MA Game Theory

Mark Dean

Homework 6

Due Weds 12th April

Question 1 Consider the buyer-seller screening model with 3 types. The agents' total utility has the usual quasi-linear form$u(q, \theta) - t$, where now we assume that $u(q, \theta) = \theta q^{\frac{1}{2}}$. Costs of the monopolist are linear with $c = 1$ (so profit is $t - q$). The three types are respectively $\theta = 1$, $\theta = 2$, and $\theta = 3$. Suppose that the proportion of these three types in the population is $(\beta 1, \beta 2, \beta 3)$.

1. Describe the first-best allocation in this economy.
2. Using the single-crossing condition, prove that a necessary condition for incentive compatibility is that q_2 be larger or equal than q_1.
3. Suppose that $(\beta 1, \beta 2, \beta 3) = (0.7, 0.2, 0.1)$. Describe the equilibrium under asymmetric information.
4. Suppose that $\beta 3 = 0$. Under which condition is it optimal for the seller to exclude type $\theta = 1$ from the market?
5. Under which condition is it optimal for the seller to offer the same contract to types $\theta = 1$ and $\theta = 2$ (bunching)?

Question 2 Consider the following continuous-type screening model. The agents' total utility is $\theta q^{\frac{1}{2}} - t$, and the principal has a uniform prior $\theta \sim U[1, 2]$. The the production cost of the principal is just q.

1. Describe the first-best allocation in this economy.

2. Prove that any direct revealing mechanism $(q(\theta), t(\theta))$ gives agent θ an informational rent

$$v(\theta) = \int_1^\theta q(z)^{\frac{1}{2}} dz + v(1)$$

3. What is the principal's profit on agent θ with such a mechanism, expressed in terms of $(t(\theta), q(\theta))$?

4. Prove that the profit-maximizing mechanism has $q(\theta)$ given by

$$\max_q \left(q^{\frac{1}{2}} \theta - q - (2 - \theta) q^{\frac{1}{2}} \right)$$

5. Derive the second-best menu of contracts, and compare it with the first best solution. Plot noth in on a graph with q and θ on the axis. Measure also the informational surplus of buyers and the profit of the seller. Show that bunching is not an equilibrium.