Search and Satisficing

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Satisficing

- We introduced two types of imperfect perception
  - 'All or Nothing' (e.g. consideration sets)
  - 'Partial Understanding' (e.g. Information structures)
- Today we will focus on former case
  - 'optimal model of consideration set construction'

Satisficing as Optimal Stopping

- Satisficing model: Simon [1955]
- Very simple model:
  - Decision maker faced with a set of alternatives A
  - Searches through this set one by one
  - If they find alternative that is better than some threshold, stop search and choose that alternative
  - If all objects are searched, choose best alternative

Satisficing as Optimal Stopping

- Usually presented as a compelling description of a 'choice procedure'
- Can also be derived as optimal behavior as a simple sequential search model with search costs
- Primitives
  - A set A containing M items from a set X
  - A utility function u: $X \to \mathbb{R}$
  - A probability distribution f :decision maker's beliefs about the value of each option
  - A per object search cost k

The Stopping Problem

- At any point DM has two options

1. Stop searching, and choose the best alternative so far seen (search with recall)
2. Search another item and pay the cost k

Optimal Stopping

- Solve by backwards induction
- Choice when there is 1 more object to search and current best alternative has utility $\bar{u}$

1. Stop searching: $\bar{u} - (M-1)k$
2. Search the final item:

$$\int_{-\infty}^{\bar{u}} \bar{u} f(u) du + \int_{\bar{u}}^{\infty} u f(u) du - Mk$$

Optimal Stopping

- Stop searching if

$$\bar{u} - (M-1)k \leq \int_{-\infty}^{\bar{u}} \bar{u} f(u) du + \int_{\bar{u}}^{\infty} u f(u) du - Mk$$

- Implying

$$k \leq \int_{\bar{u}}^{\infty} (u - \bar{u}) f(u) du$$

- Cutoff strategy: search continues if $\bar{u} > u^*$ solving

$$k = \int_{u^*}^{\infty} (u - u^*) f(u) du \quad (1)$$

Optimal Stopping

- Now consider behavior when there are 2 items remaining
- $\bar{u} < u^*$ Search will continue
  - Search optimal if one object remaining
  - Can always operate continuation strategy of stopping after searching only one more option
- $\bar{u} > u^*$ search will stop
  - Not optimal to search one more item only
  - Search will stop next period, as $\bar{u} > u^*$

Optimal Stopping

- Optimal stopping strategy is satisficing:
- Find u^* that solves

$$k = \int_{u^*}^{\infty} (u - u^*) f(u) du$$

- Continue searching until find an object with $u > u^*$, then stop
- Predictions about how reservation level changes with environment
  - u^* decreasing in k
  - increasing in variance of f (for well behaved distributions)
  - Unaffected by the size of the choice set
- Comes from optimization, not reduced form satisficing model

Testing Satisficing: The Problem

- Satisficing models difficult to test using choice data alone
  - If search order is fixed, prediction is just WARP
  - If it can vary, any behavior can be explained
- Two ways out:
  - Make more assumptions
  - Enrich data set
- Consider the latter:- **Choice Process Data**
  - Campbell (1978)
  - Caplin and Dean (2010)
  - Caplin, Dean and Martin (2011)

Choice Process Data

- In order to test predictions of our model we introduce 'choice process' data
- Records how choice changes with contemplation time
  - $C(A)$: Standard choice data - choice from set A
  - $C_A(t)$: Choice process data - choice made from set A after contemplation time t

Notation

- X : Finite grand choice set
- $\mathcal{X}$: Non-empty subsets of X
- $Z \in \{Z_t\}_t^\infty$: Sequences of elements of $\mathcal{X}$
- $\mathcal{Z}$ set of sequences Z
- $\mathcal{Z}_A \subset \mathcal{Z}$: set of sequences s.t. $Z_t \subset A \in \mathcal{X}$

A Definition of Choice Process

Definition

A Choice Process Data Set (X, C) comprises of:

