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Intermediate Microeconomics W3211

Lecture 1: The Consumer Problem

Columbia University, Spring 2016

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Introduction

Constrained optimization: your tool for better shopping

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A Reminder

- The course will cover two key ideas
  1. How do people respond to incentives?
     - i.e. what choices do they make?
  2. How does equilibrium determines prices, outcomes etc.

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Modelling Consumer Behavior

- First section of the course: How do people make choices?
  - Particular type of person: consumer
  - Particular type of choice: what to consume (i.e. what stuff to buy)
- In principle there are many ways we could try to model consumers
  - Creatures of habit: always buy the same thing every week
  - Minions: Always buy what advertisers tell them to buy
  - Whimsical: choose what to buy at random
- We will make a particular assumption: that consumers are **smart**
  - Make the best purchases they can, given the prices and the amount of money they have
- Solving problems of this type is the study of **constrained optimization**

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Constrained Optimization

- One of the most powerful and useful areas of mathematics
- We will use it again and again
- If you can become comfortable with constrained optimization problems, you are half way to mastering the course
- The first thing to learn is how to set up a constrained optimization problem
- Luckily, they always look the same…..

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Constrained Optimization

- The three elements of a constrained optimization problem:

1. **CHOOSE <some alternative>**
2. **IN ORDER TO MAXIMIZE <some objective>**
3. **SUBJECT TO <some constraints>**

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Example: A Columbia student is trying to decide what courses they are going to take. They want to get the highest possible grade point average, but they also want to do the economics major

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Constrained Optimization

- The three elements of a constrained optimization problem:

1. **CHOOSE a set of courses**
2. **IN ORDER TO MAXIMIZE GPA**
3. **SUBJECT TO satisfying the econ major requirements**

Example: A Columbia student is trying to decide what courses they are going to take. They want to get the highest possible grade point average, but they also want to do the economics major

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Constrained Optimization

- The three elements of a constrained optimization problem:

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Example: The British government is signed up to reduce emissions by 25% through taxes and subsidies. However, they want to do so in a way that minimizes the damage to the economy

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Constrained Optimization

- The three elements of a constrained optimization problem:

1. **CHOOSE taxes and subsidies**
2. **IN ORDER TO MAXIMIZE economic output**
3. **SUBJECT TO reducing greenhouse gasses by 25%**

Example: The British government is signed up to reduce emissions by 25% through taxes and subsidies. However, they want to do so in a way that minimizes the damage to the economy

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Constrained Optimization

- The three elements of a constrained optimization problem:

1. **CHOOSE <some alternative>**
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Example: A customer in a restaurant has a budget of $100. They want to choose a collection of dishes that gives them the most happiness without going bankrupt

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Constrained Optimization

- The three elements of a constrained optimization problem:

1. **CHOOSE a collection of dishes**
2. **IN ORDER TO MAXIMIZE 'happiness'**
3. **SUBJECT TO not spending over $100**

Example: A customer in a restaurant has a budget of $100. They want to choose a collection of dishes that gives them the most happiness without going bankrupt

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Constrained Optimization and Consumer Behavior

- (Obviously) this last example is also an example of the consumer's problem
- We will spend the next few lectures setting up the consumer's optimization problem more thoroughly
  - Objects of choice
  - Constraints
  - Objective function

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Constrained Optimization and Consumer Behavior

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- We will spend the next few lectures setting up the consumer's optimization problem more thoroughly
  - Objects of choice (Varian Ch. 2, Feldman and Serrano Ch 3)
  - Constraints (Varian Ch. 2, Feldman and Serrano Ch 3)
  - Objective function (Varian Ch. 3 & 4, Feldman and Serrano Ch 2)

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Constrained Optimization and Consumer Behavior

- (Obviously) this last example is also an example of the consumer's problem
- We will spend the next few lectures setting up the consumer's optimization problem more thoroughly
  - Objects of choice (Varian Ch. 2, Feldman and Serrano Ch 3)
  - Constraints (Varian Ch. 2, Feldman and Serrano Ch 3)
  - Objective function (Varian Ch. 3 & 4, Feldman and Serrano Ch 2)
- We will then move on to solving the consumer's problem, and making predictions about behavior

Setting up the Consumer's Problem

1: What are we choosing?

