G5212: Game Theory

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Adverse Selection

- We have now completed our basic analysis of the adverse selection model
- This model has been applied and extended in literally thousands of ways
- e.g. in the Salanie book
- Applications
  - Regulation: firm knows more about costs than the regulator
  - Taxation: people know more about their own productivity than the government
  - Insurance: People know more about their risks than the insurer
- Extensions
  - Competition amongst principals
  - Multidimensional characteristics
  - Two sided asymmetric information

Adverse Selection

- To keep the course of finite length, we will concentrate on
- **Application**: Insurance
- **Extension:** Competition amongst principals

Insurance

- Let's think back to a simplified version of insurance example we used to motivate this section.
- There are two types of agent
  - L(ow) and H(igh)
- Each starts with the same wealth level W
- Each has the probability of suffering an accident $p_L < p_H$
- Cost of the accident is d

Insurance

- An insurance contract consists of
  - A premium q which is paid regardless of whether there was an accident
  - A reimbursement R which gets paid in case of an accident
- So an agent holding an insurance contract (q, R) will get two wealth levels

$$\begin{aligned} W_A &= W - q - d + R \\ W_N &= W - q \end{aligned}$$

Risk Aversion

- Agents are risk averse, while firms are not
- Each agent has concave utility, so, for the low type

$$V(q, R, L) = p_L U(W_A(q, R)) + (1 - p_L)U(W_N(q, R))$$

- While for the high type

$$V(q, R, H) = p_H U(W_A(q, R)) + (1 - p_H)U(W_N(q, R))$$

- The profit of a firm selling to a consumer of type p is

$$\pi = q - pR$$

Outside Option

- Notice that a key difference to the standard model is that here the outside option depends on type
- For the low type

$$\bar{U}_L = p_L U(W - d) + (1 - p_L)U(W)$$

- For the high type

$$\bar{U}_H = p_H U(W - d) + (1 - p_H)U(W)$$

First Best Solution

- First best solution
- For type H

$$\max q - pR$$

- Subject to

$$p_H U(W_A(q,R)) + (1-p_H)U(W_N(q,R)) \geq \bar{U}_H$$

- Which we can analyze graphically in W_A W_N space

First Best Solution

- Iso-utility curves

$$p_H U(W_A(q,R)) + (1-p_H)U(W_N(q,R)) = U$$

- Totally differentiate this and you get

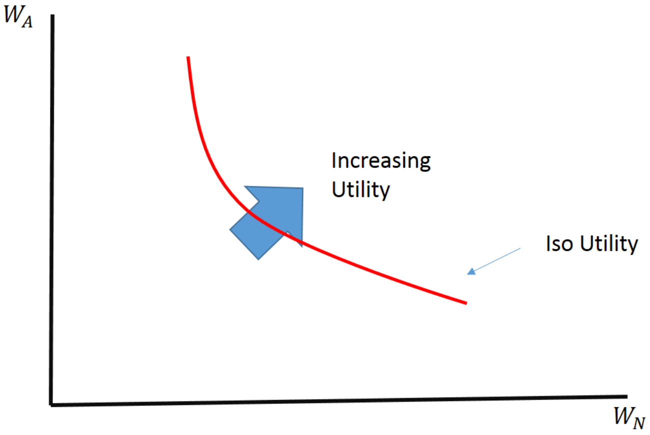
$$p_H U'(W_A)dW_A + (1-p_H)U'(W_N)dW_N = 0$$

- And so

$$\frac{dW_A}{dW_N} = -\frac{(1-p_H)U'(W_N)}{p_H U'(W_A)}$$

  - Downward sloping
  - Utility increases in NE direction
  - Convex (as we decrease W_A and increase W_N marginal utility of former rises and latter falls)

Iso-Utility Curve



First Best Solution

- Iso-profit lines

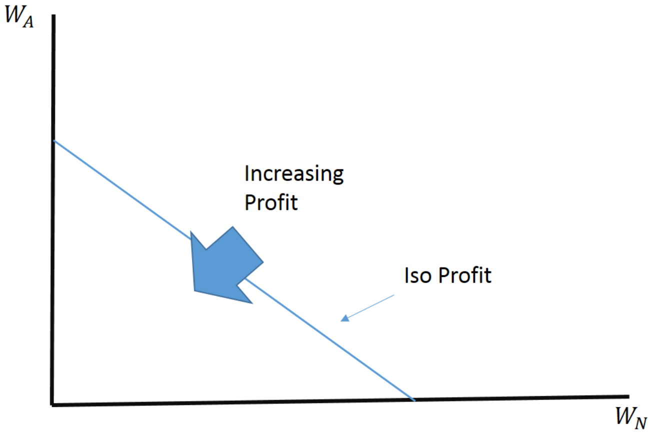
$$\begin{aligned} \pi &= q - pR \\ W_N &= W - q \\ W_A &= W - q - d + R \end{aligned}$$