- finite set X
- choice function $C : \mathcal{X} \to \mathcal{Z}$

such that $C(A) \in \mathcal{Z}_A \; \forall \; A \in \mathcal{X}$

- $C_A(t)$: choice made from set A after contemplation time t

Characterizing the Satisficing Model

- Two main assumptions

1. Search is **alternative-based**
   - DM searches through items in choice set sequentially
   - Completely understands each item before moving on to the next
2. Stopping is due to a **fixed reservation rule**
   - Subjects have a fixed reservation utility level
   - Stop searching if and only if find an item with utility above that level

Alternative-Based Search (ABS)

- DM has a fixed utility function
- Searches sequentially through the available options,
- Always chooses the best alternative of those searched
- May not search the entire choice set

Alternative-Based Search

- DM is equipped with a utility function

$$u : X \to \mathbb{R}$$

- and a search correspondence

$$S : \mathcal{X} \to \mathcal{Z}$$

with $S_A(t) \subseteq S_A(t+s)$

- Such that the DM always chooses best option of those searched

$$C_A(t) = \arg \max_{x \in S_A(t)} u(x)$$

Revealed Preference and ABS

- Finally choosing x over y does *not* imply (strict) revealed preference
  - DM may not know that y was available
- Replacing y with x *does* imply (strict) revealed preference
  - DM must know that y is available, as previously chose it
  - Now chooses x, so must prefer x over y
- Choosing x and y at the same time reveals indifference
- Use $\succ^{ABS}$ to indicate ABS strict revealed preference
- Use $\sim^{ABS}$ to indicate revealed indifference

Characterizing ABS

- Choice process data will have an ABS representation if and only if $\succ^{ABS}$ and $\sim^{ABS}$ can be represented by a utility function u

$$x \succ^{ABS} y \Rightarrow u(x) > u(y)$$
$$x \sim^{ABS} y \Rightarrow u(x) = u(y)$$

- Necessary and sufficient conditions for utility representation **Only Weak Cycles**
  - Let $\succeq^{ABS} = \succ^{ABS} \cup \sim^{ABS}$
  - $xT(\succeq^{ABS})y$ imples not $y \succ^{ABS} x$

Theorem 1

Theorem

Choice process data admits an ABS representation if and only if $\succ^{ABS}$ and $\sim^{ABS}$ satisfy Only Weak Cycles

Proof.

(Sketch of Sufficiency)

1. *Generate U that represents $\succeq^{ABS}$*
2. *Set $S_A(t) = \cup_{s=1}^{t} C_A(s)$*

□

Satisficing

- Choice process data admits an **satisficing representation** if we can find
  - An ABS representation (u, S)
  - A reservation level ρ
- Such that search stops if and only if an above reservation object is found
  - If the highest utility object in $S_A(t)$ is above ρ, search stops
  - If it is below ρ, then search continues
- Implies complete search of sets comprising only of below-reservation objects

Revealed Preference and Satisficing

- Final choice can now contain revealed preference information
  - If final choice is **below-reservation** utility
- How do we know if an object is below reservation?
- If they are **non-terminal**: Search continues after that object has been chosen

Directly and Indirectly Non-Terminal Sets

- Directly Non-Terminal: $x \in X^N$ if
  - $x \in C_A(t)$
  - $C_A(t) \neq C_A(t+s)$
- Indirectly Non Terminal: $x \in X^I$ if
  - for some $y \in X^N$
  - $x, y \in A$ and $y \in \lim_{t \to \infty} C_A(t)$
- Let $X^{IN} = X^I \cup X^N$

Add New Revealed Preference Information

- If
  - one of x,$y \in A$ is in X^{IN}
  - x is finally chosen from some set A when y is not,
- then, $x \succ^S y$
  - If x is is in X^{IN}, then A must have been fully searched, and so x must be preferred to y
  - If y is in X^{IN}, then either x is below reservation level, in which case the set is fully searched, or x is above reservation utility
- Let $\succ = \succ^S \cup \succ^{ABS}$

