Competitive Equilibrium and Efficiency in an Exchange Economy

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1 Introduction

The first section of the course has equipped us with a model of the way consumers behave in the form of demand functions[1]

$$x_1(p_1, p_2, M)$$
$$x_2(p_1, p_2, M)$$

This is a good start, but our goals are much more grand: we want to understand how the whole *economy* operates,. rather than just one consumer within that economy. In particular, we want to know how goods will be **allocated** (i.e. who will end up with what) and what the **prices** will be of the different goods in the economy. At the moment we can say what our consumer will want to consume, given a set of prices, but we have nothing to say about where these prices have come from, or where their income has come from, or whether there is enough of each good in the economy to match their demands.

To model all this seems like a pretty daunting goal - the economy is made up of millions of different people, and millions of different goods, as well as firms who produce those goods, shops that sell them and so on. So how do we approach this problem? As you might suspect, we are going to make some fairly heroic simplifying assumptions which are going to allow us to analyze an economy. In particular, we are going to make three assumptions that are going to help a lot

1. There are only two goods in the world

[1] from now on we are going to save time by removing the $*$ from $x_1^*(p_1, p_2, M)$

2. There are only two consumers in the world

3. There are no firms or shops in the world. Instead, each consumer starts off with an **endowment** of good 1 and good 2, which they can either consume for themselves or sell it at market prices.

It is this last property that means that we are studying what we call an **endowment economy.** Imagine two hermits that live on a barren island. Both hermits have a stash of brandy and figs, and all they can do is either consumer their own stash, or sell some of it to the other hermit. In later lectures we will relax this assumption and think about an economy which has firms that produce things as well, but for now we will be able to get a lot of insight from this simplified world.

2 The Consumer's Problem Revisited.

The first thing we need to do is think about what the consumer's problem looks like in the endowment economy. In one important respect, the problem is going to be similar to that we have already covered: The consumer will treat prices as fixed, and decide what to buy and sell given those prices. However, in one important respect the problem is going to be different. We are now not going to assume that, rather than a fixed monetary income (M in the last chapter), the consumer will now have an endowment of each good (each hermit starts life with a certain amount of brandy and a certain number of figs). We will use ω_1^1 and ω_2^1 to denote the endowment of consumer one, and ω_1^2 and ω_2^2 to denote the endowment of consumer two. Note that we will always superscripts to refer to the consumer and subscripts to refer to the good.

What does the consumer's budget constraint now look like? Well, as they cannot borrow money, the total amount they get from buying goods cannot be more than the total amount they get from selling goods. For good 1, the net amount the consumer spends is given by $p_1(x_1 - \omega_1^1)$.(nor the moment, because we are only concentrating on one consumer, I will omit the superscript from x_1^1) This will be a negative number if consumer 1 consumes less than their endowment (and therefore sells the rest), or a positive amount if they consume more than their endowment, (and therefore have to buy the excess) Similarly the net amount that they spend on good two is $p_2(x_2 - \omega_2^1)$. The budget constraint is therefore given by

$$p_1(x_1 - \omega_1^1) + p_2(x_2 - \omega_2^1) \leq 0$$

Perhaps a more intuitive way of thinking about this is to rearrange this expression to give

$$p_1x_1 + p_2x_2 \leq p_1\omega_1^1 + p_2\omega_2^1$$

The interpretation here is that the first thing the consumer does is sell their endowment at market prices to generate income, then decides how to spend that income.

What does the budget set look like now? Intuitively, the budget line is a straight line, and it must be the case that it goes through the endowment (i.e. the consumer must always be able to afford the endowment). Rearranging the above again gives us

$$\begin{aligned} x_2 &= \frac{(p_1\omega_1^1 + p_2\omega_2^1)}{p_2} - \frac{p_1}{p_2}x_1 \\ &= (\frac{p_1}{p_2}\omega_1^1 + \omega_2^1) - \frac{p_1}{p_2}x_1 \end{aligned}$$

In fact, the only important change in the budget constraint is that a change in prices now causes the budget line to 'pivot' round the endowment (see figure 1), so an increase in the price of one of the goods is not necessarily a bad thing for the consumer. In fact, if the consumer's endowment is made up entirely of good 1, an increase in the price of good 1 will make them unambiguously better off (you should check your understanding by ensuring that you agree with this statement).

Let's go through an example of solving the consumer's problem in the endowment economy for an individual. We will assume that they have Cobb Douglas preferences of the following form:

Choose: *a consumption bundle* (x_1, x_2)
In order to maximize: $u(x_1, x_2) = x_1^{\frac{1}{3}}x_2^{\frac{2}{3}}$
Subject to: $p_1x_1 + p_2x_2 = p_1\omega_1^1 + p_2\omega_2^1$

The procedure that we use to solve this problem is exactly the same as that covered in the consumer choice problem. In fact, as this is a case of Cobb-Douglas preferences, the procedure is easy: we find the point of tangency, then use that and the budget constraint to solve for their optimal bundle (x_1^*, x_2^*) (remembering, as I am sure you do, that because Cobb-Douglas preferences are convex, the optimal consumption bundle always occurs at the point of tangency, if such a point exists)

Remember that the point of tangency occurs when

$$\frac{p_1}{p_2} = MRS(x_1, x_2)$$

and that

$$MRS(x_1,x_2) = \frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}}$$

Thus, taking the necessary derivatives gives us

$$\begin{aligned}
\frac{\partial u(x_1,x_2)}{\partial x_1} &= \frac{1}{3}x_1^{-\frac{2}{3}}x_2^{\frac{2}{3}} \\
\frac{\partial u(x_1,x_2)}{\partial x_2} &= \frac{2}{3}x_1^{\frac{1}{3}}x_2^{-\frac{1}{3}}
\end{aligned}$$

