

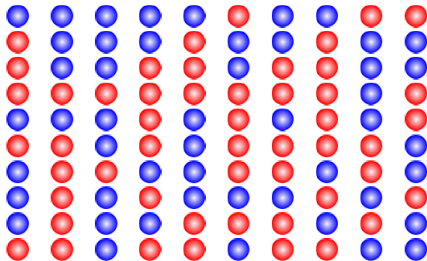
Rational Inattention, and Costly Information Acquisition

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ECON 2090 Spring 2015

- (Hopefully) convinced you that attention costs are important
- Introduced the 'satisficing' model of search and choice
- But, this model seems quite restrictive:
 - Sequential Search
 - 'All or nothing' understanding of alternatives
- Seems like a good model for choice over a large number of simple alternatives
- Not for a small number of complex alternatives

A Non-Satisficing Situation

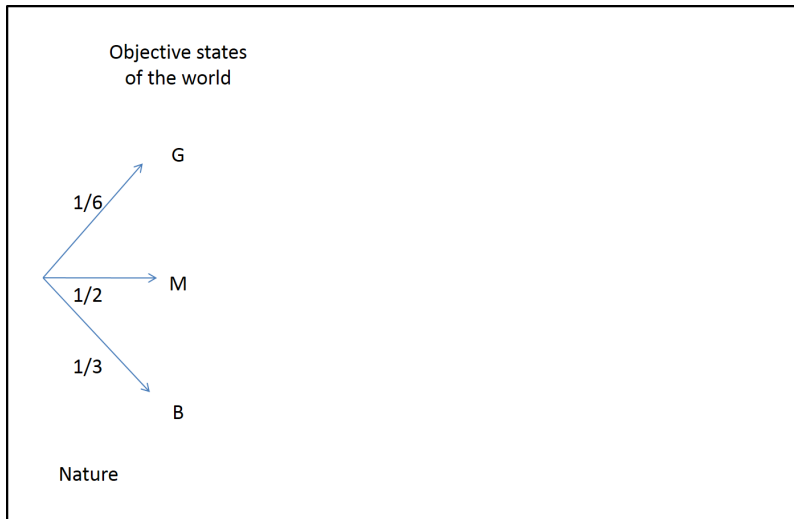


Act	Payoff 47 red dots	Payoff 53 red dots
a	20	0
b	0	10

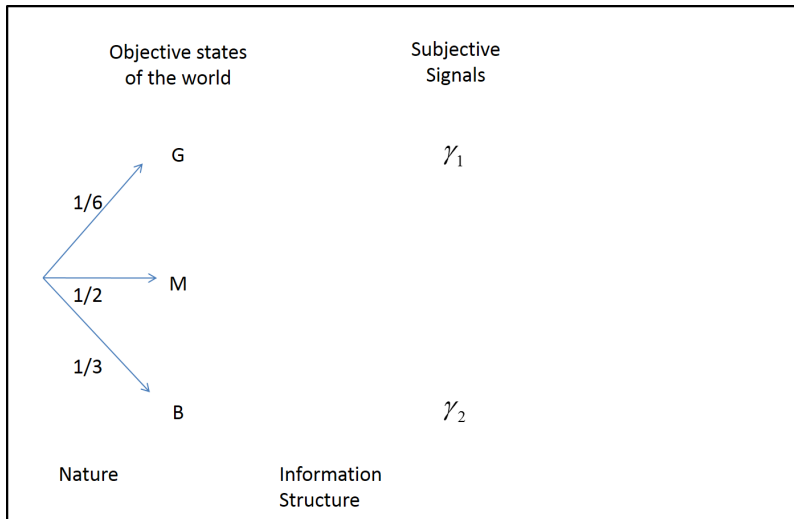
- Objective states of the world
 - e.g. Demand could be 'good', 'medium' or 'bad'
- Decision maker chooses an action
 - e.g. Set price to be high, medium, or low
- Gross payoff depends on action and state
 - e.g. Quantity sold depends on price and demand
- Decision maker get to learn something about the state before choosing action
 - e.g. Could do market research, focus groups, etc.
- Can choose what to learn conditional on the problem

- The specifics of the process of information acquisition may be very complex
- We model the choice of information in an *abstract* way
- The decision maker chooses an *information structure*
 - Set of signals to receive
 - Probability of receiving each signal in each state of the world
- Choose action conditional on signal received
- Value of strategy given by
 - Expected value of actions taken given posterior beliefs
 - Minus cost of information
- Flexible enough to cover all commonly used models
 - via restriction on the cost function