- Implies

$$W_A = \frac{W}{p} - \frac{(1-p)}{p} W_N - d - \frac{\pi}{p}$$

- Which are
  - Linear
  - Downward Sloping
  - Profit increasing in a Southwest direction

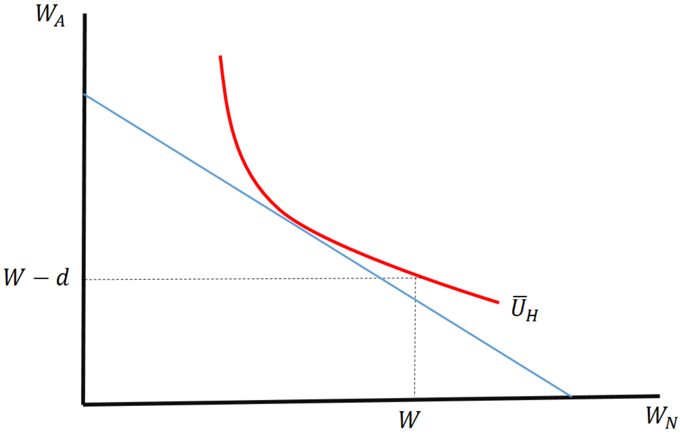
Iso-Profit Curves



First Best Solution

- First best solution
    - Fix the Iso Utility line based on U_H
    - Get on the highest possible iso profit line

First Best Solution



First Best Solution

- Where does this happen?
- Where the slopes of the two lines are equal
- Iso profit:

$$W_A = \frac{W}{p_H} - \frac{(1-p_H)}{p_H} W_N + d$$

- Iso utility

$$\frac{dW_A}{dW_N} = -\frac{(1-p_H)U'(W_N)}{p_H U'(W_A)}$$

- So

$$\frac{(1-p_H)U'(W_N)}{p_H U'(W_A)} = \frac{(1-p_H)}{p_H}$$

First Best Solution

- At the first best, each type gets full insurance

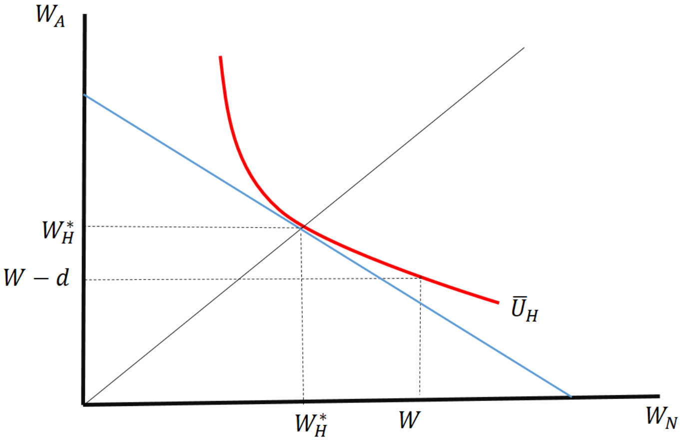
$$\frac{U'(W_N)}{U'(W_A)} = 1 \Rightarrow W_N = W_A$$

- Insurer extracts all the rent
- Note also that the slope of the iso indifference curve is