And so taking the relevant ratio gives

$$\frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}} = \frac{\frac{1}{3}x_1^{-\frac{2}{3}}x_2^{\frac{2}{3}}}{\frac{2}{3}x_1^{\frac{1}{3}}x_2^{-\frac{1}{3}}} = \frac{1}{2}\frac{x_2}{x_1}$$

setting this equal to prices gives

$$\frac{1}{2}\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

and so

$$x_2 = 2\frac{p_1}{p_2}x_1$$

Up to here, what we have done has been completely identical to solving the standard consumers problem. Things start to look slightly different when we substitute into the budget constraint

$$\begin{aligned}
p_1x_1 + p_2 2\frac{p_1}{p_2}x_1 &= p_1\omega_1^1 + p_2\omega_2^1 \\
&\Rightarrow \quad 3x_1p_1 = p_1\omega_1^1 + p_2\omega_2^1 \\
&\Rightarrow \quad x_1 = \frac{1}{3}\left(\omega_1^1 + \frac{p_2}{p_1}\omega_2^1\right)
\end{aligned}$$

and so

$$x_2 = \frac{2}{3}\left(\frac{p_1}{p_2}\omega_1^1 + \omega_2^1\right)$$

A couple of things to note here. First, the demand for each good now depends on the prices and the endowments in the economy. These are the new *parameters* of the consumer's problem. Second, now it is clear that the only thing that matters is the *ratio* of price of good one to good two (look back at the equations if you don't believe me). This is a general property, not specific to the utility function we have chose. Because of this, we can *normalize* the price of good 2 to one, and so rewrite the demand functions as

$$\begin{aligned} x_1^1(p_1, \omega_1^1, \omega_2^1) &= \frac{1}{3}\left(\omega_1^1 + \frac{1}{p_1}\omega_2^1\right) \\ x_2^1(p_1, \omega_1^1, \omega_2^1) &= \frac{2}{3}\left(p_1\omega_1^1 + \omega_2^1\right) \end{aligned}$$

You will therefore not see p_2 again for the remainder of each chapter.

The final tool we need before going on to think about equilibrium is the **net demand** of each consumer for each good. This is just the actual demand, minus their endowment, so the amount they want to buy and sell of each good. We will use $z_1^1(p_1, \omega_1^1, \omega_2^1)$ to denote the net demand of consumer 1 for good 1 and so on, so

$$\begin{aligned} z_1^1(p_1, \omega_1^1, \omega_2^1) &= x_1^1(p_1, \omega_1^1, \omega_2^1) - \omega_1^1 \\ z_2^1(p_1, \omega_1^1, \omega_2^1) &= x_2^1(p_1, \omega_1^1, \omega_2^1) - \omega_2^1 \end{aligned}$$

3 The Edgeworth Box

We are now going to introduce a graphical tool that is going to be very useful in thinking about our two good - two consumer economy. It is called the Edgeworth Box.

In constructing the Edgeworth box, the first thing to think about is the total amount of stuff in the economy - how much is there of each good? The answer is the sum of the endowments of the two consumers in the economy, so the total amount of good 1 is $\omega_1^1 + \omega_1^2$, while the total amount of good 2 is $\omega_2^1 + \omega_2^2$. We are therefore going to start off by drawing a box with length $\omega_1^1 + \omega_1^2$ and height $\omega_2^1 + \omega_2^2$ - shown in figure 2.

We can now use this box to represent any **feasible allocation -** in other words any consumption bundle (x_1^1, x_2^1) for consumer 1 and (x_1^2, x_2^2) for consumer two such that

$$\begin{aligned} \omega_1^1 + \omega_1^2 &= x_1^1 + x_1^2 \\ \omega_2^1 + \omega_2^2 &= x_2^1 + x_2^2 \end{aligned}$$

or in other words the total amount of each good available in the economy is equal to the total amount of each good consumed in the economy. In fact, each point in the Edgeworth box represents a feasible allocation, as shown in figure 3: We measure consumer 1's consumption bundle from the bottom-left hand corner and consumer 2's bundle from the top-right hand corner. Thus, we know that any achievable allocation of goods between the two consumers has to lie somewhere in the box. Note that, every point in the box represents a consumption bundle for consumer 1 *and* consumer 2. The bottom left hand point is the one in which consumer 1 gets zero of both goods and consumer 1 gets everything, while the top right hand point is where consumer 2 gets nothing and consumer 3 gets everything.

We can also use the Edgeworth box to represent consumer 1's consumption problem for a price p_1. This is shown in figure 4. We can draw the consumer's budget set by locating the initial endowment in the Edgeworth box, and using the prices to draw the budget line. We can also draw the consumer's indifference curves on the Edgeworth box (as each point is a consumption bundle for consumer 1). Thus, for any given price, we can locate the optimal consumption bundle for consumer 1 in the Edgeworth box (figure 5).

What makes the Edgeworth box so useful is that we can simultaneously use it to represent consumer 2's consumption problem. To do this, we 'flip' the graph for consumer 2, so it's origin is in the top right hand corner. For a given budget line, the budget set for consumer 2 is the area *above* the budget line, as depicted in figure 6. We can also depict the indifference curves of consumer 2 in the Edgeworth box (as each point in the box is a consumption bundle for consumer 2). In this case, if consumer 2 has monotonic preferences, their indifference curves will move on to higher preferences as we move in a South Easterly direction, as it is in this direction that consumer 2 gets more stuff (figure 7). Thus, we can use the Edgeworth box to show consumer 2's optimal bundle (figure 8).

4 A Competitive Equilibrium

The next thing we need to do is define an equilibrium. I will start by defining it formally, then we will think about what it means.

Definition 1 *An equilibrium is a consumption bundle for each consumer* (x_1^1, x_2^1), (x_1^2, x_2^2) *and a price* p_1 *such that*

1. *The allocation is feasible*

$$\begin{aligned}
\omega_1^1 + \omega_1^2 &= x_1^1 + x_1^2 \\
\omega_2^1 + \omega_2^2 &= x_2^1 + x_2^2
\end{aligned}$$

2. *The consumption bundles solve the consumers optimization problem, given the price and initial allocations*

$$\begin{aligned}
x_1^1(p_1, \omega_1^1, \omega_2^1) &= x_1^1 \\
x_2^1(p_1, \omega_1^1, \omega_2^1) &= x_2^1
\end{aligned}$$

$$\begin{aligned}
x_1^2(p_1, \omega_1^1, \omega_2^1) &= x_1^2 \\
x_2^2(p_1, \omega_1^1, \omega_2^1) &= x_2^2
\end{aligned}$$

So how do we interpret this? Well first of all, what things to go to make up an equilibrium? It consists of an *allocation,* or a list of who gets what in the economy, and a price for good 1 (remember that we have set $p_2 = 1$). Thus, we can think of it as a prediction about the things we care about in the economy: who gets what, and at what prices. What makes an allocation and price an equilibrium is two things. Firstly - *feasibility* - the amount of each good that is being consumed is the same as the amount there actually is in the economy (the amount of figs that the two hermits eat between them is the same as the total amount of figs that they brought to the island, and the same with the brandy). Second, *optimality* - given the prices, each consumer is choosing their most preferred bundle of the ones in their budget set.