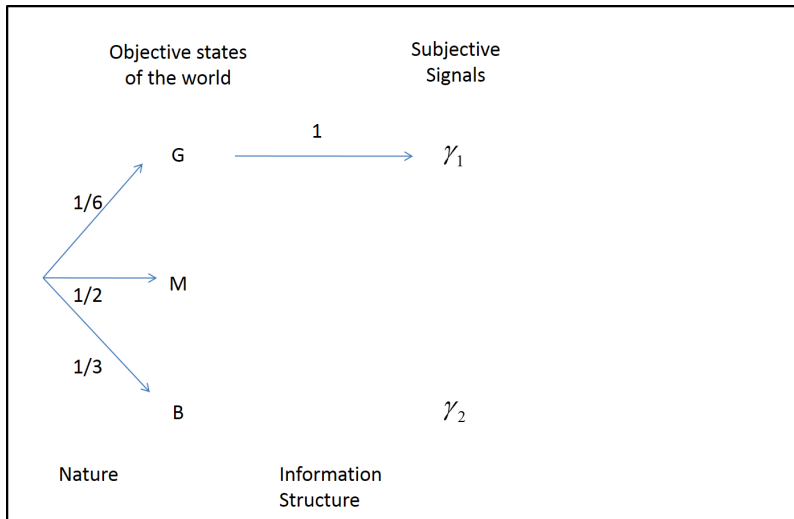
The Choice Problem



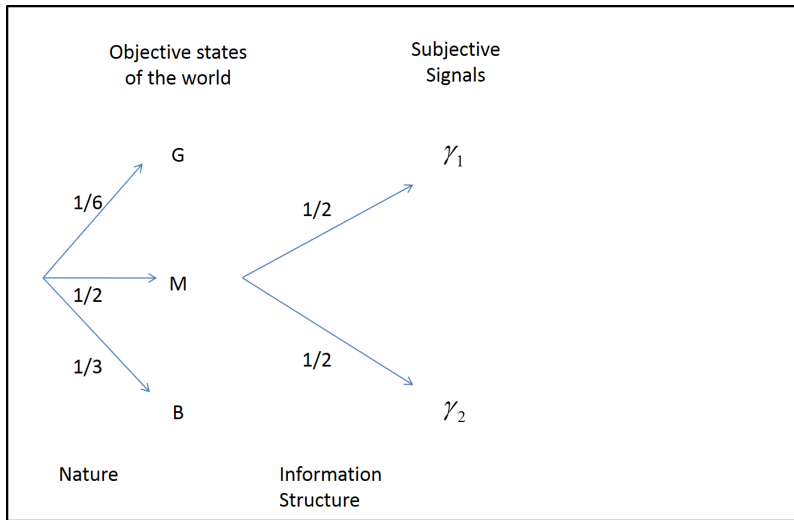
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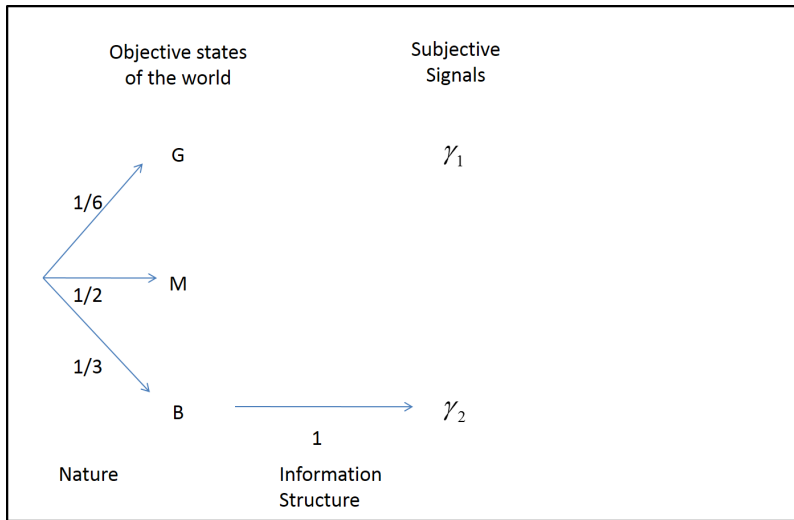
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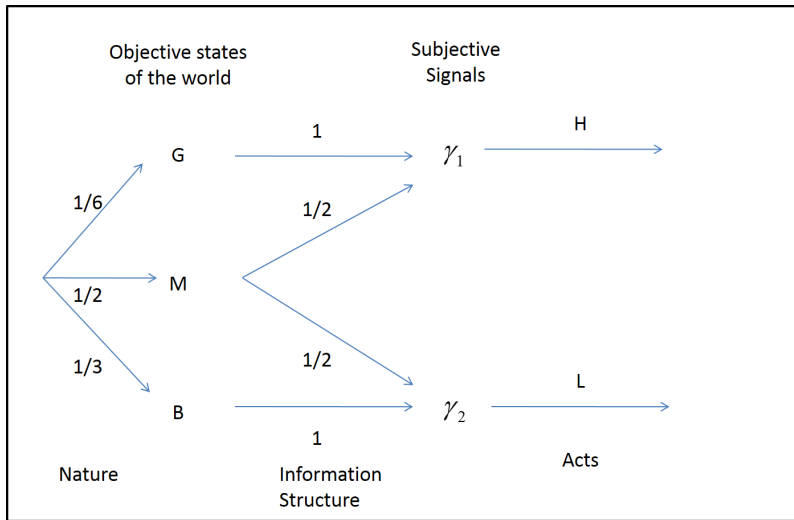
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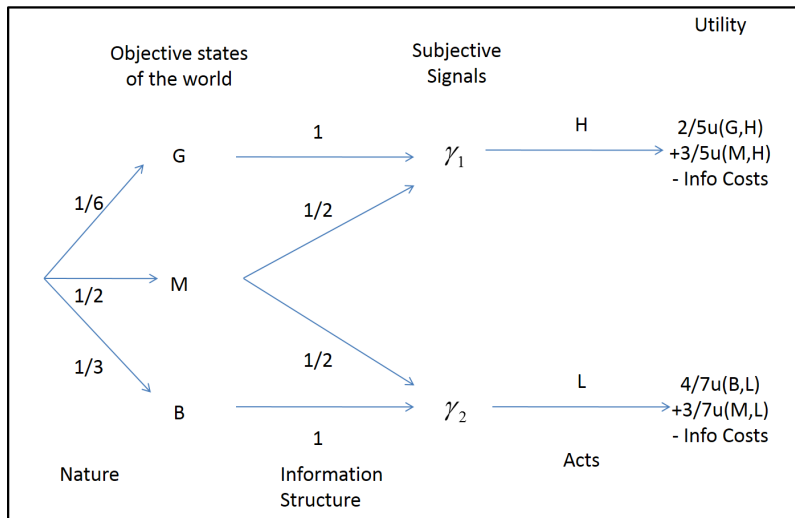
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The Choice Problem