Theorem 2

Theorem

Choice process data admits an satisficing representation if and only if $\succ$ and $\sim^{ABS}$ satisfy Only Weak Cycles

Experimental Design

- Experimental design has two aims
  - Identify choice 'mistakes'
  - Test satisficing model as an explanation for these mistakes
- Two design challenges
  - Find a set of choice objects for which 'choice quality' is obvious but subjects do not always choose best option
  - Find a way of eliciting 'choice process data'
- We first test for 'mistakes' in a standard choice task...
- ... then add choice process data in same environment
- Make life easier for ourselves by making preferences directly observable

Choice Objects

- Subjects choose between 'sums'

four plus eight minus four

- Value of option is the value of the sum
- 'Full information' ranking obvious, but uncovering value takes effort
- 6 treatments
  - 2 x complexity (3 and 7 operations)
  - 3 x choice set size (10, 20 and 40 options)
- No time limit

Size 10, Complexity 3

Size 20, Complexity 7

Results

Failure rates (%) (22 subjects, 657 choices)

Failure rate		
	Complexity	
Set size	3	7
10	7%	24%
20	22%	56%
40	29%	65%

Results

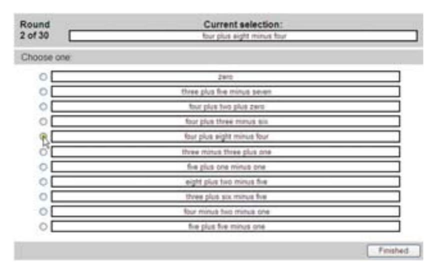
Average Loss ($)

Average Loss ($)		
	Complexity	
Set size	3	7
10	0.41	1.69
20	1.10	4.00
40	2.30	7.12

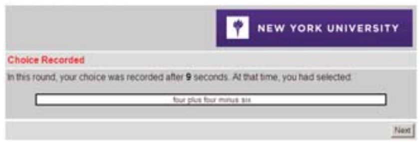
Eliciting Choice Process Data

1. Allow subjects to select any alternative at any time
   - Can change selection as often as they like
2. Choice will be recorded at a random time between 0 and 120 seconds unknown to subject
   - Incentivizes subjects to always keep selected current best alternative
   - Treat the sequence of selections as choice process data
3. Round can end in two ways
   - After 120 seconds has elapsed
   - When subject presses the 'finish' button
   - We discard any rounds in which subjects do not press 'finish'

Stage 1: Selection

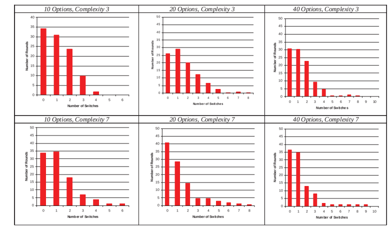


Stage 2: Choice Recorded



Do We Get Richer Data from Choice Process Methodology?

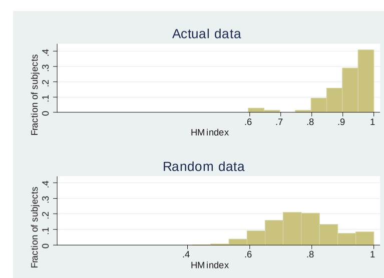
978 Rounds, 76 Subjects



Testing ABS

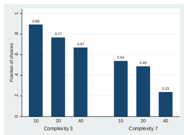
- Choice process data has ABS representation if $\succ^{ABS}$is *consistent*
- Assume that more money is preferred to less
- Implies subjects must always switch to higher-valued objects **(Condition 1)**
- Calculate Houtman-Maks index for Condition 1
  - Largest subset of choice data that is consistent with condition