So what does an equilibrium look like in an Edgeworth box? Well, we know that any point in the box is a feasible allocation, so what we are looking for is a price line such that the optimal bundle for each consumer falls at the same point. Figure 9 shows an example in which the prices make it impossible for there to be an equilibrium: At these prices, the optimal bundle for consumer 2 is A, while the optimal bundle for consumer 2 is B. Thus, any feasible bundle (point in the Edgeworth box), must be non-optimal for either consumer 1 or consumer 2. Another way to say this is that we cannot give both consumers their optimal bundles, because if we did, their consumption of good two would be greater than the endowment of good two in the economy, as $x_1^1(p_1, \omega_1^1, \omega_2^1) + x_1^2(p_1, \omega_1^1, \omega_2^1) > x_1^1 + x_1^2$.

In contrast, we can find an equilibrium in figure 10. At these prices, the optimal consumption bundles for consumer 1 and consumer 2 lie *at the same point* in the box. Put another way, at these prices, the sum of the optimal consumption of good 1 across both consumers is the same as the total endowment of good 1, and the same for good two. Thus, the price represented by that budget line, plus the bundles (x_1^1, x_2^1), (x_1^2, x_2^2) form an equilibrium, because (a) (x_1^1, x_2^1), (x_1^2, x_2^2) is feasible (b) (x_1^1, x_2^1) is optimal for consumer 1 given the prices, and (c) (x_1^2, x_2^2) is optimal for consumer 2 given the prices.

One important property of the equilibrium is that the slope of the indifference curve of consumer 1 is equal to the slope of the indifference curve of consumer 2 (as both are equal to the slope of the price line). This should not be surprising, as we know that at an interior solution the slope of each consumer's utility function (the MRS) should be the same as the slope of the budget line - which is the same for both consumers. This is an important result that we will come back to: in a competitive equilibrium, the MRS of both consumers is the same.

Fining the equilibrium graphically is one thing. But how do we find an equilibrium of an endowment economy *algebraically*? We can follow the simple algorithm, which is guaranteed to find a solution.

1. Solve the optimization problem for each consumer to get their excess demand as a function of prices (and endowments)

2. Find a price such that total net demand for each good in the economy is zero.

Will the resulting prices p_1, and allocation $\left(x_1^1(p_1, \omega_1^1, \omega_2^1), x_2^1(p_1, \omega_1^1, \omega_2^1)\right)$ and $\left(x_1^2(p_1, \omega_1^1, \omega_2^1), x_2^2(p_1, \omega_1^1, \omega_2^1)\right)$ be an equilibrium? Yes! First of all, it will be feasible, as, if total net demand for each good is zero, then

$$\begin{aligned} z_1^1(p_1, \omega_1^1, \omega_2^1) + z_1^2(p_1, \omega_1^2, \omega_2^2) &= 0 \\ \text{so } x_1^1(p_1, \omega_1^2, \omega_2^2) - \omega_1^1 + x_1^2(p_1, \omega_1^2, \omega_2^2) - \omega_1^2 &= 0 \\ \text{so } x_1^1(p_1, \omega_1^2, \omega_2^2) + x_1^2(p_1, \omega_1^2, \omega_2^2) &= \omega_1^1 + \omega_1^2 \end{aligned}$$

and the same for good two. Second, is the allocation optimal for each consumer? Again yes! We have defined the demand functions such that they *are* the solution to the consumers optimal problem at those prices.

As an example, lets assume that we have two consumers who each have preferences of the type we described in section 2. Their excess demand functions are therefore given by

$$
\begin{aligned}
z_1^1(p_1,\omega_1^1,\omega_2^1) &= \frac{1}{3}\left(\omega_1^1+\frac{1}{p_1}\omega_2^1\right)-\omega_1^1\\
z_2^1(p_1,\omega_1^1,\omega_2^1) &= \frac{2}{3}\left(p_1\omega_1^1+\omega_2^1\right)-\omega_2^1\\
z_1^2(p_1,\omega_1^2,\omega_2^2) &= \frac{1}{3}\left(\omega_1^2+\frac{1}{p_1}\omega_2^2\right)-\omega_1^2\\
z_2^2(p_1,\omega_1^2,\omega_2^2) &= \frac{2}{3}\left(p_1\omega_1^2+\omega_2^2\right)-\omega_2^2
\end{aligned}
$$

Let's start by looking at the market for good 1. The total excess demand for good 1 is given by

$$\frac{1}{3}\left(\omega_1^1+\frac{1}{p_1}\omega_2^1\right)-\omega_1^1+\frac{1}{3}\left(\omega_1^2+\frac{1}{p_1}\omega_2^2\right)-\omega_1^2$$

For an equilibrium, this has to equal zero, so rearranging, we get

$$
\begin{aligned}
\frac{1}{p_1}(\omega_2^1+\omega_2^2)-2(\omega_1^1+\omega_1^2) &= 0\\
&\Rightarrow\ p_1=\frac{1}{2}\frac{(\omega_2^1+\omega_2^2)}{(\omega_1^1+\omega_1^2)}
\end{aligned}
$$

Thus $\frac{1}{2}\frac{(\omega_2^1+\omega_2^2)}{(\omega_1^1+\omega_1^2)}$ is what we call the market clearing price for good 1. This makes sense - the price of good 1 goes up as it becomes rarer relative to good 2.

At this stage you may be a little bit worried: we have found a price for that ensure the market clears for good 1 (i.e. the excess demand is 0), but what about for good 2? Well, lets do the same thing for good 2. Here, the total excess demand is given by

$$\frac{2}{3}\left(p_1\omega_1^1+\omega_2^1\right)-\omega_2^1+\frac{2}{3}\left(p_1\omega_1^2+\omega_2^2\right)-\omega_2^2$$

thus we need this to equal zero. Again, rearranging, we get

$$2p_1(\omega_1^1+\omega_1^2)-(\omega_2^1+\omega_2^2)=0$$

or

$$p_1=\frac{1}{2}\frac{(\omega_2^1+\omega_2^2)}{(\omega_1^1+\omega_1^2)}$$

Amazing! We get the same result - so the price that clears market one also clears market 2. A miracle! Actually, it is not a miracle, but an example of what is called Walras Law:

Definition 2 *Walras law states that the price that leads to excess demand in the market for good 1 to equal zero will lead to excess demand in the market for good 2 to also equal zero.*

Practically, this means that you only have to worry about finding the market clearing price for one of the goods, and the other will follow.