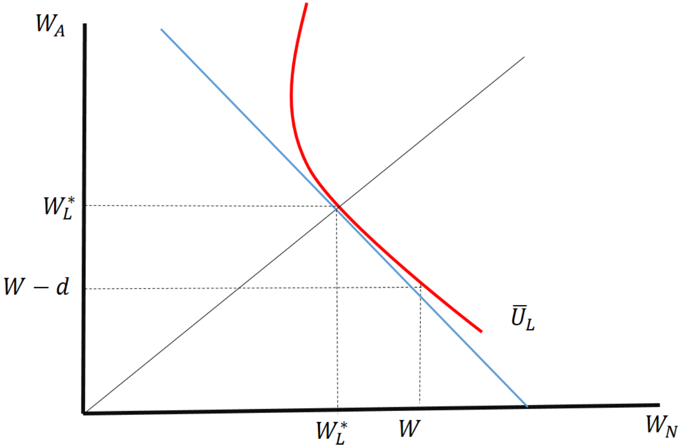
$$\frac{(1-p_H)U'(W_N)}{p_H U'(W_A)}$$

- Steeper for the low type

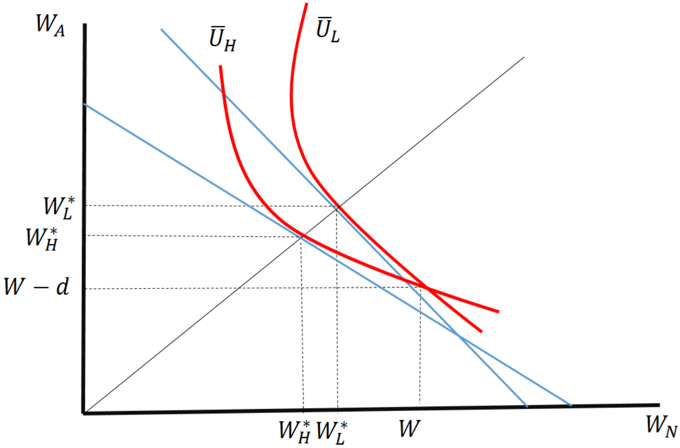
First Best Solution



First Best Solution



First Best Solution



Second Best Solution

- Clearly the first best solution won't work if the insurer can't differentiate types
  - Low type gets more in both states, so high type would want to deviate
- Have we any hope?
- Yes, because we have a 'Spence Mirrlees' condition

Second Best Solution

- Think of the marginal ratio of substitution between premium and reimbursement

$$\begin{aligned} & pU(W_A(q,R)) + (1-p)U(W_N(q,R)) \\ = \; & pU(W-q-d+R) + (1-p)U(W-q) \end{aligned}$$

- So

$$\begin{aligned} \frac{\partial V}{\partial q} &= -pU'(W_A) - (1-p)U'(W_N) \\ \frac{\partial V}{\partial R} &= pU'(W_A) \end{aligned}$$

- And

$$\frac{\frac{\partial V}{\partial q}}{\frac{\partial V}{\partial R}} \equiv -\frac{pU'(W_A) + (1-p)U'(W_N)}{pU'(W_A)}$$

Second Best Solution

- This is **higher** for **lower** p
- Agents with a higher p are prepared to pay more in terms of premium for a 1 unit increase in R
- Should be able to separate the two types
  - P_H gets more coverage at higher premium
- Notice that we have already confirmed that high types have shallower indifference curves than low types
  - Single crossing condition

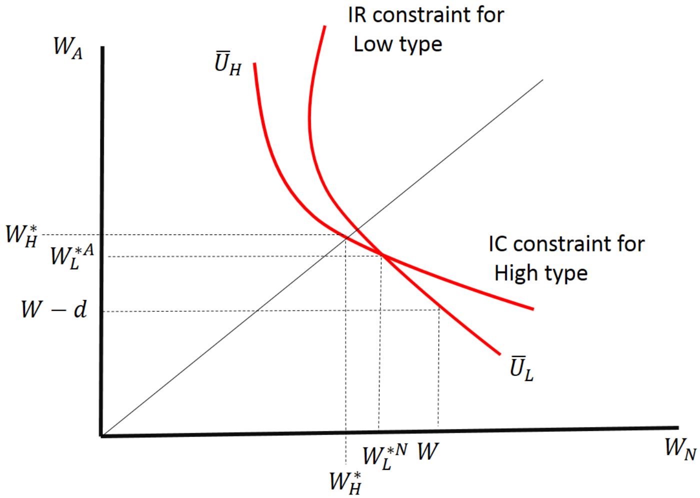
Second Best Solution

- Properties of the solution
- First, we know that one of the types must be on their IR constraint
- Which type?
- It must be the Low type
  - We need to pay the high type not to pretend to be the low type
  - If we put the low type on its IR constraint
  - And the High type obeys its IC constraint
  - High type will also satisfy the IR constraint

Second Best Solution

- Second, the high risk type will receive their first best allocation between income in the two states
- i.e full insurance
    - This is equivalent to the finding in the price discrimination model that the high type get's their first best quantity
- Third, the high type's IC constraint binds
    - We worry about the high type pretending to be the low type, not the other way round
- Solution will therefore look like....

