Microeconomic Analysis

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Homework 3

Due Thursday 27th September

Question 1 Prove the following simplified version of Sziplrajn's theorem

> **Theorem 1** *Let $\succeq$ be a partial order on a finite set X. Show that there exists a linear order on X which is an extension of $\succeq$.*

Question 2 Consider a binary relation $\succeq$ on some finite set X which is transitive and reflexive but not necessarily complete.

1. Show that there exists a utility function $u : X \to \mathbb{R}$ that represents $\succeq$ in the sense that

$$x \succeq y \to u(x) \geq u(y)$$
$$x \succ y \to u(x) > u(y)$$

2. Clearly this representation is 'worse' than the standard one, in the sense that we cannot recover $\succeq$ from the utility function. To get round this problem, we can use a **multi-utility representation**. A multi-utility representation of the relation $\succeq$ on X is is a set of functions $\mathcal{U}$, where each $u \in \mathcal{U}$ is a function $u : X \to \mathbb{R}$, and these functions represent $\succeq$ in the sense that

$$x \succeq y \text{ if and only if } u(x) \geq u(y) \; \forall \; u \in \mathcal{U}$$

Show that a multi utility representation has the same information as the original binary relation -i.e. there is a unique preference relation that is consistent with any multi-utility representation.

3. One interpretation of the multi utility representation is that each object can be ranked on a number of dimensions, and you are only prepared to say that x is better than y if it is at least as good along all dimensions, and better on one. With that in mind, show how you can construct a multi-utility representation for the partial order $\geq$ on $\mathbb{R}^n$. (i.e. $x \geq y$ if and only if $x_i \geq y_i \ \forall \ i \in \mathbb{N}$)
4. Show that any transitive, reflexive relation on a finite set X admits a multi utility representation.

Question 3 One modification of the standard economic model used to capture some idea of bounded rationality is to introduce the notion of consideration sets: rather than maximizing across all possible alternatives, the decision maker only considers a subset, and chooses the utility maximizing option in that subset.

Definition 1 *We say a set of choice data can be explained as choice with consideration sets if there is (i) a utility function* $u : X \to \mathbb{R}$ *and (ii) a consideration set correspondence* $\Gamma : 2^X/\emptyset \to 2^X/\emptyset$ *such that* $\Gamma(A) \subset A$ *and*

$$C(A) = \max_{x \in \Gamma(A)} u(x)$$

For simplicity, let's assume that we are dealing with choice functions (not correspondences), there is no indifference and X is finite

1. Show that a model of choice from consideration sets can explain any choice function
2. Now add the restriction

$$\Gamma(A) = \Gamma(A/x) \text{ if } x \notin \Gamma(A)$$

   We will call the model in Definition 1 with this restriction added 'Model A'.

   Determine whether the following methods of constructing consideration sets would satisfy this property

   (a) Top N: The decision maker is choosing between cars, and in any choice set considers the top 3 according to safety rating

   (b) The decision maker is choosing between jobs and has three criteria: wage, holiday and proximity. In any choice set they consider only the alternatives that are best in each category

(c) The decision maker is choosing between hats, and considers all which are at or below the median price

3. Find a set of choices that cannot be explained by model A.

4. Show that, if choices are produced by model A then they must satisfy the following condition:

$$\begin{aligned} \text{For any non-empty set } S\text{, there exists } x^* &\in S\text{, such that, for any set } T \text{ including } x^* \\ C(T) &= x^* \text{ whenever} \\ (i)\ C(T) &\in S \text{ and} \\ (ii)\ C(T) &\neq c(T \backslash x^*) \end{aligned}$$

5. Show that α implies this property, but not visa versa

6. Show that, under model A, if $x = C(A)$ and $y \in A$, then it is not necessarily the case that $u(x) > u(y)$. Define the relation xPy such that if, for some choice set A such that $x, y \in A$ and $x \neq y$ it is the case that $C(A) = x \neq C(A \backslash y)$. Show that under model A xPy implies $u(x) > u(y)$

7. Show that the condition in part 4 guarantees that the relation P is acyclic

8. (Hard) Show that the condition in part (4) is enough to guarantee the existence of a representation of the form of model A