To help you with the intuition, I'll provide a couple of other ways of thinking about what it means to find an equilibrium First, let's think want a change in prices looks like in the Edgeworth box. Figure 11 shows how consumer 1's optimal bundle changes as the price of good 1 increases. In other words, it shows how $x_1^1(p_1, \omega_1^1, \omega_2^1)$ and $x_2^1(p_1, \omega_1^1, \omega_2^1)$ change with p_1. The line that connects all these points (i.e. connects all the points $x_1^1(p_1, \omega_1^1, \omega_2^1)$ $x_2^1(p_1, \omega_1^1, \omega_2^1)$ for some p_1) is called consumer 1's **offer curve**. It is shown here in yellow. Figure 12 performs the same exercise with good 2. We can think of the finding an equilibrium as the process of finding the point where the two offer curves intersect (figure 13). This is the point at which the demanded bundle of consumer 1 and consumer 2 add up exactly to the sum of the endowments for each good.

A second way of thinking of what we are doing is in terms of good old fashioned supply and demand, that you may remember from Econ 11. Let's think first about supply. In this economy, that is easy: The supply of each good is fixed at the level of the endowments. So the supply of good 1 is $\omega_1^1 + \omega_1^2$, whatever the price. Demand, however, *is* a function of price. In particular, the total demand for good 1 is given by

$$x_1^1(p_1, \omega_1^1, \omega_2^1) + x_1^2(p_1, \omega_1^2, \omega_2^2)$$

From consumer theory we know that each individual demand function is generally (though not always - remember Giffen goods!) downward sloping. Thus, in equilibrium, we are trying to find the price at which the downward sloping demand curve crosses the (flat) supply curve. This point is depicted in figure 14: the equilibrium price is the price such that demand equals supply for good 1. The beauty of Walras' law tells us the same price will also equilibrate supply and demand for good 2.

Lets do one other example. In this case we will assume that both our consumers see good 1 and good 2 as perfect compliments. We will also assume that the endowments of each good are as follows.

$$\begin{aligned}\omega_1^1 &= 2\\ \omega_2^1 &= 1\\ \omega_1^2 &= 2\\ \omega_2^2 &= 1\end{aligned}$$

What does this look like in the Edgeworth box? Well we know that both consumer 1 and consumer 2's indifference curves are 'L' shaped, with the kink occurring at the point were the amount of good 1 equals the amount of good two. Thus the Edgeworth box looks like it does in figure 15. This also shows that amount of each good that is demanded for a 'typical' price p_1.

Immediately, this looks like we have a problem in finding an equilibrium. For any positive price p_1, there is excess demand for good 2, as we can see in figure 16. For any positive price p_1, each consumer is going to want to trade some of their endowment of good 1 in exchange for some of good 2. This makes sense: if the consumer only has one unit of good 2, then any amount of good 1 above 1 unit is essentially worthless to them. As there initial endowment of good 1 is greater than 1, they will trade good 1 for good 2 at any price.

We can also see this by calculating the excess demand functions for each consumer for good 1. Remember that, for perfect compliments, each consumer will buy exactly the same amount of each good, so, for consumer 1 and good 1

$$\begin{aligned}p_1^1 x_2^1 + x_2^1 &= p_1\omega_1^1 + \omega_2^1\\ x_2^1 &= \frac{p_1\omega_1^1 + \omega_2^1}{1+p_1}\\ &= \frac{1+2p_1}{1+p_1}\end{aligned}$$

Excess demand is given by

$$\begin{aligned}z_2^1(p_1,\omega_1^1,\omega_2^1) &= \frac{1+2p_1}{1+p_1} - \omega_2^1\\ &= \frac{1+2p_1}{1+p_1} - 1\\ &= \frac{p_1}{1+p_1}\end{aligned}$$

By symmetry (i.e., because consumer 1 and consumer 2 are exactly the same), the excess demand for consumer 2 and good 1 is given by

$$z_2^2(p_1, \omega_1^2, \omega_2^2) = \frac{p_1}{1 + p_1}$$

Thus, for equilibrium we need

$$\begin{aligned} z_1^1(p_1, \omega_1^1, \omega_2^1) + z_1^2(p_1, \omega_1^2, \omega_2^2) &= 0 \\ &\Rightarrow \frac{p_1}{1+p_1} + \frac{p_1}{1+p_1} = 0 \end{aligned}$$

Which cannot hold for any positive p_1. However, it *can* hold if $p_1 = 0$, or in other words if the budget line in the Edgeworth box is horizontal. At this price, consumers are unable to get any more units of good two by giving up units of good 1, so their demand for good 2 is simply their endowment. For good 2, supply equals demand. This is shown in fig 17.

What is happening with good 1 at this stage? If the price of good 1 is zero, then surely demand for it should be infinite? Well, think back to the utility function of each consumer:

$$u^1(x_1^1, x_2^1) = \min(x_1^1, x_2^1)$$

We have already established that, if the price of good 1 is zero, then each consumer will consume 1 unit of good 2. As long as they consumer at least 1 unit of good 1, they are indifferent about how many extra units of good 1 they have. They will be equally happy with 1, 1,000 or 1,000,000 units of good 1. (If you only have one left shoe, then you don't benefit from having any more than one right shoe). Thus, the demand function for each consumer for good 1 is not well defined. they are happy to demand any amount of good 1 that is at least 1. However, this is enough for an equilibrium: The price $p_1 = 0$, along with the bundles

$$\begin{aligned} x_1^1 &= 2 \\ x_2^1 &= 1 \\ x_1^2 &= 2 \\ x_2^2 &= 1 \end{aligned}$$

is an equilibrium as it is (a) feasible and (b) given prices, the bundles solve the consumer's optimization problem, as there is no bundle they can afford that is better than this one.

This raises the question: does an equilibrium always exist? (as you progress in your economics education, you will find out that economists are always obsessed with existence and uniqueness). The answer is 'almost always, but not in every case'. Unfortunately, we don't quite have the maths to answer this question formally, but to give an intuition of when there might not be an equilibrium look at figure 18. This is a case where the demand curve has a 'jump' in it: for prices below p_1^*, demand is above supply, whereas for prices of p_1^* and above, demand is below supply. This is a case in which we could not find an equilibrium price. The problem here is that the demand function is not *continuous.* Continuous functions are ones that you can draw without taking your pen off the page.

A final point to note about competitive equilibria is that they depend on the initial endowments that each consumer gets: in general, if we change the endowments then the equilibrium outcome will change (for example, if we give more endowment to consumer 1, we would expect them to end up with more stuff in equilibrium. In the Edgeworth Box, we can locate the equilibrium point for any initial endowment by finding a price line that gives a point of tangency between the indifference curves of the two consumers.

