G5212: Game Theory

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Moral Hazard

- We will finish off our discussion of Moral Hazard with a couple of extensions and an application
  - The Continuous Case
  - Insurance
  - Limited Liability

The Continuous Case

- In the above analysis we assumed that there were a discrete number of actions
  - Meant that the number of IC and IR constraints was finite (but possibly large)
- What about if there is a continuum of actions?
  - Now there are an infinite number of such constraints
- We can use the trick from the screening model
  - Use the first order approach
  - Assume only ‘local’ constraints bind

The Continuous Case

- Say that a lives on some interval $[a_1, a_2]$
- There are still m possible outcomes
  - We will assume that $p : [a_1, a_2] \rightarrow \Delta^M$ is a differentiable function
- Assume wage schedule w
- The utility of choosing action a is given by

$$\sum_m p_j(a) u(w_j) - a$$

- So first order conditions give us

$$\sum_m p'_j(a) u(w_j) = 1$$

The Continuous Case

- In the case of two outcomes x_s and x_f, we therefore get two equations
    - Let $p(a)$ be the probability of success following action a
- IC constraint

$$p'(a)u(w_s) - p'(a)u(w_f) = 1$$

- IR constraint

$$p(a)u(w_s) + (1 - p(a))u(w_f) - a = \bar{u}$$

- Jiggling around gives

$$\begin{aligned} u(w_f) &= \bar{u} + a - \frac{p(a)}{p'(a)} \\ u(w_s) &= \bar{u} + a + \frac{1 - p(a)}{p'(a)} \end{aligned}$$

The Continuous Case

- Are the first order conditions enough?
- Typically no
- One could also try to make use of the second order conditions

$$\sum_{m} p_j''(a)u(w_j) \leq 0$$

- However Rodgerson [1985] showed that, in fact under the MLRC and CDFC the first order conditions are necessary and sufficient

Insurance

- We can apply the continuous action model to the case of insurance
    - Classic application of the Moral Hazard model
    - In fact, where the term ‘Moral Hazard’ came from
    - If you are well insured, then you are less incentivized to take costly actions that will protect you from a loss
        - Careful driving
        - Locking your door
        - Preventative health care
- Notice that this is a different insurance problem to the one we have studied previously
    - Before, the problem was that insurers may end up with the wrong **type** of insurees
    - Here the problem is that those that are insured may take the **wrong action**
    - This can be a problem even if types are perfectly observable

Insurance

- Consider a driver
  - Initial wealth W
  - If they have an accident they face cost d
  - Have an insurance contract with premium q
  - Receive reimbursement R in the case of an accident
- They an take an action $a \in [a_*, a^*]$
  - Cost is a
  - Probability of accident is $p(a)$, decreasing and convex
- Expected profit of the firm is

$$q - p(a)R$$

Insurance

- What does the driver do?
- They will choose a to maximize

$$p(a)u(W-d-q+R)+(1-p(a))u(W-q)-a$$

- This gives FOC

$$p'(a)\left(u(W-d-q+R)-u(W-q)\right)=1$$

- In the two outcome case
  - p decreasing implies MLR
  - p convex implies CDFC
  - $\Rightarrow$ FOC are necessary and sufficient

Insurance

- Assume that the profit maximizing effort level is above a_*
- Clearly full insurance will not work
- Coinsurance required to incentivize the driver to be careful
- We can get a full solution using the participation constraint

$$
\begin{aligned}
& p(a)u(W-d-q+R)+(1-p(a))u(W-q)-a \\
= \; & \bar{u} \\
= \; & p(\bar{a})u(W-d)+(1-p(\bar{a}))u(W)-\bar{a}
\end{aligned}
$$

where $\bar{a}$ is the optimal effort level given no insurance

Insurance

- Combining these two equations give

$$\begin{aligned} u(W-d-q+R) &= \bar{u}+a+\frac{1-p(a)}{p'(a)} \\ u(W-q) &= \bar{u}+a-\frac{p(a)}{p'(a)} \end{aligned}$$