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Consumption Bundles

- Imagine you are in a (very boring) supermarket
  - Sells only apples, bananas and cantaloupes
- What options are you choosing between?
- Number of apples, number of bananas and number of cantaloupes
- We can represent your choices as a vector

$$x = \begin{pmatrix} x_a \\ x_b \\ x_c \end{pmatrix}$$

- x_a number of apples, x_b number of bananas, x_c number of cantaloupes
- For example:

$$x = \begin{pmatrix} 3 \\ 7 \\ 4 \end{pmatrix}$$

- Is 3 apples, 7 bananas and 4 cantaloupes
- More generally, if there are n goods, you are choosing between vectors of n numbers
  - i.e choosing $x \in \mathbb{R}^N$
  - Can choose fraction of apples

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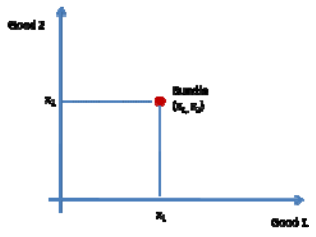
Consumption Bundles

- In many cases we will concentrate on an *extremely* boring supermarket with only two goods
- Makes it easy to represent graphically



Setting up the Consumer's Problem

2: What are our constraints?

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The Budget Constraint

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- Let's go back to our very boring supermarket
- Say you have $100 in your pocket
- Can you choose **any** bundles of apples, bananas and cantaloupes?
- No, you can only choose bundles that you can afford
- Also cannot choose to consume negative amounts of apple etc.

Budget Constraints

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- We denote the commodity prices by $p_1, p_2, \dots, p_n$
- Denote by m the disposable income/wealth of the consumer
- Question:

When is a consumption bundle $(x_1, \dots, x_n)$ **affordable** at given prices $p_1, \dots, p_n$?

- Answer:

When

$$p_1x_1 + \dots + p_nx_n \leq m$$

Budget Set and Budget Constraint

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- We will call the set of all bundles the consumer can choose the **budget set**

$$B(p_1, \dots, p_n, m) = \{ (x_1, \dots, x_n) \mid x_1 \geq 0, \dots, x_n \geq 0 \text{ and } p_1x_1 + \dots + p_nx_n \leq m \}$$

- The **budget constraint** is the set of bundles that are only just affordable

$$\{ (x_1,\dots,x_n) \mid x_1 \geq 0, \dots, x_n \geq 0 \text{ and } \mathbf{p_1x_1 + \dots + p_nx_n = m} \}.$$

It is the upper boundary of the budget set

Graphically: Two Commodities

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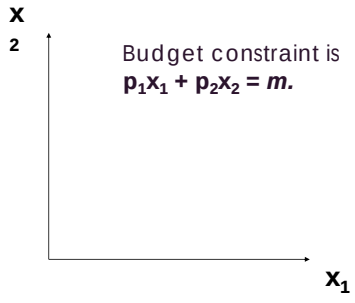

Budget constraint is
$\mathbf{p_1x_1 + p_2x_2 = m.}$

Graphically: Two Commodities

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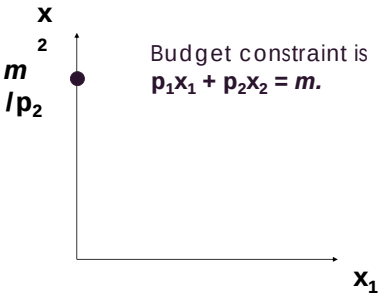

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Graphically: Two Commodities

