MA Game Theory

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Homework 8

Due Monday 1st May

Question 1 Consider a competitive credit market with risk-neutral banks. Suppose that entrepreneurs are also risk-neutral. They contemplate the possibility to invest in a risky project that generates a gross payoff of R at with probability π_θ (and 0 otherwise), against a cost of 1. There are two types of entrepreneurs, $\theta = L$ or M, with $\pi_L < \pi_H$. The proportion of type H in the economy is β. Entrepreneurs must finance the initial cost 1 of their project by a standard debt contract supplied by banks. A debt contract requires the borrower to repay r (which can be interpreted as the principal plus interest) if the project is successful, but the bank can recover a collateral from the entrepreneur if the project fails (implying the entrepreneur's default). The entrepreneur can offer to provide an amount of collateral $c \in \mathbb{R}_+$ to the bank. Collateral has a value c for the entrepreneur, but has no value at all for banks. Having observed how much collateral the entrepreneur offers, banks set the interest rate they will lend at. Assume that there are many banks, so that banks receive zero profit. We assume that $\pi_L R > 1$

1. Characterize the competitive equilibrium in the absence of asymmetric information and no collateral ($c = 0$).
2. Suppose now that banks do not observe the entrepreneur' type but that entrepreneurs can offer collateral. Characterize the least-cost separating competitive equilibrium in this signalling game. Explain why collateral can serve as a credible signal of the entrepreneurs' type.
3. Describe a pooling equilibrium in this context. Can you eliminate it by using Cho-Kreps' intuitive criterion?

Question 2 Consider the Crawford-Sobel model we described in class. Assume now that the state is uniformly distributed between 0 and 10. Assume $a = \frac{1}{20}$. Find the partition equilibrium for $p = 4$. What is the maximum number of partitions that could be supported with these parameters?

Question 3 If a worker exerts standard effort, he produces an output x that is uniformly distributed on $[0, 1]$. If he does exert no effort at all, the output x has a density function $2(1 - x)$ on $[0, 1]$. The worker's cost of standard effort is denoted c, so their utility if they receive a wage w is $u(w) - c$ if they exert effort and $u(w)$ otherwise, with u concave. The employer is risk-neutral. Because she does not observe the worker's effort, the worker's wage w can only depend upon the observed output x. If output is x and they pay wage $w(x)$ their profit is $x - w(x)$

1. Assuming that the worker is risk-averse, show that the profit maximizing wage contract $w(x)$ is weakly increasing in x.
2. From now on, suppose that the worker's utility function is $u(w) = 2\sqrt{x}$, and that his reservation utility is $U = 1$. Suppose first that the employer considers that the cost of effort is too large to induce the worker to exert it. Describe the optimal wage contract in that case. What is the employer's expected profit in that case?
3. Assume that $c \leq 0.5$. Suppose alternatively that the employer wants to induce the worker to exert effort. Show that the optimal wage contract $w(.)$ is quadratic in x in that case. Determine the equilibrium wage contract as a function of c. Compute the expected profit of the employer in that case.