- This allows us to solve for $R(a)$ and $q(a)$ as functions of a
- Plug this in to the insurer's profit function

$$q(a)-p(a)R(a)$$

- And maximize to get complete solution

Limited Liability

- We can use the 2 outcome, continuous action model to analyze an interesting variant of the standard moral hazard model
- Remember that we said at the start that one way to make the problem boring was to assume the agent was risk neutral
  - Then the principal can just ‘sell the firm’ to the agent
- There is another way to make the problem interesting, even with risk neutral agents
  - Limited liability
- Selling the firm may mean that the agent has to suffer very bad outcomes
  - Maybe there is a limit to the badness of the outcome that the principal can impose on the agent
- For example, if Lehman Brothers goes bankrupt, cannot force the manager to pay $300 billion !

Limited Liability

- We now have an additional constraint which is

$$w \geq \bar{w}$$

- So the general problem is

$$\max_{a_i, w} \sum_{j=1}^{m} p_{ij}(x_j - w_j)$$

subject to

$$\begin{aligned}
\sum_{j=1}^{m} p_{ij} w_j - a_i &\geq \bar{u} \\
a_i &\in \arg\max_{a_l} \sum_{j=1}^{m} p_{lj} w_j - a_l \\
w_j &\geq \bar{w} \text{ for all } j
\end{aligned}$$

Limited Liability

- What does the solution look like?
- If our unconstrained solution never set $w_j < \bar{w}$ for any j then the new constraint makes no difference
- If it does bind, we could always just raise all wages by the same amount to ensure that

$$\min_j w_j = \bar{w}$$

  - IC constraints would still hold
- However in general this will not be optimal
- What is true is that in general a binding liquidity constraint will mean that the IR constraint does **not** bind
  - Agent makes rents at the optimum
  - Unlike the risk averse case

Limited Liability - A Simple Example

- We will illustrate the limited liability model in the two state two action case
- Assume

$$x_s = 1 \quad x_f = 0$$

$$\begin{aligned} p_1(x_s) &= 1 \\ p_0(x_s) &\in (0,1) \\ a_0 &= 0 \\ a_1 &< 1 - p_0(x_s) \\ \bar{u} &= 0 \end{aligned}$$

- And, crucially, $u(w) = w$
  - Both principal and agent are risk neutral

Limited Liability - A Simple Example

- Without limited liability, the solution is simple
  - Pay the agent a_1 in the case of high outcome, so $w_s = a_1$
  - Punish low outcome enough to ensure IC constraint binds
- This requires

$$\begin{aligned} p_0(x_s)a_1 + (1 - p_0(x_s))w_f &\leq 0 \\ &\Rightarrow w_f \leq -\frac{p_0(x_s)a_1}{(1 - p_0(x_s))} \end{aligned}$$

- Or for w_f to be less than zero

Limited Liability - A Simple Example

- Let's make the problem interesting by adding the constraint that $w \geq 0$
- Assume we still want to implement a_1
- The problem is now

$$\min_{w_s, w_f} w_s$$

subject to

$$\begin{aligned} w_s - a_1 &\geq p_0(x_s)w_s + (1 - p_0(x_s))w_f \\ w_s - a_1 &\geq 0 \\ w_f &\geq 0 \end{aligned}$$

Limited Liability - A Simple Example

- Notice that there are three constraints and only two unknowns
- In general they cannot all hold with equality
- IC constraint has to bind
- So which of the other two?
- Can't be the IR constraint, as we know that this pushes w_f below zero
- Must be that $w_f = 0$
- And so, by the IC constraint

$$\begin{aligned} w_s - a_1 &= p_0(x_s)w_s \\ &\Rightarrow w_s = \frac{a_1}{(1 - p_0(x_s))} \end{aligned}$$

Limited Liability - A Simple Example

- This means that the agent gets rents

$$\begin{aligned} & w_s - a_1 \\ = \quad & \frac{a_1}{1 - p_0(x_s)} - a_1 = \frac{p_0(x_s)}{1 - p_0(x_s)} a_1 \end{aligned}$$