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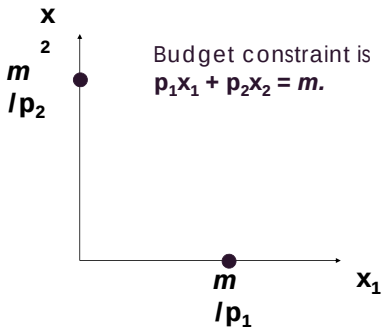

Graphically: Two Commodities

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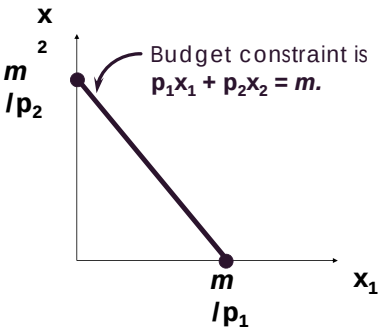

Graphically: Two Commodities

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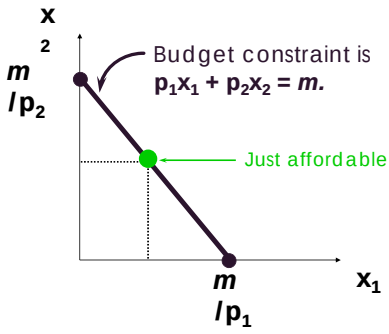

Graphically: Two Commodities

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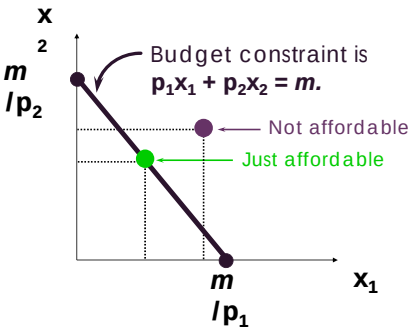

Graphically: Two Commodities

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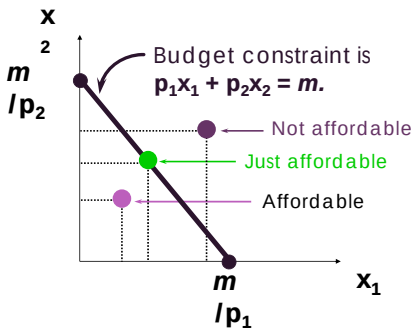

Graphically: Two Commodities

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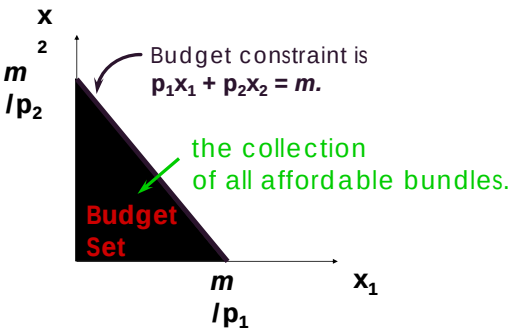

Graphically: Two Commodities

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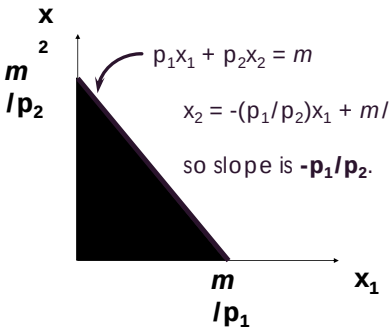

Budget Constraints

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- For n = 2 and x_1 on the horizontal axis, the constraint's slope is $-p_1/p_2$. What does it mean?

$$x_2 = -\frac{p_1}{p_2}x_1 + \frac{m}{p_2}$$

- If we want to increasing x_1 by 1 unit, we must reduce x_2 by p_1/p_2.

