Microeconomic Analysis

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Homework 1

Due Thursday 20th September

Question 1 Show that the two definitions of continuity we introduced in class are identical. Show they are also identical to the following conditions

- For any x the upper and lower contours $\{y|y \succeq x\}$ and $\{y|x \succeq y\}$ are closed
- For any x the sets $\{y|y \succ x\}$ and $\{y|x \succ y\}$ are open

Question 2 Prove the claim from class that if C is a continuous choice correspondence and $\succeq$ represents C then $\succeq$ is continuous.

Question 3 A preference relation $\succeq$ on $\mathbb{R}_+^N$ is strictly increasing if $x > y$ (i.e. $x_n \geq y_n \ \forall \ n$ and $x_n > y_n$ some n) implies $x \succ y$ for all $x, y \in \mathbb{R}_+^N$. Let $\succeq$ be a continuous and strictly increasing preference relation on $\mathbb{R}_+^N$. Define $u(x) = \max\left[\alpha \geq 0 | x \succeq (\alpha, \alpha, ..., \alpha)\right]$. Show that u is well defined, strictly increasing, continuous and represents $\succeq$.

Question 4 We say that a preordered set $(X, \succeq)$ is $\succeq -$separable if there exists a countable set $Y \subset X$ such that, for every $x, z \in X$ such that $x \succ z$, there exists a $y \in Y$ such that $x \succeq y \succeq z$. Prove the following theorem. (hint - you may find it useful to know that any subset of $\mathbb{R}$ is separable - i.e. it has a countable, dense subset.)

Theorem 1 *Let $\succeq$ be a complete preference relation on X. $\succeq$ admits a utility representation if and only if $(X, \succeq)$ is $\succeq -$separable*

Question 5 Question 6 in problem set 2 from the Rubinstein book (note he isn't joking about part 3 being difficult. Have a go but if you get stuck ask for a hint).