

# Behavioral Economics

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Homework 2 - Autumn 2022

Due Friday 2nd December

**PLEASE ANSWER 3 OF THE 4 QUESTIONS**

**Question 1 (Quasi-Hyperbolic Discounting)** Consider an infinitely lived quasi-hyperbolic decision maker who is trying to lose weight. Their preferences in each period are given by

$$u(\gamma_t, w_t, w_{t-1}, x_t) = \gamma_t w_t - (w_t - w_{t-1})^2 - n w_t^2 - m x_t$$

Where  $w_t$  is weight in period  $t$  and  $\gamma_t$  is a preference parameter drawn randomly from some distribution  $f$  (for the sake of argument let's assume it is  $N(0, \sigma^2)$ ) and  $x_t$  is any amount of money spent in period  $t$ . The interpretation is that the first term is the utility from consumption (which varies randomly), the second term is an adjustment cost, the third term is the cost of excess weight while the fourth term is the utility from any money spent (initially we will assume that this is zero)

1. Model the behavior of the agent as a game played between different 'agents' in each period. Guess and verify that the game has a solution of the form

$$w_t = a w_{t-1} + b \gamma_t + c$$

Solve for  $a$ ,  $b$  and  $c$ .

2. Imagine that you observed the weight path of this agent. Which of the parameters of the original model could you recover?
3. Now assume that in some time period  $t$ , the agent faces a 'commitment contract' such that they have to pay an amount  $x_t$  if their weight is above a threshold  $y_t$ . Characterize the behavior of the agent in this period.

4. Now assume that, after setting their weight in period  $t - 1$ , the agent is (unexpectedly) given the option of setting a target weight for period  $t$  (assume that the amount they forfeit  $x$  is fixed, but they can choose  $y$ ). What will their optimal target be? How will it vary with the parameters of the problem?

**Question 2: (An Alternative Model of Menu Preferences)** Consider someone who is going to have to choose between jobs, which will vary in their pay and the number of days holiday they will get. When asked to choose between jobs, they will do so in order to maximize  $p(x) + d(x)$ , where  $p(x)$  is the pay of job  $x$  and  $d(x)$  is the number of days holiday given by job  $x$ . However, they will also feel sad if the job they choose is not the one that gives the highest pay, or the most days holiday. So, for example, if you are asked to choose between a job with good pay and good holidays and one that has great pay and terrible holidays, you may end up choosing the former, but you will be upset because you have turned down the great pay available from the latter.

Thus preferences over *menus* of jobs is given by

$$U(A) = \max_{x \in A} [p(x) + d(x)] + \theta \left[ p(x) - \max_{y \in A} p(y) \right] + \theta \left[ d(x) - \max_{z \in A} d(z) \right]$$

where  $A$  is the menu,  $x$  is the job that they will eventually choose,  $y$  is the available job which gives the highest pay and  $z$  is the job that gives the most days off.  $\theta \geq 0$  is a parameter that measures how sad they will be for forgoing the highest possible pay and number of days off.

1. Can this model give rise to a preference for commitment?
2. Will decision makers who obey this model obey sophistication and set betweenness?
3. If not, come up with a replacement for set betweenness which captures the implications of the model
4. Would you consider this a model of temptation? If not, how would you think about distinguishing between behavior driven by the psychological factors that this model is trying to capture and behaviors driven by temptation?

**Question 3 (An Alternative Data Set)** Consider a data set consisting of choices from sets of alternatives of the form  $(a, t)$ , where  $a \in A$  (with  $A$  finite) is some task that can be completed and  $t$  is a time between 0 and some upper bound  $T$ . Choosing  $(a, t)$  therefore means completing

a task at time  $t$ . From any given choice set only one completion option can be chosen. Let  $\Gamma$  be the set of all such  $(a, t)$ . Say we observe a choice function on subsets of  $\Gamma$ .

Consider a model of decision making consisting of a series of one-to-one utility functions for each time  $t$  such that  $U_t : A_{\geq t} \rightarrow \mathbb{R}$  where  $A_{\geq t} = \{(a, s) | a \in A \text{ and } s \geq t\}$ . A **sophisticated** strategy for any  $B \in \Gamma$  is a mapping  $q$  from  $\tau = \{t \in 0, \dots, T | (a, t) \in B\}$  to  $A \cup \{\text{wait}\}$  such that

1. If  $s = \max_{t \in \tau} t$  then  $q(s) = \arg \max_{a \in A} \{U_s(a, s) | (a, s) \in B\}$
2. Otherwise, if  $\max_{a \in A} \{U_s(a, s) | (a, s) \in B\} \geq U_s(\hat{a}_s, \hat{t}_s)$  then  $q(s) = \arg \max_{a \in A} \{U_s(a, s) | (a, s) \in B\}$
3. Otherwise  $q(s) = \{\text{wait}\}$   
 where  $\hat{t}_s = \min \{s' > s | q(s') \neq \{\text{wait}\}\}$  and  $\hat{a}_s = q(\hat{t}_s)$

The interpretation of this strategy is that the DM at each stage solves the game using backwards induction. In any period  $s$  the DM figures out what will happen if they do not choose a completion option available to them at that time. They will choose to wait if and only if the utility of the completion option that will occur is better (according to period  $s$  utility) than what they could currently obtain.

1. Define an equivalent notion of a **naive** strategy
2. We say a choice function can be represented by the sophisticated model if we can find a set of utility functions such that, for every  $B \in \Gamma$   $c(B) = \{(a, t) = \{q(s), s\} | s = \min s \in \tau | q(s) \neq \text{wait}\}$ . It is time consistent if it can be represented by a sophisticated model in which the utility function is the same at every  $s$ . Show that  $c$  is time consistent if and only if it satisfies the independence of irrelevant alternatives
3. A reversal occurs if for  $B \in \Gamma$ ,  $c(B) = (a_1, t_1)$  yet  $C(B \cup (a_2, t_2)) = (a_3, t_3)$ . A reversal is called a doing it later reversal if  $t_1 < t_3$  and either  $t_2 \leq t_3$  or  $(a_1, t_1) = c(\{(a_1, t_1), (a_3, t_3)\})$  or both. A doing it earlier reversal is one in which  $t_3 < t_1$ . Explain why these are appropriate terms for this type of reversal.
4. Show that the naive model you defined in stage 1 does not exhibit any doing-it-earlier reversals
5. Show that the sophisticated model does not exhibit any doing-it-later reversals

**Question 3 (An Experimental Design Question)** I discussed in class how I thought that one reason that the menu preference literature is not as popular as it once was is the lack of accompanying experimental data. This is a shame, as the theoretical literature makes it clear that menu preferences are a useful way of identifying not only temptation and self control but also stuff like regret, preference uncertainty and rational inattention. Having read the Toussaert paper (and looked at some of the others we discussed in class) I would like you to write a 3-5 page paper outlining the design of an experiment. The aim of the experiment will be for us to learn something about the way temptation and self control work. Ideally you would start with something of general interest - (for example: "is self control a depletable resource"?) rather than a technical question ("does set betweenness hold?"). You will then want to figure out theoretically what type of behavior would answer that question, and then the experimental environment in which you will collect the relevant data (ideally one in which you think there is a good chance that there will be a lot of temptation and self control.