Budget Constraints

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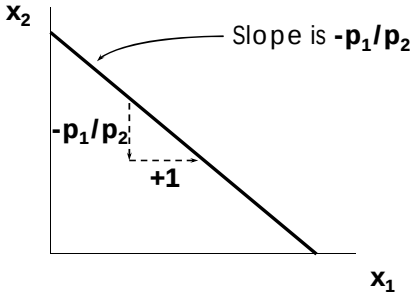

Budget Constraints

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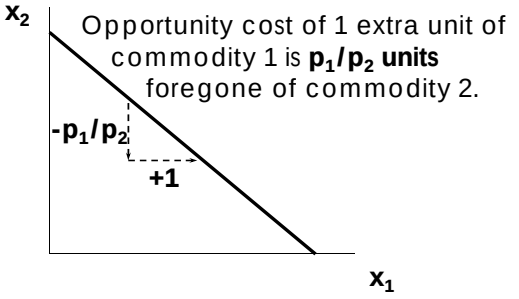

Budget Constraints

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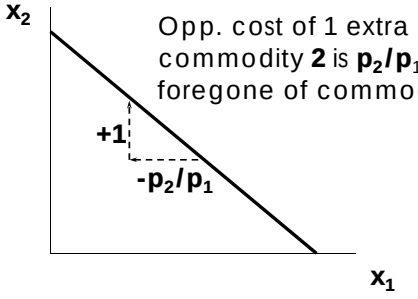

Budget Constraints

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- If n = 3 what do the budget constraint and the budget set look like?

Budget Constraint for Three Commodities

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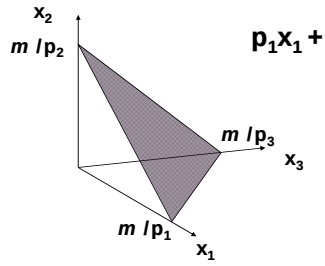

$$p_1x_1 + p_2x_2 + p_3x_3 = m$$

Budget Set for Three Commodities

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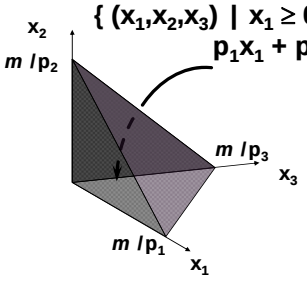

$$\{(x_1,x_2,x_3) \mid x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \text{ and } p_1x_1 + p_2x_2 + p_3x_3 \leq m\}$$

Budget Sets & Constraints; Income and Price Changes

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- The budget constraint depends on prices and income
- These are the **parameters** of the consumer's problem
- What happens to the budget constraint if they change?
- This is the first step in asking what happens to **consumer choices** when prices and incomes change

What if **income *m* increases**?

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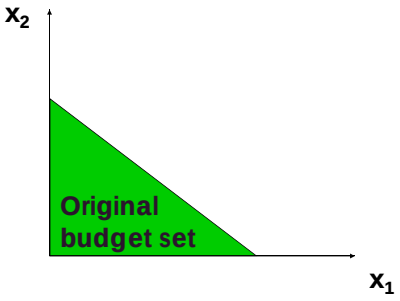

Higher income gives more choice

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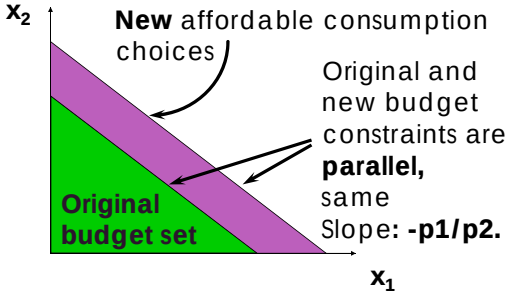

What if **income *m* decreases**?