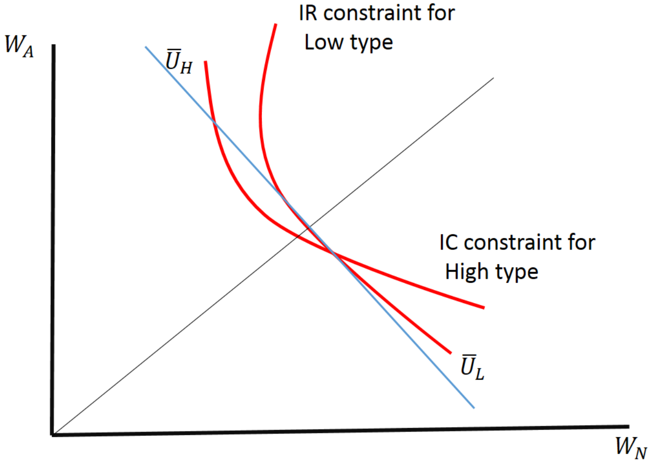
Second Best Solution



Second Best Solution

- As with the price discrimination model we still have one more unknown
- Three constraints
    - IR(L)
    - IC(H)
    - Full insurance for the high type
- Four unknowns
- Notice that, given the low type gets partial insurance, their IR constraint is flatter than the iso-profit line

Second Best Solution



Second Best Solution

- This means that, as we move the contract for the low type towards no insurance we will
  - Increase profits from the high types
  - Decrease profits from the low types
- How we want to make that trade off depends on the proportion of the two types

Competition Amongst Principals

- So far we have assumed that the principal acts as a monopolist
- Either the agent takes their offer, or they get their outside option
- This seems extreme
  - Given that principals are making positive profit, we would expect other principals to enter the market
- There are many ways to model such competition
- We will look at the simplest: Perfect competition

Profit!

- Let's think back to the simple price discrimination model with two agents
- How much profit did the principal make of each type?
  - Remember θ_1 was the low marginal utility guy
  - IC constraint binds

$$\begin{aligned}\Pi_1 &= t_1 - C(q_1) \\ &= \theta_1 q_1 - C(q_1) \\ &= \int_0^{q_1} (\theta_1 - C'(q))dq\end{aligned}$$

- Remember that θ_1 gets less than his first best allocation so $C'(q) < C'(q_1) < \theta_1$ for all $q \in [0, q_1]$ (as costs are convex)
- Positive profits

Profit!

- What about for type 2?
- Recalling that

$$\begin{aligned} t_2 &= \theta_1 q_1 + \theta_2(q_2 - q_1) \\ &= t_1 + \theta_2(q_2 - q_1) \end{aligned}$$

We can also write

$$\begin{aligned} \Pi_2 - \Pi_1 &= t_2 - C\left(q_2\right) - \left(t_1 - C\left(q_1\right)\right) \\ &= \theta_2(q_2 - q_1) - C\left(q_2\right) + C\left(q_1\right) \end{aligned}$$

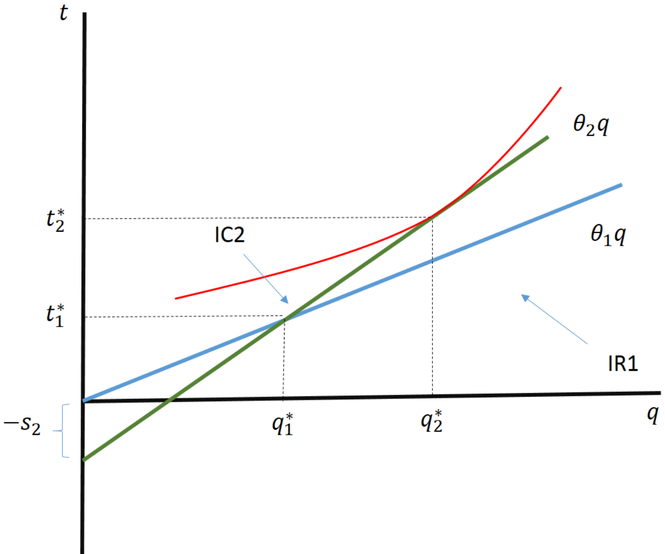
- Defining $f(q) = \theta_2 q - C(q)$, the above is

$$\begin{aligned} & f(q_2) - f(q_1) \\ = & \int_{q_1}^{q_2} f'(q)d(q) \\ = & \int_{q_1}^{q_2} (\theta_2 - C'(q))d(q) > 0 \end{aligned}$$