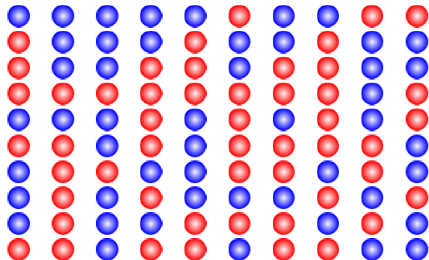
- Ω : Objective states of the world (finite)
 - with prior probabilities μ
- a : An action - utility depends on the state
 - $U(a(\omega))$ utility of action a in state ω
 - \mathcal{A} : Set of actions:
- $A \subset \mathcal{A}$: Decision problem (finite)

- For each decision problem
 - 1 Choose information structure (π)
 - Defined by:
 - Set of signals: $\Gamma(\pi)$
 - Probability of receiving each signal γ from each state ω : $\pi(\gamma|\omega)$
 - 2 Choose action conditional on signal received (C)
 - $C(\gamma)$ probability distribution over actions given signal γ
- In order to maximize
 - Expected value of actions taken given posterior beliefs
 - Minus cost of information K

$$\sum_{\Omega} \mu(\omega) \sum_{\gamma \in \Gamma(\pi)} \pi(\gamma|\omega) \left(\sum_{a \in A} C(a|\gamma) U(a(\omega)) \right) - K(\pi)$$

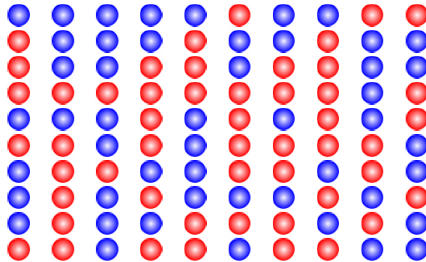
- Let D be a collection of decision problems
- For each $A \in D$ we observe **state dependent stochastic choice data** P_A
 - $P_A(a|\omega)$ probability of choosing action a conditional on state ω
- Also assume we observe:
 - Prior probabilities μ
 - Utilities U
- Do **not** observe
 - Information structures π_A
 - Subjective signals γ
 - Information costs K

An Experimental Example



- Subjects presented with 100 balls
- State is determined by the number of red balls
- Prior distribution of red balls known to subject

An Experimental Example



Action	Payoff 49 red balls	Payoff 51 red balls
a	10	0
b	0	10

- No time limit: trade off between effort and financial rewards

An Experimental Example

- Data: State dependant stochastic choice
 - Probability of choosing each action in each objective state of the world

Action	State = 49 red balls	State = 51 red balls
Prob choose a	$P(a 49)$	$P(a 51)$
Prob choose b	$P(b 49)$	$P(b 51)$

- Observe subject making same choice 50 times
- Can use this to estimate P_A

- What type of stochastic choice data $\{D, P\}$ is consistent with optimal information acquisition?
- i.e. there exists a cost function K
- For each decision problem $A \in D$ an information structure π_A and choice function C_A s.t.
 - C_A is optimal for each γ
 - π_A is optimal given K
 - C_A and π_A are consistent with P_A

$$P_A(a|\omega) = \sum_{\gamma \in \Gamma(\pi_A)} \pi_A(\gamma|\omega) C_A(a|\gamma).$$

- What 'mistakes' are consistent with optimal behavior in the face of information costs?

A Comparison to Existing Approaches

- The problem we study is very flexible
 - No in principle restriction on information structures
 - No restrictions on costs
- Nests other models of information acquisition
 - Shannon Mutual Information (fixed or costly)
 - Shannon Capacity
 - Fixed signals
 - Partitions
- Can mimic a hard constraint by setting costs to ∞
- Conditions we provide are necessary and sufficient in finite data sets
 - Easily applied to laboratory data
 - Possible to apply it to non-experimental data

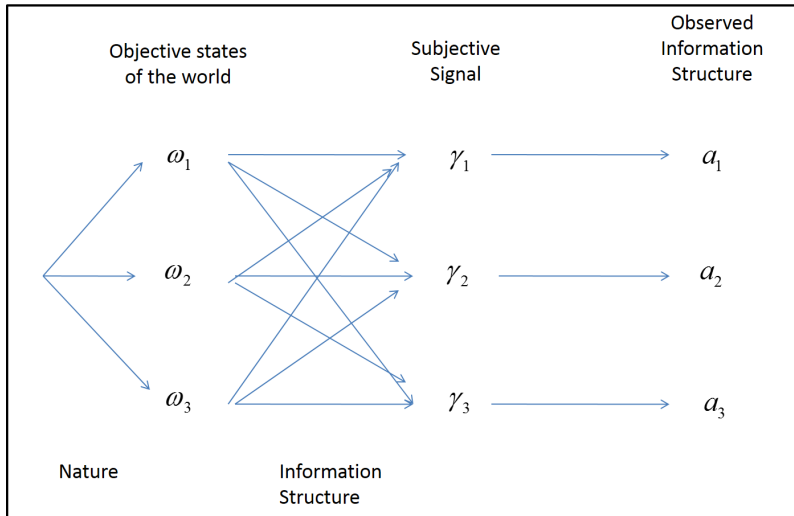
Observing Information Structures

- Key observation: State dependent stochastic choice data tells us a lot about the information structure a decision maker has used
- Assume that decision maker is 'well behaved'
 - Chooses each action in response to at most one signal
 - No mixed strategies - one action per signal
- Information structure can be observed directly from state dependent stochastic choice
 - For each chosen action a there is an associated signal $\bar{\gamma}^a$
 - Probability of signal $\bar{\gamma}^a$ in state ω is the same as the probability of choosing a in ω

