MA Game Theory

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Homework 1

Due Mon 13th February

NOTE: Please answer question 3 and 4 on a separate sheet to questions 1 and 2

Question 1 Consider the following two-player game:

	L	M	N
A	$4,2$	$0,0$	$5,0$
B	$1,4$	$1,4$	$0,5$
C	$0,0$	$2,4$	$1,2$

Find the set of all Nash equilibria (including all mixed strategy Nash equilibria) of this game.

Question 2 A crime is observed by a group of $n > 1$ people. Each person would like the police to be informed but prefers that someone else make the phone call. Specifically, suppose that each person attaches the value v to the police being informed and bears the cost c if she makes the phone call, where $v > c > 0$.

1. Set it up as a game: for each player, define the strategy space and the payoff function as a function of all players' strategies.
2. Is there a symmetric pure strategy Nash Equilibrium in which all players use the same pure strategy?
3. Find a symmetric mixed strategy equilibrium in which each person calls with the same positive probability less than one. This can be done in a few steps:
   (a) Let p be the probability that each person calls in a mixed strategy equilibrium. From player i's perspective, what's the probability that no one else calls? What's the probability that at least one other person calls?

(b) In the symmetric mixed strategy equilibrium, it must be the case that player i is indifferent between calling and keeping silent. Write down her indifference condition by making use of the two probabilities derived in part (a.1).

(c) Based on the indifferent condition derived in part (a.2), express p as a function of c and v and n.

(d) How does p change as the size of the group increases? What about the probability that at least one person calls? Explain intuitively why the probability that crime is reported increases/decreases with group size

Question 3 Here are the two games we used in class to demonstrate evolutionary stability

	X	Y
X	$3,3$	$3,0$
Y	$0,3$	$10,10$

	Hawk	Dove
Hawk	$-1,-1$	$2,0$
Dove	$0,2$	$1,1$

1. Show that the mixed strategy of the first game is evolutionarily unstable, while that of the latter game is evolutionarily stable (using the definition in class)

2. Imagine that these games are played repeatedly by populations of agents. In period t, the fraction of agents playing strategy s is given by

$$\sigma^{t+1}(s) = \sigma^t(s) + \rho(u(s, \sigma^t(s)) - u_i(s', \sigma^t(s)))$$

for some small ρ (e.g. less that $\frac{1}{2}$) , $u(s, \sigma^t(s))$ is the utility of playing strategy s (for example X) given the mixed strategy played in the last period, and $u(s', \sigma^t(s))$ is the utility of playing the other strategy (e.g. Y) (to make things easier, we can ignore problems with the boundary - i.e. when this would push probabilities above 1 or below 0)

Show that under these rules, the mixed strategy Nash equilibrium in game 1 is unstable, while in game 2 it is stable - in other words in game 1, if there is a small pertubation away from the mixed strategy then the distance between played strategies will increase over time, while in game 2 it will decrease

Question 4 (The MinMax theorem). A two player game is called zero sum if, for any strategy profile s, $u_1(s) = -u_2(s)$. Define w_i as player i's maxmin value - i.e. the maximal expected value she can guarantee that she can achieve. Define v_i as the minmax value - i.e. the minimal expected value that player j can enforce on player i. So

$$\begin{aligned} w_i &= \max_{\sigma_i} \min_{\sigma_j} u_i(\sigma_i, \sigma_j) \\ v_i &= \min_{\sigma_j} \max_{\sigma_i} u_i(\sigma_i, \sigma_j) \end{aligned}$$

1. Prove that $v_i \geq w_i$
2. Prove that, in any Nash Equilibrium σ^*, $u_i(\sigma^*) = w_i = v_i$