Competition

- So what would happen if we had competition between agents?
- i.e. allowed for free entry and exit of the market by principals?
- Remember that the monopolist will offer these contracts

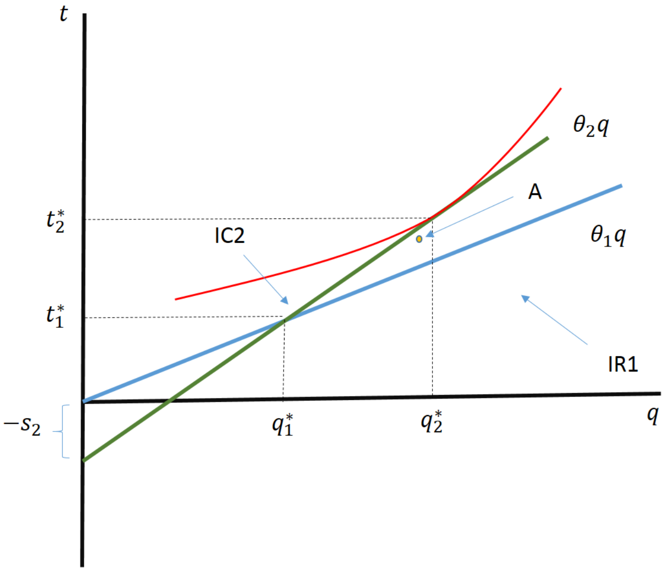
Competition



Competition

- But a cunning entrant could take away a bunch of the business from the monopolist
- And still make positive profit
- Consider contract A in the following picture

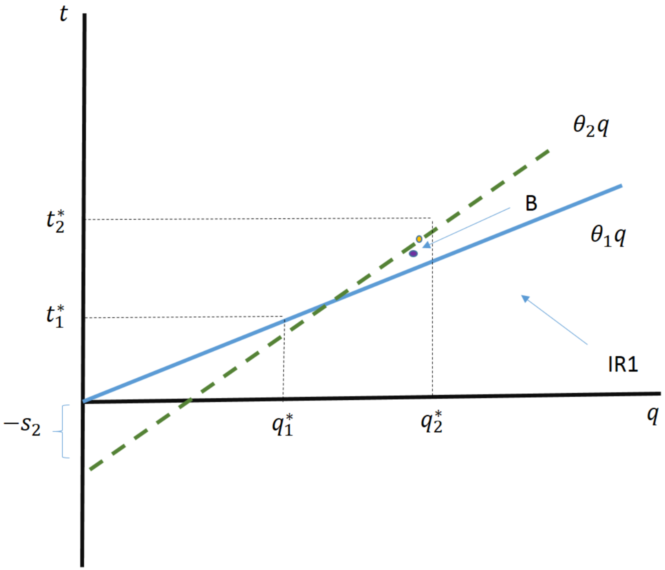
Competition



Competition

- Contract A is preferred by θ_2 to (q_2^*, t_2^*)
  - Can see this because it is to the South East of the IC constraint running through that contract
- It will also make less profit
  - However can still make positive profit
  - We know this because (q_2^*, t_2^*) makes strictly positive profit and we can move A as close as we like to that contract
- So all the θ_2 types will switch to contract A, the monopolist will lose profit and the entrant will make profit
- But if an entrant did this, how would the monopolist respond?

Competition



Competition

- The monopolist could respond with contract B
- This is preferred by θ_2 types to contract A
- And, assuming that A made positive profit, we can pick it so it gives positive profit
- The monopolist steals back all the θ_2 types....
- This is clearly getting us nowhere
- We need a concept of equilibrium to figure out what we think will happen

The Rothschild-Stiglitz Equilibrium

- We say that a set of contracts $\{t_i, q_i\}$ is an equilibrium if
  - Every contract provides non-negative profits
  - There are no contracts that can be added to the set which will make strictly positive profits
    - assuming that all other contracts stay where they are
- Can think of this as a Nash Equilibrium of a game played by identical principals