$$\bar{\pi}(\bar{\gamma}^a|\omega) = P(a|\omega)$$

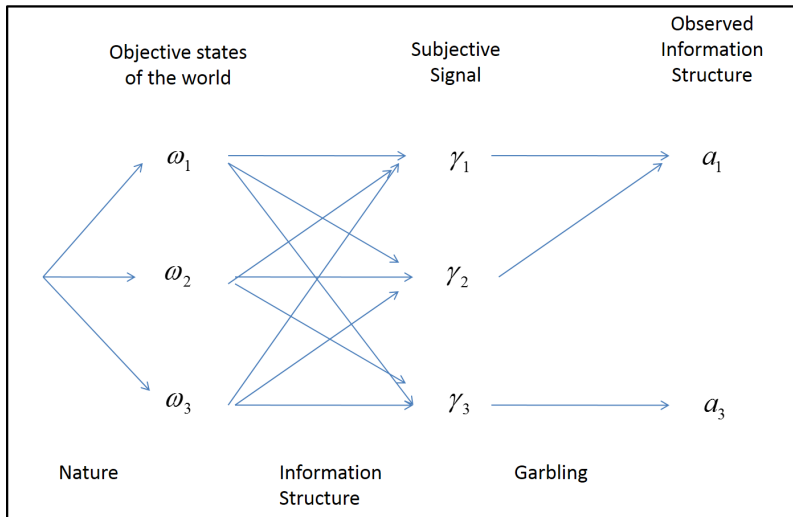
- Call $\bar{\pi}$ the 'revealed information structure'

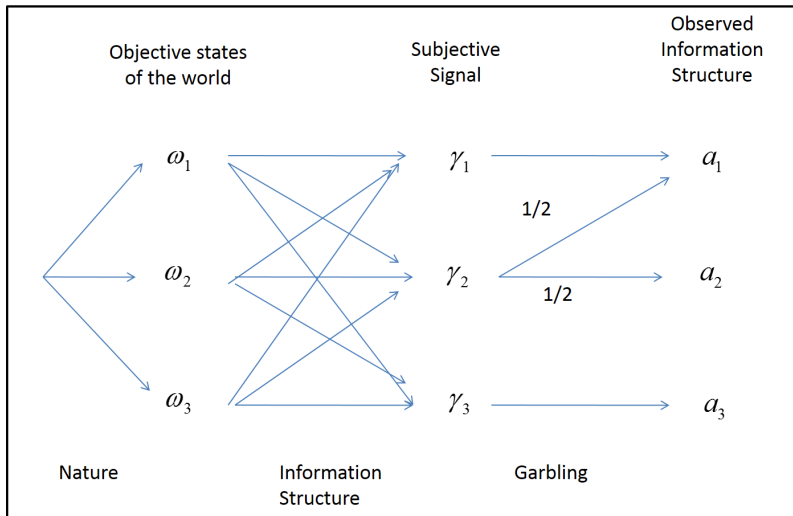
Recovering Attention Strategy



- What if decision maker is not well behaved?
 - Chooses some act in more than one **subjective** state
 - Mixed strategies - more than one act in an **subjective** state

Same Act in Different States





Observing Information Structures

- Can still recover revealed information structure $\bar{\pi}$
- Not necessarily the same as true information structure π
- But will be a **garbling** of the true information structure
 - i.e. π is statistically sufficient for $\bar{\pi}$
- There exists a stochastic $|\Gamma(\pi)| \times |\Gamma(\bar{\pi})|$ matrix B such that if we
 - Apply π
 - For each state γ^i move to state $\bar{\gamma}^j$ with probability B^{ij}
 - We obtain $\bar{\pi}$
- i.e.

$$\sum_j B^{ij} = 1 \quad \forall j$$
$$\bar{\pi}(\bar{\gamma}^j | \omega) = \sum_i B^{ij} \pi(\gamma^i | \omega) \quad \forall j$$

An Aside: Blackwell's Theorem

- Let $G(A, \pi)$ be the *gross value* of using information structure π in decision problem A

$$\begin{aligned} & G(A, \pi) \\ = & \max_{C:\Gamma(\pi)\rightarrow\Delta(A)} \sum_{\Omega} \mu(\omega) \sum_{\gamma\in\Gamma(\pi)} \pi(\gamma|\omega) \left(\sum_{a\in A} C(a|\gamma) U(a(\omega)) \right) \end{aligned}$$

- An information structure π is sufficient for information structure π' if and only if

$$G(A, \pi) \geq G(A, \pi') \quad \forall A$$

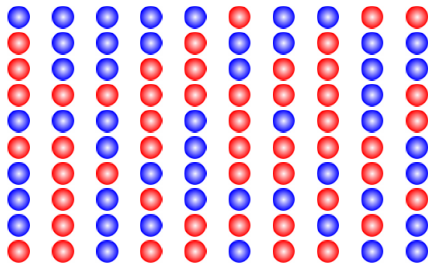
Characterizing Rational Inattention

- Choice of act optimal given attentional strategy
 - [Caplin and Martin 2014]
- Choice of attention strategy optimal