5 Pareto Efficiency

We are now going to move on to a significantly more controversial part of the course: We are going to look of questions of 'welfare'. This is where economists (either implicitly or explicitly) stop talking about what *will* happen, and start talking about what *should* happen. As we shall see, economists are very cautious about when they are prepared to make such judgements. However, even the small amount they do say is enough to cause arguments.

The key to welfare comparisons as made by economists is the idea of *Pareto Efficiency.* To understand the concept, think about the following two allocations

$$\text{Allocation A:} \qquad (2,4)\ \ (2,2)$$

$$\text{Allocation B:} \qquad (2,4)\ (2,4)$$

It seems that, assuming that both consumers have monotonic preferences, allocation B is better than allocation A: Consumer 1 is just as well off in both cases, while consumer 2 is better off in the second case. Now consider the following two bundles

$$\begin{array}{ll} \text{Allocation C:} & (2,4)\ \ (2,2) \\ \text{Allocation D:} & (1,4)\ (10,2) \end{array}$$

Can we rank bundles C and D? The traditional economic argument is ‘no, we cannot’. While consumer 2 prefers D, consumer 1 prefers C, we cannot unambiguously say which of these two bundles. This is, of course, a very specific stance. Other moral thinkers would have no trouble ranking bundles C and D. For example, a utilitarian might say that allocation D has more total ‘stuff’ than allocation C, and therefore is likely to give more ‘total utility’. However, economists traditionally shy away from such ‘interpersonal comparisons’. We don’t like saying that giving 8 extra units of good 1 to consumer 2 offsets the cost of taking away 1 unit from consumer 1. Why not? One reason is that we don’t trust the magnitude of differences in utility as being meaningful: we know that it doesn’t mean anything to say that the extra 8 units of good 1 will double consumer 2’s utility, or increase it by 7, or anything of that sort. Thus, we don’t really know whether the utility gained by person 2 is bigger than that lost by person 1 in going from allocation C to allocation D. For example, what if good 1 is insulin, good 2 is Ferrari’s, and person 1 is diabetic?

This leads us to our definition of Pareto efficiency.

Definition 3 *Let* (x_1^1, x_2^1), (x_1^2, x_2^2) *be an allocation. We say that another feasible allocation* (y_1^1, y_2^1), (y_1^2, y_2^2) ***Pareto dominates*** (x_1^1, x_2^1), (x_1^2, x_2^2) *if*

$$\begin{aligned} u^1(y_1^1, y_2^1) &\geq u^1(x_1^1, x_2^1) \\ \textit{and } u^2(y_1^2, y_2^2) &\geq u^2(x_1^2, x_2^2) \end{aligned}$$

and

$$\begin{aligned} u^1(y_1^1, y_2^1) &> u^1(x_1^1, x_2^1) \\ \textit{or } u^2(y_1^2, y_2^2) &> u^2(x_1^2, x_2^2) \end{aligned}$$

We say that an allocation is pareto optimal (or pareto efficient) if it is not pareto dominated by any other feasible bundle.

Another way to say this is that a bundle is pareto optimal if the only way to make one of the consumers better off is to make the other worse off in utility terms.

Before we move on, it is worth noting some of the limitations with the concept of pareto optimality. As we have already discussed, not all bundles can be compared using the pareto criterion, even we might think that they should be comparable. Consider the following examples.

Allocation E:	$(2, 4)$ $(2, 2)$
Allocation F:	$(1.99999, 4)$ $(100000, 2)$

Can we use say that one of these bundles is better than the other in the pareto sense? The answer is no (again, assuming monotonicity) - even though F is much better for consumer 2, the fact that it is worse for consumer 1 (even though it is only a little bit worse) means that they cannot be compared. What about the following two bundles?

Allocation G:	$(100, 100)$ $(0, 0)$
Allocation H:	$(100, 99)$ $(0, 1)$

Again, the answer is no.- although bundle H looks a lot 'fairer' than bundle G, the pareto criterion does not allow us to judge between the two. In fact, in general, any allocation in which one consumer gets everything will be pareto optimal. This is something that we will come back to later.

What do pareto efficient allocations look like in the Edgeworth box? In order to think about this, it is worth first noting that the problem of finding a pareto optima can be thought of as a constrained maximization problem of maximizing one consumer''s utility (say consumer 1) subject to the other consumer achieving utility $\bar{u}$.[2]

The Pareto Problem

Choose: *a feasible allocation* (x_1^1, x_2^1), (x_1^2, x_2^2)

In order to maximize: $u^1(x_1^1, x_2^1)$

Subject to: $u^2(x_1^2, x_2^2) \geq \bar{u}$

Now, we can think about finding pareto optima by looking at figure 20. Remember, the question is asking whether there is any way of making one consumer better off (say consumer 2) while not making the other consumer (consumer 1) worse off. Is point A a pareto optima? The answer is

[2] Assuming monotonicity

clearly no - we can move consumer 2 onto a better indifference curve, while keeping consumer 1 on the same indifference curve - for example by moving to point C. Therefore point C pareto dominates point A, meaning A is not pareto optimal. The same is true for point B. What about point C? Here we *are* looking at a pareto optima. It should be obvious that the only way to move one of the consumers onto a higher indifference curve is by moving the other consumer onto a lower one

What is special about point C? Thinking back to the lectures on consumer choice the answer should be obvious: at point C the indifference curve of consumer 1 is tangential to that of consumer 2. Just as any interior solution to the consumer problem must be a point of tangency between the budget line and the consumer, so any interior pareto optima must be a point of tangency between the indifference curves of the two consumers.

This also makes sense intuitively. If the two indifference curves are not tangent, then the two consumers have difference marginal rates of substitution at that point - so (for example) the rate at which consumer 1 trades good 1 for good 2 while remaining indifferent is higher than the rate at which consumer 2 does - for example point A in figure 20 . If this were the case, it is clear that we could give consumer 1 less of good 2 and more of good 1, while doing the reverse to consumer 2, making them both better off - this is what happens when we move from point A to point C.

So we know that any interior point has to be a point of tangency to be a pareto optima. Is it also true that any point of tangency is also a pareto optima? If you understood the materiel on the consumer's problem, you should understand that the answer is no - if preferences are not convex, then there may be tangency points that are not optima. More generally, we may find pareto optima in corner solutions as well (Think about perfect substitutes). However, if preferences are strictly convex, then any point of tangency *will* be an pareto optima.