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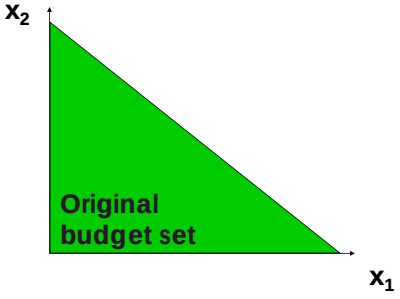
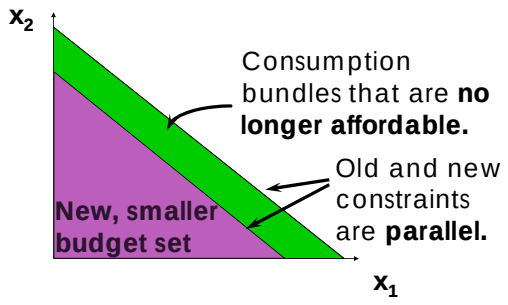


What if **income *m* decreases**? 43



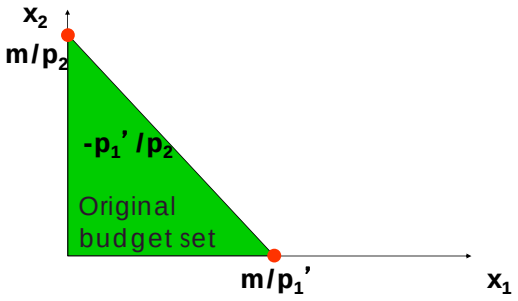
Budget Constraints - Income Changes 44

- **Increase in income** *m:*
  - Shift the constraint **outward** in a **parallel** manner
  - Enlarge budget set
  - Everything available before is still available, and more
  - Consumer is 'better off'
- **Decrease in income** *m:*
  - Shift the constraint **inward** in a **parallel** manner
  - Reduce budget set
  - Consumer is 'worse off' (in general)

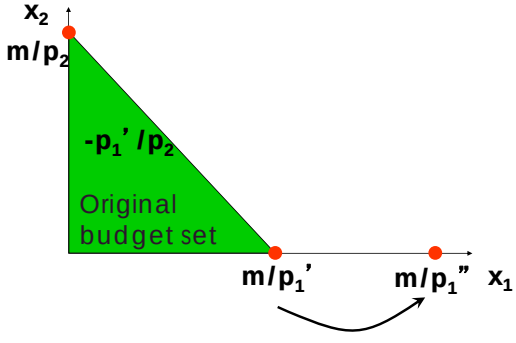
Budget Constraints - Price Changes 45

- What happens if just **one price decreases**?
- Suppose p_1 decreases.

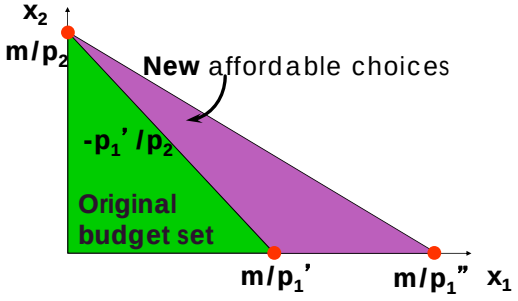
p_1 decreases from p_1'to p_1" ? 46



p_1 decreases from p_1'to p_1" ? 47



p_1 decreases from p_1'to p_1" ? 48



p_1 decreases from p_1'to p_1" ?

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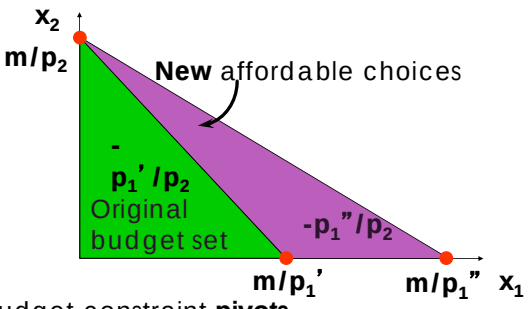