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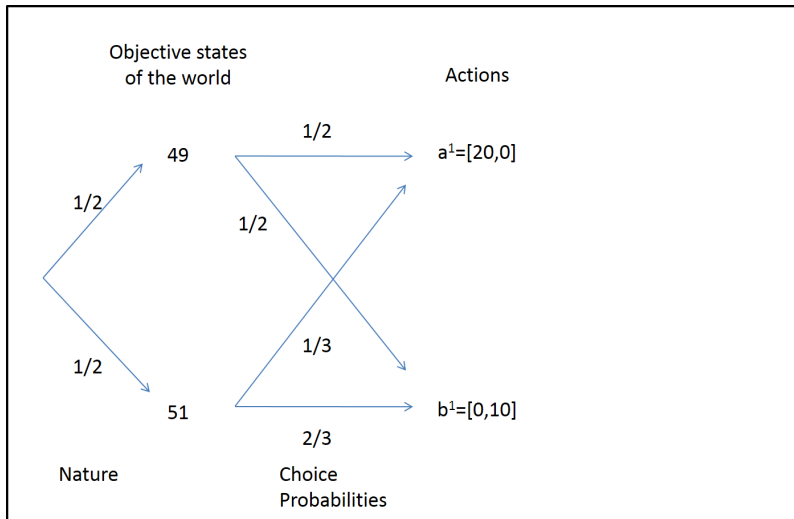
Optimal Choice of action



Action	Payoff 49 red balls	Payoff 51 red balls
a^1	20	0
b^1	0	10

Prior: $\{0.5, 0.5\}$

Optimal Choice of actions



- Posterior probability of 49 red balls when action b was chosen

$$\begin{aligned} \Pr(\omega = 49 | b \text{ chosen}) &= \frac{\Pr(\omega = 49, b \text{ chosen})}{\Pr(b \text{ chosen})} \\ &= \frac{\frac{1}{4}}{\frac{1}{4} + \frac{2}{6}} = \frac{3}{7} \end{aligned}$$

- But for this posterior

$$\begin{aligned} \frac{3}{7}U(a(49)) + \frac{4}{7}U(a(51)) &= \frac{3}{7}20 + \frac{4}{7}0 = 8.6 \\ \frac{3}{7}U(b(49)) + \frac{4}{7}U(b(51)) &= \frac{3}{7}0 + \frac{4}{7}10 = 5.7 \end{aligned}$$

- To avoid such cases requires

$$a \in \arg \max_{a \in A} \sum_{\Omega} \Pr(\omega|a) U(a(\omega))$$

- Which implies

Condition 1 (No Improving Action Switches) For every chosen action a

$$\sum \mu(\omega) P_A(a|\omega) [u(a(\omega)) - u(b(\omega))] \geq 0.$$

for all $b \in A$

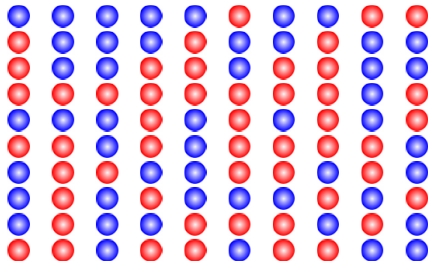
- If $\bar{\pi}$ not true information structure, condition still holds
 - a optimal at all posteriors in which it is chosen
 - Must also be optimal at convex combination of these posteriors

Characterizing Rational Inattention

- Choice of act optimal given attentional strategy
- Choice of attention strategy optimal