Obviously there is not just one pareto optima in an Edgeworth box. In general, for each indifference curve of consumer 2, there will be at least one pareto optima (if indifference curves are well behaved, this will occur at the point of tangency with the indifference curve of consumer 1. The set of all pareto optima is called the **contract curve,** and is shown in figure 21. Note that you can have pareto optima that are extremely unfair - in fact, if preferences are monotonic, then the point where consumer 1 gets *nothing* and consumer 2 gets *everything* is a pareto optima - as is the opposite situation.

In order to fix ideas, lets calculate the set of contract curve for consumers with Cobb Douglas preferences. In particular, lets assume that

$$\begin{aligned} u^1(x_1^1, x_2^1) &= \left(x_1^1\right)^{\frac{1}{3}} \left(x_2^1\right)^{\frac{2}{3}} \\ u^2(x_1^2, x_2^2) &= \left(x_1^2\right)^{\frac{1}{2}} \left(x_2^2\right)^{\frac{1}{2}} \end{aligned}$$

Because Cobb Douglas preferences are convex and monotonic, we know that the set of pareto optima is the same as the set of tangency points between consumer 1's and consumer 2's indifference curves. thus, to find such points, we need to set

$$\begin{aligned} MRS_{1,2}^1 &= MRS_{1,2}^2 \\ \frac{\frac{\partial u^1(x_1^1,x_2^1)}{\partial x_1^1}}{\frac{\partial u^1(x_1^1,x_2^1)}{\partial x_2^1}} &= \frac{\frac{\partial u^2(x_1^2,x_2^2)}{\partial x_1^2}}{\frac{\partial u^2(x_1^2,x_2^2)}{\partial x_2^2}} \end{aligned}$$

The marginal utilities are given by

$$\begin{aligned} \frac{\partial u^1(x_1^1, x_2^1)}{\partial x_1^1} &= \frac{1}{3}\left(x_1^1\right)^{-\frac{2}{3}} \left(x_2^1\right)^{\frac{2}{3}} \\ \frac{\partial u^1(x_1^1, x_2^1)}{\partial x_2^1} &= \frac{2}{3}\left(x_1^1\right)^{\frac{2}{3}} \left(x_2^1\right)^{\frac{1}{3}} \\ \frac{\partial u^2(x_1^2, x_2^2)}{\partial x_1^2} &= \frac{1}{2}\left(x_1^2\right)^{-\frac{1}{2}} \left(x_2^2\right)^{\frac{1}{2}} \\ \frac{\partial u^2(x_1^2, x_2^2)}{\partial x_2^2} &= \frac{1}{2}\left(x_1^2\right)^{\frac{1}{2}} \left(x_2^2\right)^{-\frac{1}{2}} \end{aligned}$$

Thus we get

$$\frac{1}{2}\frac{x_2^1}{x_1^1} = \frac{x_2^2}{x_1^2}$$

This tells us what has to be true for a point of tangency. However, if this is a pareto optima, it also has to be feasible, so

$$\begin{aligned} \omega_1^1 + \omega_1^2 &= x_1^1 + x_1^2 \\ \omega_2^1 + \omega_2^2 &= x_2^1 + x_2^2 \end{aligned}$$

Substituting in gives

$$\frac{1}{2}\frac{x_2^1}{x_1^1} = \frac{(\omega_2^1 + \omega_2^2 - x_2^1)}{(\omega_1^1 + \omega_1^2 - x_1^1)}$$

and rearranging gives

$$x_2^1 = \frac{2(\omega_2^1 + \omega_2^2)x_1^1}{\omega_1^1 + \omega_1^2 - x_1^1}$$

This is the equation of the contract curve.

6 The First Fundamental Theorem of Welfare Economics.

We will now move on to one of the most beautiful, powerful and misused theories in economics, the first fundamental theorem of welfare economics. What this theorem does is relate the notion of competitive equilibria to that of pareto optima. Those of you that have been paying attention should have started to get the feeling that these two might have something to do with each other. One hint comes from thinking about MRS: in the case of the competitive equilibria, we noted that the MRS of both consumers must equal the slope of the price line, so they must also be equal to each other. In the case of the pareto optima, we noted that it had to be the case that the slope of each consumers indifference curves had to be equal to each other, and so the two consumers must have the same MRS. Hmmm. These sound like very similar conditions.

In fact, the relationship goes deeper than that. It turns out that *any* competitive equilibrium, whether or not it is a point of tangency, must be a pareto equilibria. In other words, for any competitive equilibrium, it is impossible to make one consumer better off without making the other worse off. This is a truly astonishing result, and not only that, it is easy to prove. Moreover, the only condition that we need for it to be true is (basically) monotonicity - we do not even need convexity!

Theorem 1 *If preferences are monotonic, any competitive equilibrium is a pareto optimum.*

Proof. *Let (x_1^1, x_2^1), (x_1^2, x_2^2), p be a competitive equilibrium. If it isn't a pareto optimum, then there must be another bundle (y_1^1, y_2^1), (y_1^2, y_2^2) which pareto dominates (x_1^1, x_2^1), (x_1^2, x_2^2), meaning that*

$$\begin{aligned} u^1(y_1^1, y_2^1) &\geq u^1(x_1^1, x_2^1) \\ \text{and } u^2(y_1^2, y_2^2) &\geq u^2(x_1^2, x_2^2) \end{aligned}$$

and

$$\begin{aligned} u^1(y_1^1, y_2^1) &> u^1(x_1^1, x_2^1) \\ \text{or } u^2(y_1^2, y_2^2) &> u^2(x_1^2, x_2^2) \end{aligned}$$

Lets assume that $u^1(y_1^1, y_2^1) \geq u^1(x_1^1, x_2^1)$ and $u^2(y_1^2, y_2^2) > u^2(x_1^2, x_2^2)$ for simplicity.