Houtman-Maks Measure for ABS

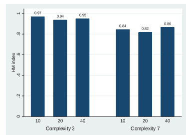


Traditional vs ABS Revealed Preference

Traditional



ABS

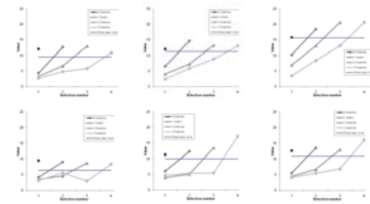


Satisficing Behavior

	10	20	40

3

7



Estimating Reservation Levels

- Choice process data allows observation of subjects
  - Stopping search
  - Continuing to search
- Allows us to estimate reservation levels
- Assume that reservation level is calculated with some noise at each switch
- Can estimate reservation levels for each treatment using maximum likelihood

Estimated Reservation Levels

	Complexity			
Set size	3		7	
10	9.54	(0.20)	6.36	(0.13)
20	11.18	(0.12)	9.95	(0.10)
40	15.54	(0.11)	10.84	(0.10)

Estimating Reservation Levels

- Increase with 'Cost of Search'
  - In line with model predictions
- Increase with size of choice set
  - In violation of model predictions

HM Indices for Estimated Reservation Levels

	Complexity	
Set size	3	7
10	0.90	0.81
20	0.87	0.78
40	0.82	0.78

Complete Search with Calculation Errors

- An alternative explanation for suboptimal choice
- Subjects look at all objects, but make calculation errors
- Estimate logistic random error model of choices
  - Scale factor allowed to vary between treatment
- Select scale factor to maximize likelihood of observed choices

Calculation Errors

- Extremely large errors needed to explain mistakes

Estimated standard deviations

	Complexity	
Set size	3	7
10	1.90	3.34
20	2.48	4.75
40	3.57	6.50

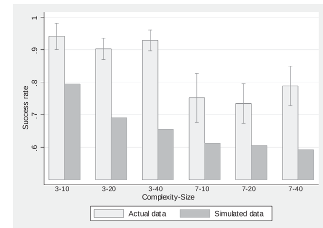
Calculation Errors

- Still underpredict magnitude of losses

Set size	Failure Rate	Complexity 3	Complexity 7
10	Actual choices	11.38	46.53
	Simulated choices	*8.35*	*32.47*
20	Actual choices	26.03	58.72
	Simulated choices	*20.13*	*37.81*
40	Actual choices	37.95	80.86
	Simulated choices	*25.26*	*44.39*

Calculation Errors

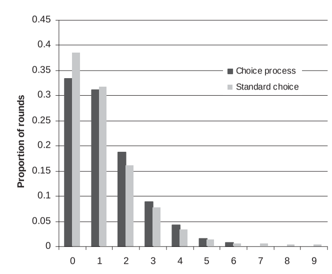
- But overpredict violations of ABS



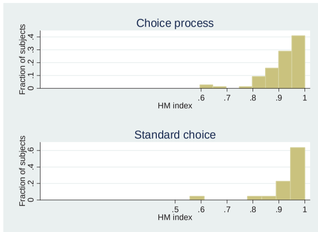
Does Choice Process Elicitation Change Behavior?

- In 'standard choice' experiment subjects could make intermediate selections
- Were not incentivized to do so, but did so anyway
- Can use this to explore the effect of choice process elicitation

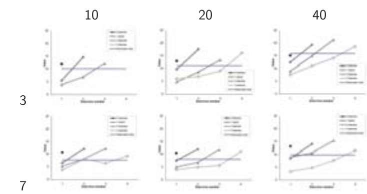
Question 1: Does Choice Process Elicitation Change Behavior?



Does Standard Choice Experiment Also Have Sequential Search?



Satisficing Behavior in Standard Choice Environment



How Does Choice Process Elicitation Change Incentives?