Optimal Choice of Attention Strategy

Decision Problem 1

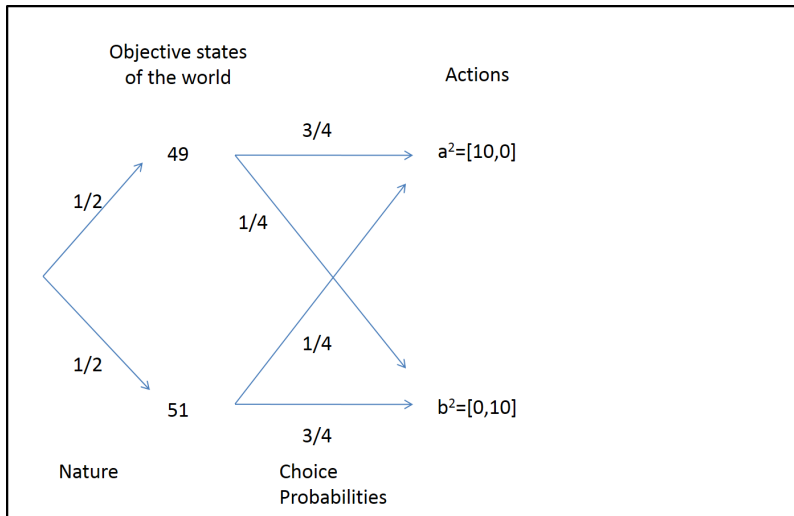


Action	Payoff 49 red balls	Payoff 51 red balls
a^2	10	0
b^2	0	10

Prior: $\{0.5, 0.5\}$

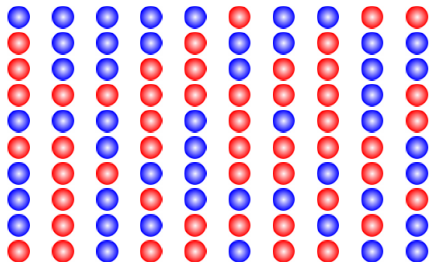
Optimal Choice of Attention Strategy

Decision Problem 1



Optimal Choice of Attention Strategy

Decision Problem 2

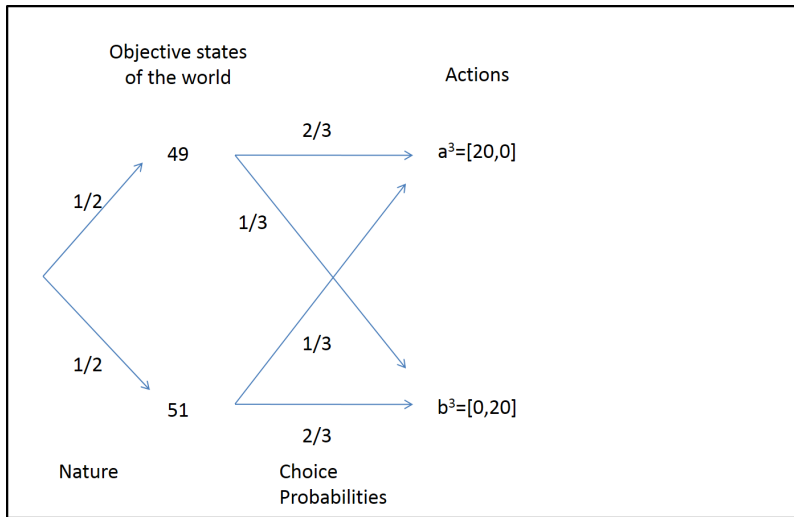


action	Payoff 49 red balls	Payoff 59 red balls
a^3	20	0
b^3	0	20

Prior: $\{0.5, 0.5\}$

Optimal Choice of Attention Strategy

Decision Problem 2



Optimal Choice of Attention Strategy

- $G(A, \pi)$ is the gross value of using information structure π in decision problem A

G	$\bar{\pi}^1$	$\bar{\pi}^2$
$\{a^1, b^1\}$	$7\frac{1}{2}$	$6\frac{2}{3}$
$\{a^2, b^2\}$	15	$13\frac{1}{3}$

- Cost function must satisfy

$$G(\{a^1, b^1\}, \pi^1) - K(\pi^1) \geq G(\{a^1, b^1\}, \pi^2) - K(\pi^2)$$

$$G(\{a^2, b^2\}, \pi^2) - K(\pi^2) \geq G(\{a^2, b^2\}, \pi^1) - K(\pi^1)$$

- Which implies

$$\frac{5}{6} = G(\{a^1, b^1\}, \pi^1) - G(\{a^1, b^1\}, \pi^2) \geq$$

$$K(\pi^1) - K(\pi^2) \geq$$

$$G(\{a^2, b^2\}, \pi^1) - G(\{a^2, b^2\}, \pi^2) = 1\frac{2}{3}$$

Optimal Choice of Attention Strategy

- Surplus must be maximized by correct assignments

$$\begin{aligned} & G(\{a^1, b^1\}, \pi^1) + G(\{a^2, b^2\}, \pi^2) \\ & \geq G(\{a^1, b^1\}, \pi^2) + G(\{a^2, b^2\}, \pi^1) \end{aligned}$$

- What if $\bar{\pi} \neq \pi$?
- We know that revealed and true information structure must give same value in DP it was observed

$$G(A^i, \bar{\pi}^i) = G(A^i, \pi^i)$$

- Also, as π weakly Blackwell dominates $\bar{\pi}$

$$G(A^i, \bar{\pi}^j) \leq G(A^i, \pi^j)$$

Optimal Choice of Attention Strategy

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- To guarantee the existence of a cost function requires a stronger condition

Condition 2 (No Improving Attention Cycles) For an observed sequence of decision problems $A^1 \dots A^K$ and associated revealed information structures $\bar{\pi}^1 \dots \bar{\pi}^K$

$$\begin{aligned} & G(A^1, \bar{\pi}^1) - G(A^1, \bar{\pi}^2) \\ & + G(A^2, \bar{\pi}^2) - G(A^2, \bar{\pi}^3) \\ & + \dots \\ & + G(A^K, \bar{\pi}^K) - G(A^K, \bar{\pi}^1) \\ \geq & 0 \end{aligned}$$

- Note that this condition relies only on observable objects

Theorem

For any data set $\{D, P\}$ the following two statements are equivalent

- 1 $\{D, P\}$ satisfy NIAS and NIAC
- 2 There exists a $K : \Pi \rightarrow \mathbb{R}$, $\{\pi^A\}_{A \in D}$ and $\{C^A\}_{A \in D}$ such that π^A and $C^A : \Gamma(\pi^A) \rightarrow A$ are optimal and generate P^A for every $A \in D$

Proof.

2 \rightarrow 1 Trivial

1 \rightarrow 2 Rochet [1987] (literature on implementation) □

- This problem is familiar from the implementation literature
- Say there were a set of environments $X_1 \dots X_N$ and actions $B_1 \dots B_M$ such that the utility of each environment and each state is given by

$$u(X_i, B_j)$$

- Say we want to implement a mechanism such that action $Y(X_i)$ is taken at in each environment.
- We need to find a taxation scheme $\tau : B_1 \dots B_M \rightarrow \mathbb{R}$ such that

$$u(X_i, Y(X_i)) - \tau(Y(X_i)) \geq u(X_i, B) - \tau(B) \\ \forall B_1 \dots B_M$$

- This is the same as our problem.