Budget constraint **pivots**

Slope flattens: from $-p_1'/p_2$ to $-p_1''/p_2$

Budget Constraints - Price Changes

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- **Reducing** the price of **one** commodity:
  - **pivots** the constraint outward
  - No old choice is lost and new choices are added,
  - So reducing one price cannot make the consumer 'worse off'

Budget Constraints - Price Changes

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- **Similarly, INCREASING** the price of **one** commodity:
  - **pivots** the constraint inward
  - Some options are lost and
  - no choices are added,
  - So increasing one price cannot make the consumer better off
- Think about (for homework):
  - Suppose both prices change by the same proportion (e.g., 10%)
  - Suppose price increase by 10% but also income increases by 10%

Price and Taxes

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- We can also use the budget constraint to start to think about **policy experiments**
- For example, what are the effects of taxes?
- Different kinds:
  - **Quantity** tax: amount *t* for each unit
    - E.g., gasoline: we pay about 15*c* per gallon
  - **Value** tax: tax on the price of the good
    - E.g., sale taxes
  - **Income** tax: taxes income m
- Taxes can apply to both goods or only one
- There also **subsidies**: opposite of tax, same idea opposite sign

Effects of Taxes on Budget Constraints

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- Quantity tax *t* on good 1 (and not 2):
  - Budget constraint goes from
    $p_1x_1 + p_2x_2 = m$
    to
    $(\mathbf{p_1 + t})\ x_1 + p_2x_2 = m$
- Value tax *t* on good 1 (and not 2):
  - Budget constraint goes from
    $p_1x_1 + p_2x_2 = m$
    to
    $\mathbf{(1+t)}p_1x_1 + p_2x_2 = m$
- Income tax: t% of income has to paid in taxes
  - Budget constraint goes from
    $p_1x_1 + p_2x_2 = m$
    to
    $p_1x_1 + p_2x_2 = m\ \mathbf{(1-t)}$

Effects of Taxes on Budget Constraints

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- What if both goods are taxed?
- Value tax on both goods
- Budget constraint goes from

$$p_1x_1 + p_2x_2 = m$$

to

$$\mathbf{(1+t)}p_1x_1 + \mathbf{(1+t)}p_2x_2 = m$$

i.e.

$$p_1x_1 + p_2x_2 = m/\mathbf{(1+t)}$$

- **Slope doesn't change!**

Uniform Value Tax

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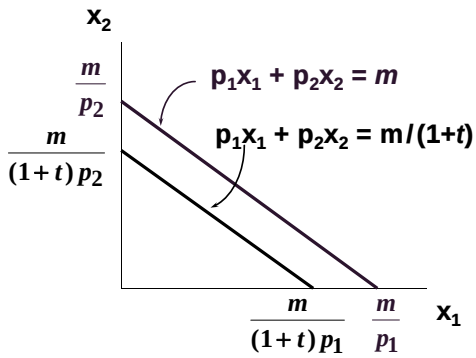

Uniform Value Tax

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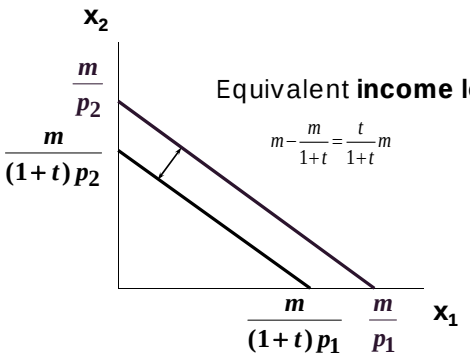

Uniform Value Tax

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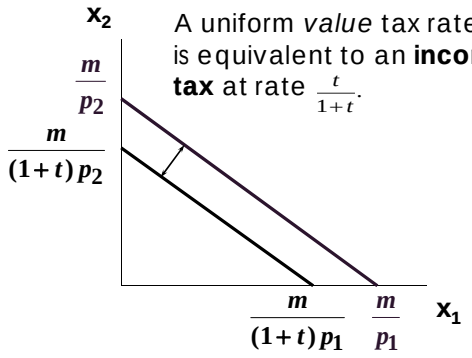

More complicated forms: The Food Stamp Program

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- Food stamps are **coupons** that can be legally **exchanged only for food**
- Suppose m = \$100, p_F = \$1 and the price of "other goods" is p_G = \$1.
- The budget constraint is then

$$F + G = 100$$

- Suppose the government distributes **40** Food stamps
- What changes?