Now, as (x_1^2, x_2^2) is part of a competitive equilibrium, it must have been the optimal bundle for consumer 1 to chose, given the price p. Thus, if $u^2(y_1^2, y_2^2) > u^2(x_1^2, x_2^2)$, then it must be the case that the price of (y_1^2, y_2^2) is higher than that of (x_1^2, x_2^2) (otherwise consumer 2 would have chosen (y_1^2, y_2^2)) Furthermore, if $u^1(y_1^1, y_2^1) \geq u^1(x_1^1, x_2^1)$, then it must be the case that (y_1^1, y_2^1) is at least as expensive as (x_1^2, x_2^2) (this is where we use monotonicity, yes?). Thus we get

$$\begin{aligned} p_1 y_1^1 + y_2^1 &\geq p_1 x_1^1 + x_2^1 = p_1 \omega_1^1 + \omega_2^1 \\ p_1 y_1^2 + y_2^2 &> p_1 x_1^2 + x_2^2 = p_1 \omega_1^2 + \omega_2^2 \end{aligned}$$

Adding the two sides of these equations gives

$$\begin{aligned} p_1 y_1^2 + y_2^2 + p_1 y_1^1 + y_2^1 &> p_1 \omega_1^1 + \omega_2^1 + p_1 \omega_1^2 + \omega_2^2 \\ p_1(y_1^2 + y_1^1) + (y_2^2 + y_2^1) &> p_1(\omega_1^2 + \omega_1^1) + (\omega_2^2 + \omega_2^1) \end{aligned}$$

Thus, either $(y_1^2 + y_1^1) > (\omega_1^2 + \omega_1^1)$ or $(y_2^2 + y_2^1) > (\omega_2^2 + \omega_2^1)$. In either case, it must be the case that (y_1^1, y_2^1), (y_1^2, y_2^2) is not a feasible bundle. ■

So what this truly astonishing result is telling us is that, in the simple environment we have been considering, a competitive equilibrium has to lie somewhere on the contract curve, as shown by figure 22. This really is quite impressive - a very simple mechanism is able to allocate goods in such a way that no-one can be made better off without making someone else worse off. Not only that, this result generates to any (lets say finite) number of goods, and any number of consumers!

To see the power of this theorem, it is worth thinking about some other allocation mechanisms that are *not* necessarily pareto efficient. You should check that you understand why (in general) these mechanisms do not lead to pareto efficient outcomes.

1. Don't let people trade - people are only allowed to consume their endowments (a situation sometimes called autarky)
2. Complete equality: Give both consumers the same amount of each good
3. Let someone else choose the allocation for each consumer
4. Fixing a price in the market, letting one person choose the bundle they want, then rationing the other consumer

7 The Second Fundamental Theorem of Welfare Economics

The first fundamental theorem tells us that any competitive equilibrium is a pareto optima. This is all well and good. However, as we have seen, some pareto optima are pretty nasty (e.g. the ones in which one consumer gets zero). So we may be interested in a second question: is every pareto optima a competitive equilibrium? Specifically, for any pareto optima, is it the case that we can find an endowment such that, if the consumers start with that endowment they will get to that pareto optima as a competitive equilibrium? This is a very important question, because it means that if we can change people's endowments, we can choose which pareto optima we end up with, still only using the market system.

Another way to think about this question is to look at figure 23. It is clear that the indicated pareto optimum cannot be supported as a competitive equilibrium from endowment A. Any price line that goes through endowment and the pareto optimum is not going to be tangent to the indifference curves at that point, and so is not going to lead to a competitive equilibrium. The question is therefore can we change the endowment in such a way such that the indicated pareto optimum is a competitive equilibrium.

The answer to this question is a guarded 'yes'. In the above example we certainly can, as shown in figure 24. Say we changed the initial endowment from A to B. The clearly the pareto optimum is now a competitive equilibrium for the prices that we have drawn, as the indifference curves for both consumers are tangent to the price line at that point.

What is special about endowment B? The key thing is that the straight line between B and the Pareto optimum has the same slope as the MRS of both consumers at that point (if it is a pareto optimum then the MRS of both consumers has to be the same, right?). Thus, any price line that goes through the pareto optimum will be tangent to each consumers' indifference curve at that point, and therefore give rise to an equilibrium at that point. In fact, any endowment that has this property will support that pareto optimum as a competitive equilibrium, as you can see from figure 25.

So can we always pull this trick? The answer is no, as we can see from figure 26. This shows a pareto optimum (point A) that cannot be supported as a competitive equilibrium. As we have discussed above, for this to be a competitive equilibrium, the line between the endowment and A would have to be equal to the MRS of each consumer at A. The price line would then connect the endowment and the pareto optimum. However, the graph shows that, for such a budget set, consumer 1 would rather consume at point B, which puts them on a higher indifference curve. A can therefore not be supported as a competitive equilibrium.

What has gone wrong? Well in the above example I have made use of non-convex preferences, and this is in fact the key condition.

Theorem 2 *If preferences are convex, monotonic (and continuous, maths fans), then any pareto optimum can be supported as a competitive equilibrium*

The proof of this theorem lies outside the scope of this course.

8 Discussion

So between them, the two fundamental theorems of welfare economics seem to offer a pretty powerful argument in favor of using markets to allocate goods. And the results are pretty amazing. It seems pretty clear that pareto optimality is a good idea - if we are not pareto optimal we can make someone better off 'for free', in the sense that it doesn't make anyone else worse off. And markets (it seems) guarantee that we will be at a pareto optimum. It is worth taking a minute to realize quite how impressive this is. We have shown this to be true for a market consisting of two consumers, but mathematically the result also holds for 400 million consumers and a billion goods. Say that you were a planner that had to allocate goods between all these people. Imagine how much information you would need to ensure your allocation was Pareto optimal - you would have to calculate each person's MRS between each good, then use all of this information to find points where the MRSs are equal for all consumers. A seemingly impossible task, and yet one that markets do for free! Moreover the second welfare theorem suggests we can select amongst different pareto optima. Wow!

However, there are, I believe, serious problems with taking either of these theorems at face value - i.e. as always indicating that markets are the best option. This is not to say that the theorems are meaningless - far from it - but there are issues that we need to think about, which I will go through now.

8.1 Equality

Look at the two allocations in figure 27. Do you prefer allocation A or allocation B? Allocation A is pareto efficient, but seems to be very unfair, while allocation B is not pareto efficient, but is much fairer. Personally, if given the choice, I would say allocation B is better than allocation A. This reveals an important feature of Pareto optimality: just because one bundle is Pareto optimal and another is not, this does not mean that we prefer to the former to the latter. Of course, there will be some other point that I like even more than B (as B is pareto dominated), but it might be the case that such a point might not be available for some other reason.

For example, consider the following scenario. I am a government official deciding how to allocate healthcare (vs all other goods). At the moment, I know that endowments are very uneven - some people are very wealthy, and some are not. Thus, if I just allow the market to operate, it will be the case that some people end up with great healthcare, while some end up with none. On the other hand, I can choose who gets how much healthcare. In this case, I don't know exactly what everyone's MRSs are between healthcare and other goods, but I can have a guess, which will be wrong, but close. In this case, I might say that it is 'better' for me to select the allocation than to allow the market to do it - even though my selection will not be pareto optimal. Of course this is a massive simplification of the healthcare debate (not least because you may disagree with me about what 'better; is) but it does make a point that is worth thinking about.