- Our problem is equivalent to finding $\theta : D \rightarrow \mathbb{R}$, such that, for all $A_i, A_j \in D$

$$G(A_i, \pi^i) - \theta(A_i) \geq G(A_i, \pi^j) - \theta(A_j)$$

- Just define $K(\pi) = \theta(A_i)$ if $\pi = \pi^i$ for some i , or $= \infty$ otherwise
- We can apply a proof from Rockerfellar [1970] to show that NIAC gives us this condition

- Pick some arbitrary A_0 and define

$$T(A) = \sup_{\text{all chains s.t } A_0 \text{ to } A=A_M} \sum_{n=1}^{M-1} G(A_{i+1}, \pi^i) - G(A_i, \pi^i)$$

- NIAC implies that $T(A_0) = 0$
- Also note that

$$T(A_0) \geq T(A_i) + G(A_0, \pi^i) - G(A_i, \pi^i)$$

- So $T(A_i)$ is bounded

- Furthermore, for any A_i, A_j we have

$$T(A_i) \geq T(A_j) + G(A_i, \pi^j) - G(A_j, \pi^j)$$

- So, setting $\theta(A_j) = G(A_j, \pi^j) - T(A_j)$, we get

$$G(A_i, \pi^i) - \theta(A_i) \geq G(A_i, \pi^j) - \theta(A_j)$$

Restrictions on the Cost Function

- What about additional conditions on cost function?
 - Weakly Monotonic with respect to Blackwell?
 - Allow mixing?
 - Positive with free inattention?
- We get these 'for free'
- Any behavior that can be rationalized can be rationalized with a cost function that has these properties
- Can also extend to 'sequential rational inattention'

- Say $\bar{\pi}^A$ is the revealed attn. strategy in decision problem A .
- Assuming weak monotonicity, it must be that

$$K(\bar{\pi}^A) - K(\pi) \leq G(A, \bar{\pi}^A) - G(A, \pi)$$

- If $\bar{\pi}^B$ is used in decision problem B then we can bound relative costs

$$G(B, \bar{\pi}^A) - G(B, \bar{\pi}^B) \leq K(\bar{\pi}^A) - K(\bar{\pi}^B) \leq G(A, \bar{\pi}^A) - G(A, \bar{\pi}^B)$$

- Tighter bounds can be obtained using chains of observations

$$\begin{aligned} & \max_{\{A^1 \dots A^n \in D \mid A^1 = B, A^n = A\}} \sum \left[G(A^i, \bar{\pi}^{A^i}) - G(A^i, \bar{\pi}^{A^{i+1}}) \right] \\ & \leq K(\bar{\pi}^A) - K(\bar{\pi}^B) \\ & \leq \min_{\{A^1 \dots A^n \in D \mid A^1 = A, A^n = B\}} \sum \left[G(A^i, \bar{\pi}^{A^i}) - G(A^i, \bar{\pi}^{A^{i+1}}) \right] \end{aligned}$$

What If Utility and Priors Are Unobservable?

- Can add 'there exists' to the statement of the NIAS and NIAC conditions
- Data has an optimal costly attention representation if **there exists** $\mu \in \Delta(\Omega)$ and $U : X \rightarrow \mathbb{R}$ such that
 - NIAS is satisfied
 - NIAC is satisfied
- If μ is known but U is unknown, conditions are linear and (relatively) easy to check
- If μ and U are unknown, conditions are harder to check
 - Still not vacuous
- Alternatively, can enrich data so that these objects can be recovered

Rational Inattention vs Random Utility

- Alternative model of random choice: Random Utility
 - ① Agent receives some information about the state of the world
 - ② Draws a utility function from some set
 - ③ Chooses in order to maximize utility given information
- Key differences between Random Utility and Rational Inattention
 - ① Random Utility allows for multiple utility functions
 - ② Rational Inattention allows attention to vary with choice problem
- How can we differentiate between the two?

- Random Utility implies monotonicity
- For any two decision problems $\{A, A \cup b\}$, $a \in A$ and $b \notin A$

$$P_A(a|\omega) \geq P_{A \cup b}(a|\omega)$$

- Rational Inattention can lead to violations of monotonicity (Ergin, Matejka and McKay)

Act	Payoff 49 red dots	Payoff 51 red dots
a	23	23
b	20	25
c	40	0

- Adding act c to $\{a, b\}$ can increase the probability of choosing b in state 51

- We perform experiments to test two things
 - Whether subjects actively adjust their attention
 - Whether they do so optimally (concentrate on NIAC)
- Rule out alternative models with fixed attention
 - Signal Detection Theory
 - Random Utility Models

- Experiment 1: Extensive Margin
- Experiment 2: Spillovers
- Experiment 3: Intensive Margin

Experiment 1: Extensive Margin

Decision Problem	Payoffs			
	$U(a(1))$	$U(a(2))$	$U(b(1))$	$U(b(2))$
1	2	0	0	2
2	10	0	0	10
3	20	0	0	20
4	30	0	0	30

- Two equally likely states
- Two acts (a and b)
- Symmetric change in the value of making correct choice
- 46 subjects