The Food Stamp Program

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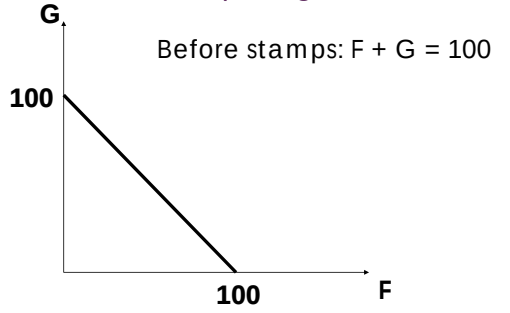

The Food Stamp Program

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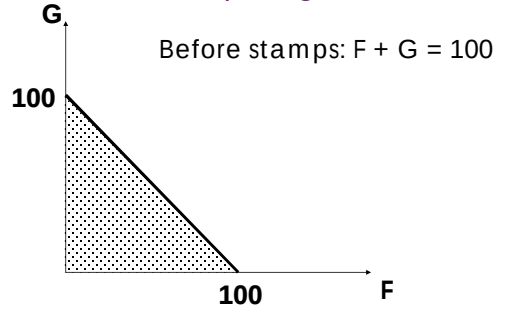

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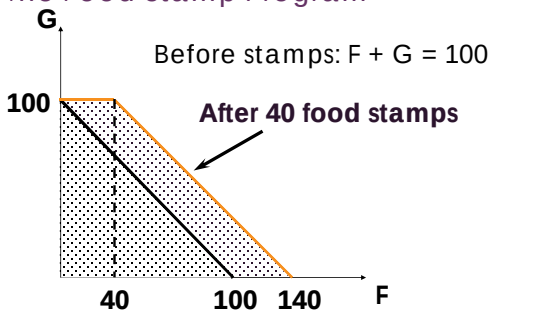
The Food Stamp Program



Setting up the Consumer's Problem

3: What are we maximizing?

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The Objectives of the Consumer

- So far we have dealt with the easy part of the problem
  - Budget constraints fairly easy to write down and fairly uncontroversial
- Now we have to think of a more difficult question
  - What is it that people want?
  - What is it that they try to maximize?
- Obviously a detailed answer to this question is incredibly complex
  - People are complex beasts!

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The Objectives of the Consumer

- How are we going to make progress?
- Make as few assumptions as possible
- Avoid being proscriptive
- e.g. we will not say that people should prefer cricket to baseball
  - Even though we know this to be true
- See how much this allows us to say

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Preferences

- We are going to assume that people maximize **preferences!**
- That was helpful, wasn't it?
- What are preferences?
- Let me ask you a question: Which bundle (of apples, bananas, cantaloupes) would you prefer to have

$$x = \begin{pmatrix} 3 \\ 7 \\ 4 \end{pmatrix} \quad \text{or} \quad y = \begin{pmatrix} 7 \\ 4 \\ 3 \end{pmatrix}$$

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Preferences

- We will allow **exactly one** of three possible answers
  - I **strictly prefer** x to y
    - Denote this $x \succ y$
  - I **strictly prefer** y to x
    - Denote this $y \succ x$
  - I am **indifferent between** x and y
    - Denote this as $x \sim y$
- It will also be useful to use $x \succsim y$ to denote that either $x \succ y$ or $x \sim y$
  - We say x is **at least preferred** to y, or x is **weakly preferred** to y
- Preferences are just a list of answers to **all** such questions between any consumption bundles

