most common class with the highest class number.

1. Is the procedure consistent with α and β?
2. Define the relation: xPy if x is chosen from $\{x, y\}$. Show that the relation P is complete, asymmetric, and transitive.

Question 5 In class we defined the $\succeq$ −maximal element of a set as

$$Max(A| \succeq) = \{x \in A | x \succeq y \text{ for all } y \in A\}$$

1. Show that if A is finite and $\succeq$ is a preferences relation then $Max(A| \succeq)$ is non-empty
2. Here is another definition: for a binary relation B, define

$$M(A|B) = \{x \in A | yBx \text{ for no } y \in A\}$$

Show that if B is equal to $\succ$ for some preference relation $\succeq$ then $Max(A|\succeq) = M(A|\succ)$

3. If $\succeq$ is transitive and reflexive, but not necessarily complete, and we still define $x \succ y$: if $x \succeq y$ but not $y \succeq x$ what is the relationship between $Max(A|\succeq)$ and $M(A|\succ)$

4. We say that a binary relation $\succ$ is **acyclic** if there exists no set $x_1, ..., x_n$ such that

$$x_1 \succ x_2 \succ ... \succ x_n \succ x_1$$

Define

$$\hat{C}(A) = M(A|B)$$

for all $A \in 2^X/\varnothing$. Show that $\hat{C}$ is a choice correspondance if and only if B is acyclic