- Surplus must be maximized by correct assignments. In two act two state case,

$$\Delta\tau_1\Delta(U(a(1)) - U(b(1))) + \Delta\tau_2\Delta(U(b(2)) - U(a(2))) \geq 0$$

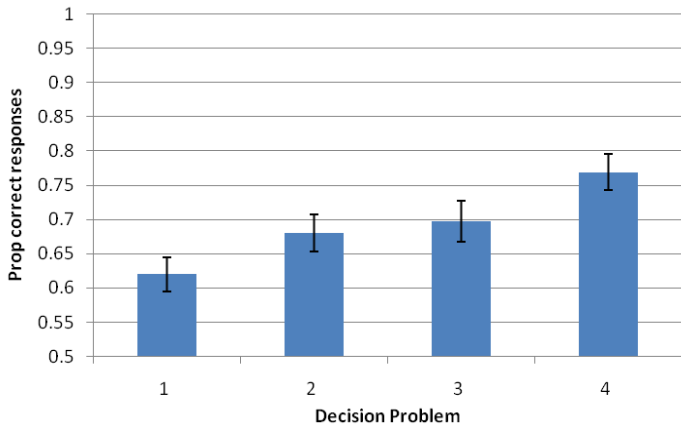
- τ_m probability of correct decision in state m
 - $U(a(m)) - U(b(m))$ benefit of correct decision in state m
- In this experiment

$$\tau_1^4 + \tau_2^4 \geq \tau_1^3 + \tau_2^3 \geq \tau_1^2 + \tau_2^2 \geq \tau_1 + \tau_2$$

Do People Optimally Adjust Attention?

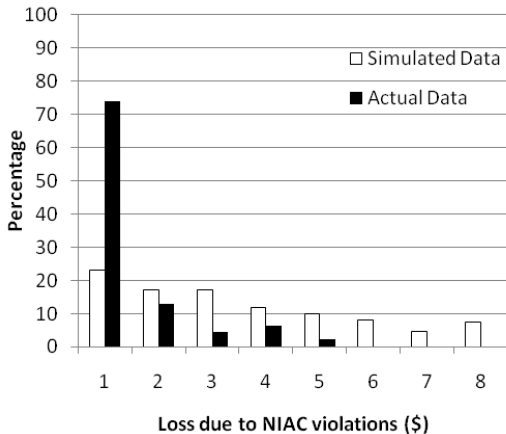
- Alternative model: Choose optimally conditional on fixed signal
 - e.g. Signal Detection theory
- In general, choices can vary with incentives
 - Changes optimal choice in posterior state
- But not in this case
 - Optimal to choose a if $\gamma_1 > 0.5$, regardless of prize
- Change in choice between decision problems rules out Signal Detection Theory
 - Also rational inattention with fixed entropy

Testing NIAC: Experiment 1



- 51% of subjects significantly increase proportion of correct choices
- 83% show no significant violation of NIAC

Testing NIAC: Experiment 1



- Individual level data
- Benchmarked against Random Choice

Experiment 2: Spillivers

Table 2: Experiment 2

DP	Payoffs					
	U(a(1))	U(a(2))	U(b(1))	U(b(2))	U(c(1))	U(c(2))
5	23	23	20	25	n/a	n/a
6	23	23	20	25	30	10
7	23	23	20	25	35	5
8	23	23	20	25	40	0

Table 3			
DP	$P(b 1)$	$P(b 2)$	$P(c 1) - P(c 2)$
5	17%	23%	n/a
6	15%	31%	18%
7	12%	33%	18%
8	13%	39%	29%

- Random utility implies

$$P_5(b|2) \geq P_j(b|2) \text{ for } j \in \{6, 7, 8\}$$

- NIAC implies

$$P_8(c|1) - P_8(c|2) \geq P_7(c|1) - P_7(c|2) \geq P_6(c|1) - P_6(c|2).$$

Experiment 3: Intensive Margin

Experiment 3								
Decision Problem	Payoffs							
	U_1^a	U_2^a	U_3^a	U_4^a	U_1^b	U_2^b	U_3^b	U_4^b
9	1	0	10	0	0	1	0	10
10	10	0	1	0	0	10	0	1
11	1	0	1	0	0	1	0	1
12	10	0	10	0	0	10	0	10

- 4 states of the world: 29, 31, 69, 71 red balls
- Change which states it is important to differentiate between

Experiment 3								
Decision Problem	Payoffs							
	U_1^a	U_2^a	U_3^a	U_4^a	U_1^b	U_2^b	U_3^b	U_4^b
9	1	0	10	0	0	1	0	10
10	10	0	1	0	0	10	0	1

- Comparing DP 9 and 10
 - DP9: important to differentiate between states 3 and 4
 - DP10: important to differentiate between states 1 and 2

$$\left(\tau_1^{10} + \tau_2^{10}\right) + \left(\tau_3^9 + \tau_4^9\right) \geq \left(\tau_3^{10} + \tau_4^{10}\right) + \left(\tau_1^9 + \tau_2^9\right),$$

- Average LHS: 73%, Average RHS: 65% (24 subjects)
- Overall 79% of subjects in line of NIAC

- We have developed simple non-parametric test for costly information acquisition
 - 'Revealed Preference' for information costs
 - Nests other models of information acquisition
- Introduced State Dependent Stochastic Choice data as an important tool for studying information acquisition
- Introduced an experimental design which allows collection of such data
 - Showed that active choice of attention is important
 - Optimal model of information acquisition passes simple tests
- Providing theoretical and experimental foundations for 'rational inattention'
 - Becomes increasingly important as amount of available information increases