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Preferences

- Are we going to allow any such list of answers to these questions?
- No! We are going to demand some basic consistency requirements
  - Reflexivity
  - Completeness
  - Transitivity

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Assumptions about Preference Relations

- **Reflexivity:** Any bundle x is always at least as preferred as itself;
- i.e $x \succsim x$

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Assumptions about Preference Relations

- **Transitivity:** If
  - x is at least as preferred as y, and
  - y is at least as preferred as z,

  Then
  - x is at least as preferred as z
- i.e. $x \succsim y$ and $y \succsim z$ implies x $\succsim z$

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Assumptions about Preference Relations

- **Completeness**: For any x and y it is always possible to make the statement that either

  x $\succsim$ y

  or

  y $\succsim$ x

  (or both)

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Well Behaved Preferences

- We call preferences well behaved if they satisfy
  - Reflexivity
  - Transitivity
  - Completeness
- [For the math fetishists, such preferences form a **complete preorder**]
- Assuming that people have well behaved preferences is going to be a very useful assumption
- But is it a **good** assumption?

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Well Behaved Preferences

- Two approaches to answering this question
  - Intuitively, do we think they are reasonable assumptions?
  - Do people actually behave as if they have?
- We will return to the second approach when we think about behavioral economics
- What about the first approach?
  - Do you intuitively think that people have well behaved preferences?

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Are Your Preferences Well Behaved?

- Notice that well behaved preferences do not allow the following answers to the question do you prefer x to y
  - I don't know what x is
  - I can't decide
  - Sometimes I prefer x and sometimes I prefer y
- In many cases this may be fine, but how about
  - Very emotional choices: Would you prefer your first or second born child to be killed?
  - When objects are very complicated: If you are feeling sick, would you rather take Bismuth subsalicylate or 8-methyl-N-vanillyl-6-nonenamide?
  - When you are addicted to a substance: Do you always say you prefer smoking to not smoking?

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What about Transitivity?

- One argument in favor of transitivity is the **money pump**
- Say that your preferences are **not** transitive
  - You strictly prefer steak to cheeseburger
  - Strictly prefer cheeseburger to hamburger
  - Strictly prefer hamburger to steak
- Then I can make an infinite amount of money from you
  - Give you a hamburger
  - Offer to trade you a cheeseburger for a hamburger and 1c
  - Offer to trade you a steak for a hamburger and 1c
  - Offer to trade you a hamburger for a steak and 1c
- You have the hamburger again and I have 3c
- Repeat ad infinitum

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What about Transitivity?

- But then think about the following example
  - Do you prefer coffee with no sugar to coffee with 1 grain of sugar?
  - Do you prefer coffee with 1 grain of sugar to coffee with 2 grains
  - ….
- Assuming that the answer to all of these questions is 'I am indifferent', then you should be indifferent between coffee with no sugar and coffee with 1 spoonful of sugar
- For an audio example see the "shepherds scale"

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Well Behaved Preferences

- So are 'well behaved preferences' a good assumption?
- As usual with economics, the answer is "often yes, sometimes no"
- So we will proceed using this assumption
- But we should proceed with caution!

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A Note on Policy

- Later in the course we will often talk as if the aim of policy should be to maximize the preference of consumers
- Often this will mean giving them the biggest choice set we can
- Is this a good assumption?
- Again, often yes, but sometimes no
- Why? At least two reasons
  1. People's preferences might be dumb: e.g. heroin addicts
  2. People may not be able to maximize their preferences
- Again, we need to proceed with caution

Summary

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Summary

- Today we have written down a model of how consumers make choices
- Based on the concept of constrained optimization

1. **CHOOSE a consumption bundle**
2. **IN ORDER TO MAXIMIZE preferences**
3. **SUBJECT TO the budget constraint**

- You should be comfortable with setting up a constrained optimization problem, and what we mean by these three elements
- Next lecture, we will set about solving the optimization problem