Microeconomic Analysis

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Homework 4

Due Thursday 11th October

Question 1 Here are four commonly used utility function for the case of two commodities

- Cobb-Douglas: $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$ for $\alpha > 0$ and $\beta > 0$
- Constant Elasticity of Substitution: $u(x_1, x_2) = (x_1^{\rho} + x_2^{\rho})^{\frac{1}{\rho}}$ for $\rho \in \mathbb{R}$
- Linear: $u(x_1, x_2) = \alpha x_1 + \beta x_2$ for $\alpha > 0$ and $\beta > 0$
- Leontief: $u(x_1, x_2) = \min(\alpha x_1, \beta x_2)$ for $\alpha > 0$ and $\beta > 0$

1. For each of these utility functions derive the Walrasian and Hicksian demand correspondences, the indirect utility function and the Slutsky matrix (where it is defined)
2. Verify for each that the properties of the two demand functions described in the lecture notes hold. Also show in each case that $x(p, w) = h(p, v(p, w))$
3. Show that the Walrasian and Hicksian demand functions of the CES preferences converge to those of the Linear and Leontief preferences as $\rho \to 1$ and $\rho \to \infty$ respectively. For simplicity, you can focus only on cases in which all three preferences give demand functions which are single valued
4. The elasticity of substitution between two goods is given by

$$\xi_{1,2}(p, w) = \frac{p_1}{p_2} \frac{\partial x(p, w)/\partial p_2}{\partial x(p, w)/\partial p_1}$$

Provide an interpretation of the elasticity, and calculate it for each of the 4 preferences above

Question 2 Rubenstein Chapter 6, problem 3

Question 3 Assume that there are two goods and that we observe $x(p_1, p_2, w)$ at the following point

$$x(1,1,8) = \begin{cases} 4 \\ 4 \end{cases}$$

Consider the budget set $B(1, 4, 26)$. Identify the set of points in this budget set which are consistent with $x(1, 1, 8)$ and the assumption of

1. Locally non-satiated preferences
2. Quasi linear preferences with respect to the first good
3. Quasi linear preferences with respect to the second good
4. Homothetic preferences

Question 4 This is a question which explores the type of preferences we used to construct the indirect utility function - sometimes called preferences over menus.

Consider the following decision maker: They have a utility function u on a set of finite alternatives X. Their preferences over sets (or menus) of these alternatives (which we indicate by $\trianglerighteq$ for weak preferences) are given by the following. For any $A \in 2^X/\emptyset$, $B \in 2^X/\emptyset$

$$\begin{aligned} A &\trianglerighteq B \text{ if and only if} \\ \max_{x \in A} u(x) &\geq \max_{x \in B} u(x) \end{aligned}$$

1. Show that the binary relation $\trianglerighteq$ is a preference relation
2. Show that it satisfies the following property: if $A \trianglerighteq B$, then $A \bowtie A \cup B$ (where $\bowtie$ indicates indifference - i.e. $A \bowtie B$ iff $A \trianglerighteq B$ and $B \trianglerighteq A$)
3. Show that if $\trianglerighteq$ is a preference relation that has this property then we can find a utility function $u : X \to \mathbb{R}$ such that $A \trianglerighteq B$ if and only if $\max_{x \in A} u(x) \geq \max_{x \in B} u(x)$ (i.e. we can find a representation of the type in part 1). *Hint: in the data we have, what observation tells us that an alternative $x \in X$ is preferred to an alternative $y \in X$?*