But wait, I hear you cry - surely the second welfare theorem tells us that we can have our cake and eat it too! Surely we can just change people's allocations, and get a fair, efficient outcome using markets. In theory, this may be true, but note that shifting people's allocations means changing what they start off with *in a way that doesn't affect prices*. This is called lump sum taxation, and rules out things such as income taxes, sales taxes (in most cases), property taxes and so on. As we will show later, taxes that change the price of any economic activity mean that market outcomes will no longer be efficient. If you look around you, you will see very few examples of lump sum taxation *in practice.* One recent attempt to introduce such a tax was the poll tax, introduced by Maggie Thatcher in the UK in the 1980s. Her attempt to do so led to rioting and, eventually, the fall of the government. Thus, while the second welfare theorem offers a way out in theory, it may not work in practice.

8.2 Revealed Preference and Preference

A second potential issue with the concept of pareto efficiency (and therefore the welfare theorems) goes back to the idea of preferences. Remember, while we may like to think of preference intuitively as what people *like*, they are in fact (for the purposes of these models) what they *choose.* And in many cases people may end up making terrible decisions - especially when things are complicated. Do we think that the people that invested their retirement savings with Bernie Madoff made good decisions? What about heroin addicts? Or people that go and see James Blunt in concert? Our definition of preferences treat all decisions as equally good. Thus, a market outcome may be pareto efficient in the sense of what people *choose* it may be that these decisions are in fact really bad.

To me, this is a less convincing argument against the market system. First of all, while there may be lots of cases where people make bad decisions, I would guess there are lots more where they make perfectly good ones (would you choose coffee over tea? eggs over bacon?). Moreover, if you did think that people were making bad decisions due to a lack of information (e.g. people start smoking because they don't know it is bad for you), then the policy prescription is clear: *tell them!* There is nothing in the 1st and 2nd welfare theorems that people would not be better off if they are given more information.

If you give them more information, and they still make a choice that you think they shouldn't, it isn't completely clear on what grounds you would force them to choose something else. For example, if I am convinced that being homosexual means that you will go to hell, I tell you this, and then you still choose to be homosexual, should I ban homosexuality? I'd argue not (though we may think of counterexamples, such as people who are addicted to heroin and so forth).

8.3 Externalities

One assumption that we have maintained throughout the course so far is that only thing that consumer 1 cares about is the amount of each good that consumer 1 consumers, and the same for consumer 2. In other words the utility function u^1 is defined on x_1^1 and x_2^1, while u^2 is defined on x_1^2 and x_2^2. u^1 does not depend on x_1^2. This assumption is (of course) wrong in all sorts of situations. One problem might be altruism - I want you to be happy, so I care about what you consume. Another problem (which has received more attention historically in the economics literature) is that of **externalities.** This is the idea that my consumption of a good might affect you directly - for example pollution from my car reduces your quality of life.

It turns out that, in the presence of externalities, market equilibria may not be efficient. We will go into this in more detail later in the course, but for now we will go through a slightly contrived example to make the point. Consider the case where we have three agents Algenon, Beth and Cindy rather than the usual two. As usual, there are two goods in the economy, whiskey and cigarettes. Algenon and Beth are both drinkers and smokers. However, Cindy hates cigarettes. Moreover, she lives next door to Algenon, and hates it when he smokes. Each starts off with an endowment of whiskey and cigarettes, and then we allow them to trade. What will happen? Well, clearly Cindy will sell all her cigarettes, and, in equilibrium, the price of whiskey will equalize the MRS of whiskey for cigarettes between Algenon and Beth. Is this going to be pareto optimal? In general, no, because the equilibrium doesn't take into account the fact that Cindy would much rather that Beth ends up with more cigarettes than Algenon. The market outcome may be Pareto dominated by one which transferred some whiskey from Cindy to Algenon and some cigarettes from Algenon to Beth. We might be able to find transfers such that Cindy is better off because the loss of whiskey is offset by the fact that Algenon is smoking less, Algenon is better off because the gain in whiskey offsets the loss of cigarettes, and Beth is better off because she has more cigarettes.

In effect, the problem here is that, in the market equilibrium, Algenon is consuming 'too many' cigarettes, because the market price does not take into account the negative impact of his smoking on Cindy. Thus there is room for pareto improvement. However the problem here may not be that there are too many markets, but that there are not enough. What would help is if there was some way for Cindy to trade whiskey to Algenon to prevent him from smoking. Thus, we could help alleviate the problem by setting up a market for 'clean air' around Cindy's house: we could say that Algenon had the right to smoke, but he could trade that right to Cindy in exchange for whiskey. Alternatively we could give Cindy the right to clean air, but allow her to sell some of this right to Algenon in exchange for whiskey. This is called assigning **property rights**, and allowing people to trade these rights - this is the idea that underlies schemes such as tradable carbon credits and so on. The Coase theorem, (which we will come back to) outlines the conditions under which this type of solution would lead to a pareto efficient outcome.

One quick caveat - do we think that people should be allowed to trade all their rights? Even if we have extreme differences in wealth? If so, you may want to track down the now thankfully banned programme 'bum fight'.

8.4 Price Taking

The final caveat that we shall look at regards a 'hidden' assumption of the above model - that both consumers are price takers. They take the price in the economy as fixed, and act accordingly. They have no power (or believe they have no power) to change the market price. In some cases this may seem like a good assumption (supermarkets? financial markets?), but in others perhaps not (ironically, the case where there are only two consumers is one in which it seems less feasible for each of them to take the price as fixed).

Lets look at an alternative assumption - that one consumer (consumer 1) decides that they have market power, and sets the price to benefit themselves. In other words, they choose a price, then allow the other consumer (consumer 2) to choose their optimal bundle given that price. Consumer 1 then takes everything else

How do we model this in the Edgeworth box? In order for consumer 1 to choose their best price, they need to know what consumer 2 will choose at each price. Luckily, we have already calculated this - it is consumer 2's offer curve (remember, this is just the curve that links together all the bundles that consumer 2 will choose at different prices). The problem of consumer 1 is therefore to choose their favorite point on consumer 2's offer curve. The solution to this problem is shown in figure 28 - consumer 1 will choose the point at which their indifference curve is tangent to consumer 2's offer curve and set prices accordingly. What will consumer do at these prices? We know that (by definition) they will choose the bundle at which the price line intersects their offer curve. As this is their optimal bundle, it must be that their indifference curves are tangent to the price line at this point (figure 29). Is this a pareto optimum? No as figure 30 shows. At this point, the indifference curves of the two consumers cross: We could make them both better off by moving in a North - Westerly direction.

We will see much more about the potential problems of the monopolist when we move on to talk about firms.