- Frame as an optimal stopping problem (within ABS framework)
- Assume
  - Fixed cost of search
  - Value of objects drawn from a fixed distribution
- Can formulate optimal strategy

Differences in Optimal Strategy

- **Fixed** reservation optimal in standard choice but **declining** reservation optimal in choice process
  - No good evidence for declining reservation level in either case
- Choice process environment should also always have lower reservation levels that standard choice
  - Weak evidence for this

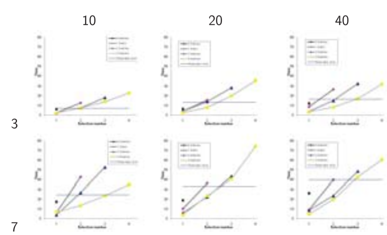
Estimated Reservation Levels

Set size		Complexity 3	Complexity 7
10	Choice process	10.17	6.34
	Standard choice	*10.05*	*8.41*
20	Choice process	11.22	8.92
	Standard choice	*11.73*	*8.39*
40	Choice process	15.15	10.07
	Standard choice	*16.38*	*10.39*

Alternative Models

- Reservation stopping time
- Complete search with calculation errors

Reservation Stopping Time?



Complete Search with Calculation Errors

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Failure Rate

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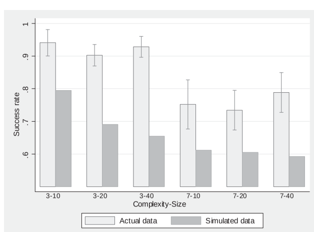

Estimating Reservation Levels

- Incomplete information search provides a good explanation for suboptimal choice in this environment
- Subjects behave in line with satisficing model
  - Search sequentially through choice set
  - Stop searching when finding object above reservation utility
- Environmental factors change behavior, but within satisficing framework

Related Literature

- Previous studies have used eye tracking/mouselab to examine process of information search
  - Payne, Bettman and Johnson [1993]
  - Gabaix, Laibson, Moloche and Weinberg [2006]
  - Reutskaja, Pulst-Korenberg, Nagel, Camerer and Rangel.[2008]
- Modelled choice data with consideration sets and ordered search
  - Rubinstein and Salant [2006]
  - Manzini and Mariotti [2007]
  - Masatlioglu and Nakajima [2008]

Results

Experiment 1

Table 1: Magnitude of Mistakes, Experiment 1

Set Size		Complexity 3	Complexity 7	Total
10	**Failure Rate (%)**	**6.78**	**23.61**	**16.03**
	Average Loss ($)	0.41	1.69	1.11
	Average Loss (%)	3.44	13.66	9.05
	Observations	*59*	*72*	*131*
20	**Failure Rate (%)**	**21.97**	**56.06**	**39.02**
	Average Loss ($)	1.10	4.00	2.55
	Average Loss (%)	7.07	24.70	15.89
	Observations	*132*	*132*	*264*
40	**Failure Rate (%)**	**28.79**	**65.38**	**46.95**
	Average Loss ($)	2.30	7.12	4.69
	Average Loss (%)	10.49	33.25	21.79
	Observations	*132*	*130*	*262*
Total	**Failure Rate (%)**	**21.98**	**52.69**	**37.60**
	Average Loss ($)	1.46	4.72	3.12
	Average Loss (%)	7.81	25.65	16.88
	Observations	*323*	*334*	*657*

Results

Experiment 2

Absolute Loss

Set Size		Complexity 3	Complexity 7	Total
10	**Choice Process**	**0.42**	**3.69**	**1.90**
	Normal Choice	*0.41*	*1.69*	*1.11*
20	**Choice Process**	**1.63**	**4.51**	**2.88**
	Normal Choice	*1.10*	*4.00*	*2.55*
40	**Choice Process**	**2.26**	**8.30**	**5.00**
	Normal Choice	*2.30*	*7.12*	*4.69*
Total	**Choice Process**	**1.58**	**5.73**	**3.43**
	Normal Choice	*1.46*	*4.72*	*3.12*

Number of Observations - Choice Process

Set Size		Complexity 3	Complexity 7	Total
10		123	101	224
20		225	172	397
40		195	162	357
Total		543	435	978