MA Game Theory

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Homework 1

Due Weds 1st February

NOTE: Please answer question 3 and 4 on a separate sheet to questions 1 and 2

Question 1 In class we showed that the set of strategies which are never a best response are equal to the set of strategies which are strictly dominated if we allow for correlated beliefs. To show that correlation is important, consider the following 3 player game. Player 1 chooses the row, player 2 chooses the column and player 3 chooses the matrix. For convenience I have shown only the payoff of player 3

Matrix 1 Player 2

Player 1

	L	R
U	10	0
D	0	0

Matrix 2 Player 2

Player 1

	L	R
U	x	0
D	0	x

Matrix 3 Player 2

Player 1

	L	R
U	0	0
D	0	10

Matrix 4 Player 2

		L	R
Player 1	U	4	4
	D	4	4

1. Imagine that the beliefs of player 3 are restricted to being uncorrelated - i.e. player 1 plays U and D with some probability and player 2 plays L and R with some probability, and the probability of any strategy profile in S_{-3} is just the product of these probabilites. Find a value for x such that strategy 2 (i.e. choosing matrix 2) is never a best response, and yet it is not strictly dominated. (Note that, for this question, and all questions, a good answer will demonstrate why what you claim is true, not just - for example - provide a number for x)
2. Show that, for your choice of x above, if we allow for correlated beliefs, then strategy 2 is a best response to some beliefs. What are those beliefs?

Question 2 Some questions about beauty contests

1. Consider the beauty contest game described in class. Replace the payoff function with

$$\begin{aligned} u(x_i, x_{-i}) &= 100 \text{ if } x_i \text{ is closest to } \frac{2}{3}\bar{x} \\ &= 0 \text{ otherwise} \end{aligned}$$

   Can this game be solved by Iterated Deletion of Strictly Dominated Strategies? What about the Iterated Deletion of Weakly Dominated Strategies?

2. Consider the following version of the beauty contest game. There are $n > 2$ players. Each player i submits a real number $x_i \in [0, 100]$. Let $\bar{x}$ be the mean of the numbers submitted. Suppose among these n players, there are m players whose payoffs are given by $(x_i - \bar{x})^2$ (i.e. they want to be as far away from the mean), where $0 < m < \frac{n}{2}$. The remaining $n - m$ players have the payoffs given by $200 - \left(x_i - \frac{2\bar{x}}{3}\right)^2$. Assume that it is common knowledge in this game that every player is maximizing his/her own payoff.
   (a) Prove that $x_i \in (0, 100)$ is a strictly dominated strategy for each of the m players.
   (b) Compute the set of rationalizable strategies that survive iterated elimination of strictly dominated strategies if $n = 4$ and $m = 1$. You need to figure out **both the** the rationalizable strategies of the first m players, and the rationalizable strategies

for the remaining $n - m$ players. Note that, for this parameterization, you should be able to come up with a unique solution.

Question 3 Consider the following two-player game:

	L	M	N	R
A	$4, 2$	$0, 0$	$5, 0$	$0, 0$
B	$1, 4$	$1, 4$	$0, 5$	$-1, 0$
C	$0, 0$	$2, 4$	$1, 2$	$0, 0$
D	$0, 0$	$0, 0$	$0, -1$	$0, 0$

1. Compute the set of strategies for each player that survive the iterated deletion of strictly dominated strategies.
2. Please explain how you eliminate a strategy and explain the minimal assumptions on rationality and (higher-order) knowledge of rationality for each elimination.
3. Show that this set of strategies for each player is indeed rationalizable (here I would like you to do more than just use the theory that the set of rationalizable strategies is the same as those thast survive IDSDS. I want you to construct the subsets $Z_i \subset S_i$ for each player and show that each action in each Z_i can be supported using beliefs on Z_{-i})

Question 4 Three game theorists work on a team project. Assume they work independently, and it is not possible for them to monitor each other. No one likes working, and the cost of efforts is measured in dollars. Each player can work up to 5 hours, and the cost per working hour is \$6. The quality and hence the profit of the project is determined by the joint efforts (total number of working hours) of the three players. For each additional hour that the team invests in the project, up to 10 hours, the total profit will increase by \$15. After 10 hours, efforts will not increase profits. Assume players divide the profit equally among them. Hence, each player's payoff is his share of the profit minus his cost. There is common knowledge of rationality.

1. Write up this scenario formally as a game
2. Compute the set of rationalizable effort levels for each player.