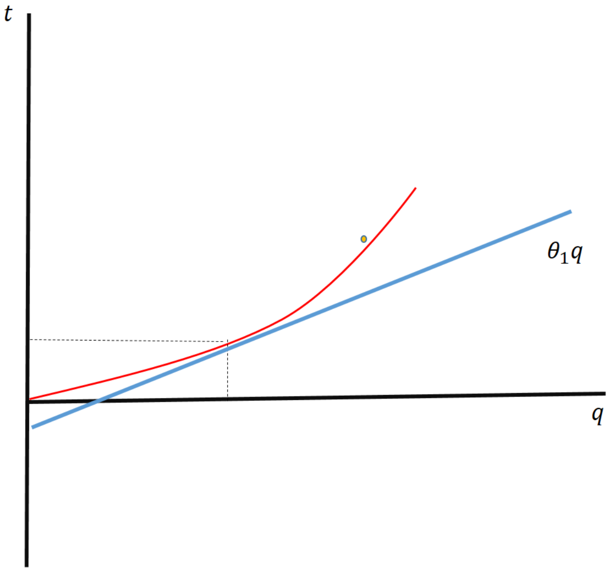
The Rothschild-Stiglitz Equilibrium

- Claim: If we have perfect competition, the set of contracts offered will be the same **regardless** of whether there is asymmetric information (!)
- First, let's consider the case of perfect information.
- Claim: For each type θ, the only equilibrium contract is one that maximizes

$$\begin{aligned} & \theta q - t \\ \text{subject to } t - C(q) & \geq 0 \end{aligned}$$

- i.e. maximizes the surplus of the agent subject to firms making non negative profits
  - Clearly, solution implies $t - C(q) = 0$

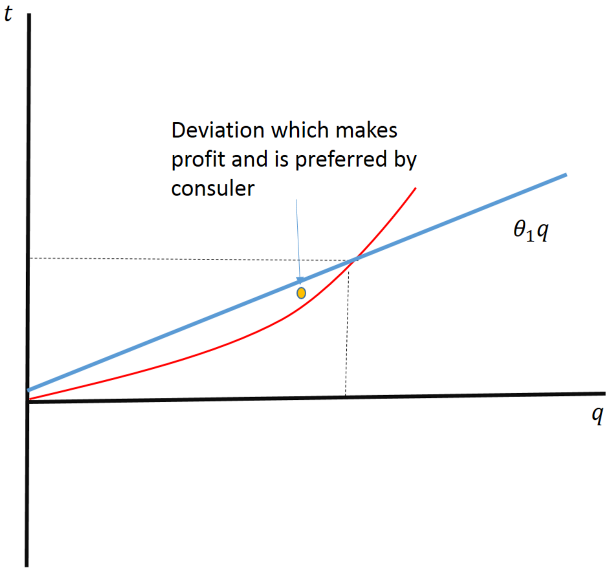
The Rothschild-Stiglitz Equilibrium



The Rothschild-Stiglitz Equilibrium

- Why?
  1. Profits must be zero
     - Otherwise an entrant could offer the same contract but with ε lower costs and steal all the business
  2. If the contract were inefficient, then someone could propose a more efficient contract, make positive profits and steal all the business

The Rothschild-Stiglitz Equilibrium



The Rothschild-Stiglitz Equilibrium

- So this implies that, if types are observable

$$
\begin{aligned}
\theta &= C'(q(\theta)) \\
t(\theta) &= C(\theta)
\end{aligned}
$$

- Each type gets their maximal surplus $S^*(\theta)$
- But it turns out the this is an RS equilibrium if types are **not** observable
  - These contracts clearly satisfy the IC constraints, as each agent is receiving the globally best contract for their type
  - By the same argument above there are no profitable deviations

The Rothschild-Stiglitz Equilibrium

- Furthermore, it is **the only** RS equilibrium
- Assume not
    - let $\{t(\theta), q(\theta)\}$ be the set of contracts in this new equilibrium
    - There must be some type θ such that, who is getting surplus $S(\theta) < S^*(\theta)$
        - Otherwise it is identical to the equilibrium on the previous slide
    - Propose a new contract for this type $\{t(\theta, \varepsilon), q(\theta, \varepsilon)\}$ which maximizes

$$\begin{aligned} & \theta q - t \\ \text{subject to } t - C(q) & \geq \varepsilon \end{aligned}$$

The Rothschild-Stiglitz Equilibrium

- For ε small enough, this contract will
  - Be strictly preferred by θ types
  - Make positive profit
- Kills the equilibrium
- Note that this result relies on some specific features of the setup
- For example, in the insurance market,
  - Perfect competition with observable types leads to complete insurance
  - This clearly can't be an equilibrium with unobservable types
- Problem is the common value nature of the problem
  - Types affect principal's payoff.